Graduate AI
Lecture 22:
Social Choice I

Teachers:
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SOCIAL CHOICE THEORY

• A mathematical theory that deals with aggregation of individual preferences
• Origins in ancient Greece
• Formal foundations: 18\textsuperscript{th} Century (Condorcet and Borda)
• 19\textsuperscript{th} Century: Charles Dodgson
• 20\textsuperscript{th} Century: Nobel prizes to Arrow and Sen
**The Voting Model**

- Set of voters $N = \{1, \ldots, n\}$
- Set of alternatives $A$; denote $|A| = m$
- Each voter has a **ranking** over the alternatives
- Preference profile = collection of all voters’ rankings

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Vote over cuisines

Indian (In)
Japanese (J)
Chinese (C)
Italian (It)
Mexican (M)
Voting rules

• Voting rule = function from preference profiles to alternatives that specifies the winner of the election

• Plurality
  o Each voter awards one point to top alternative
  o Alternative with most points wins
  o Used in almost all political elections
More voting rules

• Borda count

  o Each voter awards $m - k$ points to alternative ranked $k$’th
  o Alternative with most points wins
  o Proposed in the 18th Century by the chevalier de Borda
  o Used for national elections in Slovenia
  o Similar to rule used in the Eurovision song contest
Lordi

Eurovision 2006 winners
More voting rules

• \( x \) beats \( y \) in a {
  \textit{pairwise election} if the majority of voters prefer \( x \) to \( y \)

• Plurality with runoff
  
  o First round: two alternatives with highest plurality scores survive
  
  o Second round: pairwise election between these two alternatives
More voting rules

- Single Transferable vote (STV)
  - \( m - 1 \) rounds
  - In each round, alternative with least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in:
    - Ireland, Malta, Australia, and New Zealand
    - US: Maine (governor, US congress), cities like San Francisco and Cambridge
## STV: Example

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SOCIAL CHOICE AXIOMS

• How do we choose among the different voting rules? Via desirable properties!
• Majority consistency = if a majority of voters rank alternative $x$ first, then $x$ should be the winner
• Poll 1: Which rule is not majority consistent?
  1. Plurality
  2. Plurality with runoff
  3. Borda count
  4. STV
Marquis de Condorcet

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison
CONDORCET WINNER

• Recall: \( x \) beats \( y \) in a pairwise election if a majority of voters rank \( x \) above \( y \)

• Condorcet winner beats every other alternative in pairwise election

• Condorcet paradox = cycle in majority preferences
CONDORCET CONSISTENCY

• Condorcet consistency = select a Condorcet winner if one exists

• Poll 2: Which rule is Condorcet consistent?
  1. Plurality
  2. Borda count
  3. Both
  4. Neither
CONDORCET CONSISTENT RULES

• Copeland
  o Alternative’s score is \#alternatives it beats in pairwise elections
  o Why does Copeland satisfy the Condorcet criterion?

• Maximin
  o Score of $x$ is $\min_y |\{i \in N: x \succ_i y\}|$
  o Why does Maximin satisfy the Condorcet criterion?
DODGSON’S RULE

• Distance function between profiles: \#swaps between adjacent alternatives
• Dodgson score of $x$ = the min distance from a profile where $x$ is a Condorcet winner
• Dodgson’s rule: select alternative that minimizes Dodgson score
• The problem of computing the Dodgson score is NP-complete!
DODGSON UNLEASHED

Voter 1: a, b, c, d, e
Voter 2: b, a, c, d, e
Voter 3: e, b, c, d, a
Voter 4: e, c, d, b, a
Voter 5: b, e, d, c, a
APPLICATION: WEB SEARCH

• Generalized Condorcet: if there is a partition $X, Y$ of $A$ such that a majority prefers every $x \in X$ to every $y \in Y$, then $X$ is ranked above $Y$

• Assumption: spam website identified by a majority of search engines

• When aggregating results from different search engines, spam websites will be ranked last [Dwork et al. 2001]
APPLICATION: WEB SEARCH

Google

bing

YAHOO!

overall
Awesome example

- Plurality: $a$
- Borda: $b$
- Condorcet winner: $c$
- STV: $d$
- Plurality with runoff: $e$

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IS SOCIAL CHOICE PRACTICAL?

• UK referendum: Choose between plurality and STV as a method for electing MPs
• Academics agreed STV is better...
• ... but STV seen as beneficial to the hated Nick Clegg
• Hard to change political elections!
Computational social choice

• However:
  o in online voting...
  o in human computation...
  o in multiagent systems...

the designer is free to employ any voting rule!
Example: Robobees

- Robobees need to decide on a joint plan (alternative)
- Many possible plans
- Each robobee (agent) has a numerical evaluation (utility) for each alternative
- Want to maximize sum of utilities = social welfare
- Communication is restricted
**Example: Robobees**

- **Approach 1:** communicate utilities
  - May be infeasible
- **Approach 2:** each agent votes for favorite alternative (plurality)
  - $\log m$ bits per agent
  - May select a bad alternative

- $n/2 - 1$ agents

- $n/2 + 1$ agents
Example: RoboBees

• Approach 3: each agent votes for an alternative with probability proportional to its utility

• Theorem [Caragiannis & P 2011]: if $n = \omega(m \log m)$ then this approach gives almost optimal social welfare in expectation
AI-Driven Decisions

RoboVote is a free service that helps users combine their preferences or opinions into optimal decisions. To do so, RoboVote employs state-of-the-art voting methods developed in artificial intelligence research. Learn More.

Poll Types

RoboVote offers two types of polls, which are tailored to different scenarios; it is up to users to indicate to RoboVote which scenario best fits the problem at hand.

Objective Opinions

In this scenario, some alternatives are objectively better than others, and the opinion of a participant reflects an attempt to estimate the correct order. RoboVote’s proposed outcome is guaranteed to be as close as possible — based on the available information — to the best outcome. Examples include deciding which product prototype to develop, or which company to invest in, based on a metric such as projected revenue or market share. Try the demo.

Subjective Preferences

In this scenario participants’ preferences reflect their subjective taste; RoboVote proposes an outcome that mathematically makes participants as happy as possible overall. Common examples include deciding which restaurant or movie to go to as a group, which destination to choose for a family vacation, or whom to elect as class president. Try the demo.

Ready to get started?

CREATE A POLL
SUMMARY

• Terminology:
  o Voting rules: plurality, Borda, plurality with runoff, STV, Copeland, Maximin, Dodgson
  o Axioms: Majority consistency, Condorcet consistency

• Big ideas:
  o When we build voting systems, we are not constrained by politics and tradition!