**16-711 Kinematics, Dynamic Systems, and Control**
**Spring term, 2000**
**Problem Set 5**

Due: beginning of lecture, Thursday, March 30.

1. *Unilateral Laplace Transforms*

   (a) Find the Laplace transform of the following time functions

      i.
      $$x(t) = \begin{cases} 1 & 0 < t < T \\ 0 & t > T \end{cases}$$

      ii.
      $$x(t) = e^{-\alpha t} - e^{-\beta t}$$

      iii.
      $$x(t) = \sin(\omega_0 t) = \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$$

   (b) Find the inverse Laplace transform for the following

      i.
      $$X(s) = \frac{s+3}{s^2+s}$$

      ii.
      $$X(s) = \frac{1 - e^{-(s+\alpha)T}}{s+\alpha}$$

      *Note that this is not a rational function of s.*
      Sketch the resulting time function.

      iii.
      $$X(s) = \frac{1}{(s+1)^2(s+2)}$$

2. *"Real" actuators:* The actuators used in any robot include dynamics. Consider a simple 1-dof robot actuated by an electrical motor. The differential equations describing this system might be of the form

$$\begin{aligned} \ddot{x} &= \tfrac{1}{M} f \\ f &= K i_m \\ \dot{i}_m &= \tfrac{1}{L}\left(v - K\dot{x} - R i_m\right) \end{aligned}$$

   where $x$ is the position of the "joint", $f$ is the force applied by the motor, $v$ is the voltage applied to the motor, and $K$ and $L$ are constants (the torque constant and winding inductance) associated with the motor.

   (a) Write the transfer function from $v$ to $x$.

   (b) Consider using a PD control policy of the form $v = K_p x + K_d \dot{x}$. For what values of the gain parameters $K_p$ and $K_d$ is the system stable?

   (c) Describe what gains you would pick and argue why your choice is good.

3. Consider the system described by the following set of differential equations

$$
\begin{aligned}
\dot{x} = &\quad Ax + Bu \\
y = &\quad Cx
\end{aligned}
$$

where $x \in \mathbb{R}^3$, $u \in \mathbb{R}^1$, $y \in \mathbb{R}^1$, with

$$
\begin{aligned}
A = &\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & K \\ 0 & -K & 1 \end{bmatrix} \\
B = &\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\
C = &\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

(a) Find the transfer function between $u$ and $y$ for this system.

(b) What is the characteristic equation of this system?

(c) Is this system controllable? (Show how you reach your conclusion, and comment on the implications)

4. Consider another system of the form used above, only now $x \in \mathbb{R}^4$, with

$$
\begin{aligned}
A = &\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \\
B = &\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\
C = &\begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}
\end{aligned}
$$

(a) Find the transfer function between $u$ and $y$ for this system.

(b) What is the characteristic equation of this system?

(c) Is this system controllable? (Show how you reach your conclusion, and comment on the implications)