HOMEWORK 2 — CS 818A3 — SPRING 2005

## Problem 1

When

$$\exists \alpha, \beta.\ (\mathbf{lseg}\ \alpha\ (\mathsf{i}, \mathsf{j})\ *\ \mathbf{lseg}\ \beta\ (\mathsf{j}, \mathsf{k})) \wedge \gamma = \alpha \cdot \beta,$$

we say that $\mathsf{j}$ is an *interior pointer* of the list segment described by $\mathbf{lseg}\ \gamma\ (\mathsf{i}, \mathsf{k})$.

1. Give an assertion describing a list segment with two interior pointers $\mathsf{j}_1$ and $\mathsf{j}_2$, such that $\mathsf{j}_1$ comes before than, or at the same point as, $\mathsf{j}_2$ in the ordering of the elements of the list segment.

2. Give an assertion describing a list segment with two interior pointers $\mathsf{j}_1$ and $\mathsf{j}_2$, where there is no constraint on the relative positions of $\mathsf{j}_1$ and $\mathsf{j}_2$.

3. Prove that the first assertion implies the second.

## Problem 2

A *braced list segment* is a list segment with an interior pointer $\mathsf{j}$ to its last element; in the special case where the list segment is empty, $\mathsf{j}$ is **nil**. Formally,

$$\mathbf{brlseg}\ \epsilon\ (\mathsf{i}, \mathsf{j}, \mathsf{k}) \stackrel{\text{def}}{=} \mathbf{emp} \wedge \mathsf{i} = \mathsf{k} \wedge \mathsf{j} = \mathbf{nil}$$

$$\mathbf{brlseg}\ \alpha \cdot a\ (\mathsf{i}, \mathsf{j}, \mathsf{k}) \stackrel{\text{def}}{=} \mathbf{lseg}\ \alpha\ (\mathsf{i}, \mathsf{j})\ *\ \mathsf{j} \mapsto \mathsf{a}, \mathsf{k}.$$

1. Prove the assertion

$$\mathbf{brlseg}\ \alpha\ (\mathsf{i}, \mathsf{j}, \mathsf{k}) \Rightarrow \mathbf{lseg}\ \alpha\ (\mathsf{i}, \mathsf{k}).$$

2. Give an annotated specification of a command that changes the final pointer of a braced list segment from $\mathsf{k}$ to $\mathsf{l}$. The command should not construct or deallocate any list structure, and it should not change the value of $\mathsf{j}$. It should satisfy

$$\{\mathbf{brlseg}\ \alpha\ (\mathsf{i}, \mathsf{j}, \mathsf{k})\}\ \cdots\ \{\mathbf{brlseg}\ \alpha\ (\mathsf{i}, \mathsf{j}, \mathsf{l})\}.$$

3. Give an annotated specification of a command that concatenates two braced list segments. The command should not construct or deallocate any list structure, and it should not change the value of $\mathsf{k}_2$. It should satisfy

$$\{\mathbf{brlseg}\,\alpha\,(\mathsf{i}_1,\mathsf{j}_1,\mathsf{k}_1)\ *\ \mathbf{brlseg}\,\beta\,(\mathsf{i}_2,\mathsf{j}_2,\mathsf{k}_2)\}$$
$$\ldots$$
$$\{\mathbf{brlseg}\,\alpha{\cdot}\beta\,(\mathsf{i}_1,\mathsf{j},\mathsf{k}_2)\}.$$

4. Give an annotated specification of a command that appends an item $\mathsf{a}$ at the beginning of a braced list segment. The command should not change the value of $\mathsf{k}$, and should satisfy

$$\{\mathbf{brlseg}\,\alpha\,(\mathsf{i},\mathsf{j},\mathsf{k})\}\ \cdots\ \{\mathbf{brlseg}\,\mathsf{a}{\cdot}\alpha\,(\mathsf{i},\mathsf{j},\mathsf{k})\}.$$

5. Give an annotated specification of a command that appends an item $\mathsf{a}$ at the end of a braced list segment. The command should not change the value of $\mathsf{k}$, and should satisfy

$$\{\mathbf{brlseg}\,\alpha\,(\mathsf{i},\mathsf{j},\mathsf{k})\}\ \cdots\ \{\mathbf{brlseg}\,\alpha{\cdot}\mathsf{a}\,(\mathsf{i},\mathsf{j},\mathsf{k})\}.$$

6. Give an annotated specification of a command that deletes an item $\mathsf{a}$ from the beginning of a nonempty braced list segment. The command should not change the value of $\mathsf{k}$, and should satisfy

$$\{\mathbf{brlseg}\,\mathsf{a}{\cdot}\alpha\,(\mathsf{i},\mathsf{j},\mathsf{k})\}\ \cdots\ \{\mathbf{brlseg}\,\alpha\,(\mathsf{i},\mathsf{j},\mathsf{k})\}.$$