PID Controls
(Part II)

Howie Choset

(thanks to George Kantor and Wikipedia)

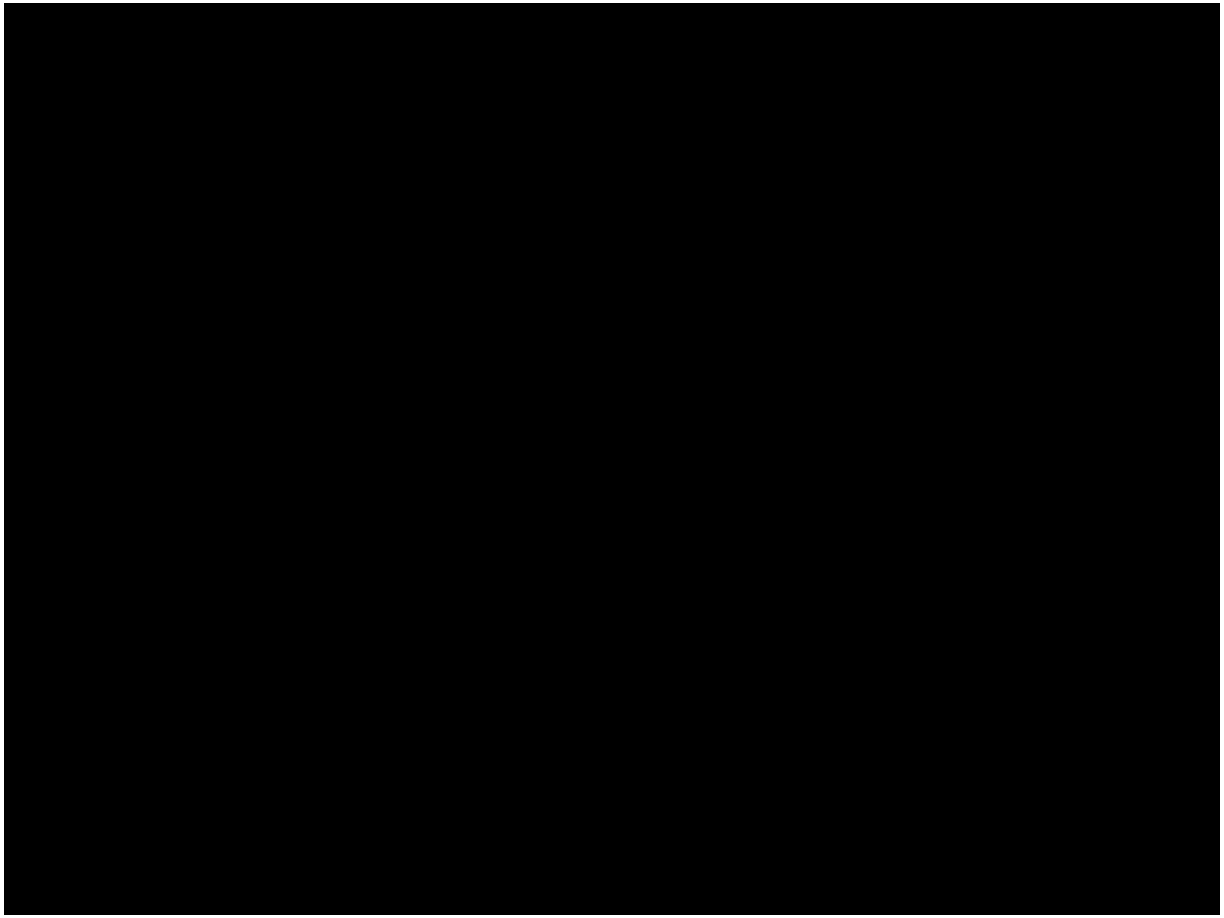
http://www.library.cmu.edu/ctms/ctms/examples/motor/motor.htm

Overview

- Mass-Spring-Damper System
- Second order ODE
  - Definition
  - Vary parameters
  - Forcing functions
- Different feedback meaning
  - Proportional
  - Derivative
- Control for Error – block diagram
- Integral Control
- Different Affects of Varying PID
- Feed Forward Term
- Vehicle Controls

Big Dog Quadruped

Boston Dynamics



Nathan Michael Quadrotors

Controls

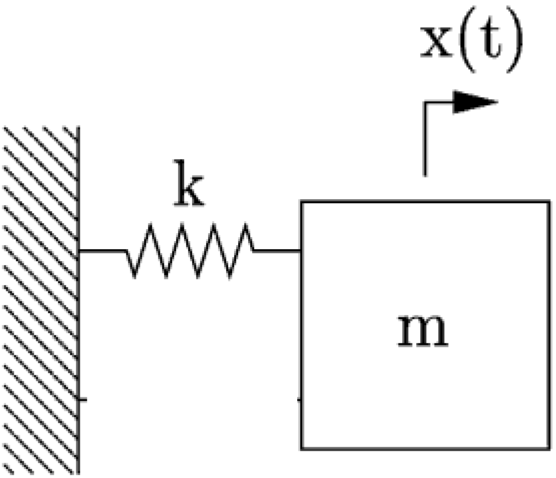
Estimation

RC Airplane - Adaptive Control

Chowdhary G., Johnson E., Chandramohan R., Kimbrell M. S., Calise A.

Mass Spring

$$F_s = -kx$$



$$F_{\text{tot}} = ma = m\frac{d^2x}{dt^2} = m\ddot{x}$$

$$F_{\text{tot}} = F_s$$

$$m\ddot{x} = -kx$$

$$\ddot{x} + \frac{k}{m}x = 0$$

Solutions/Responses

Critical damping ($\zeta = 1$)

$$x(t) = (A + Bt)\, e^{-\omega_0 t}$$

$$A = x(0)$$

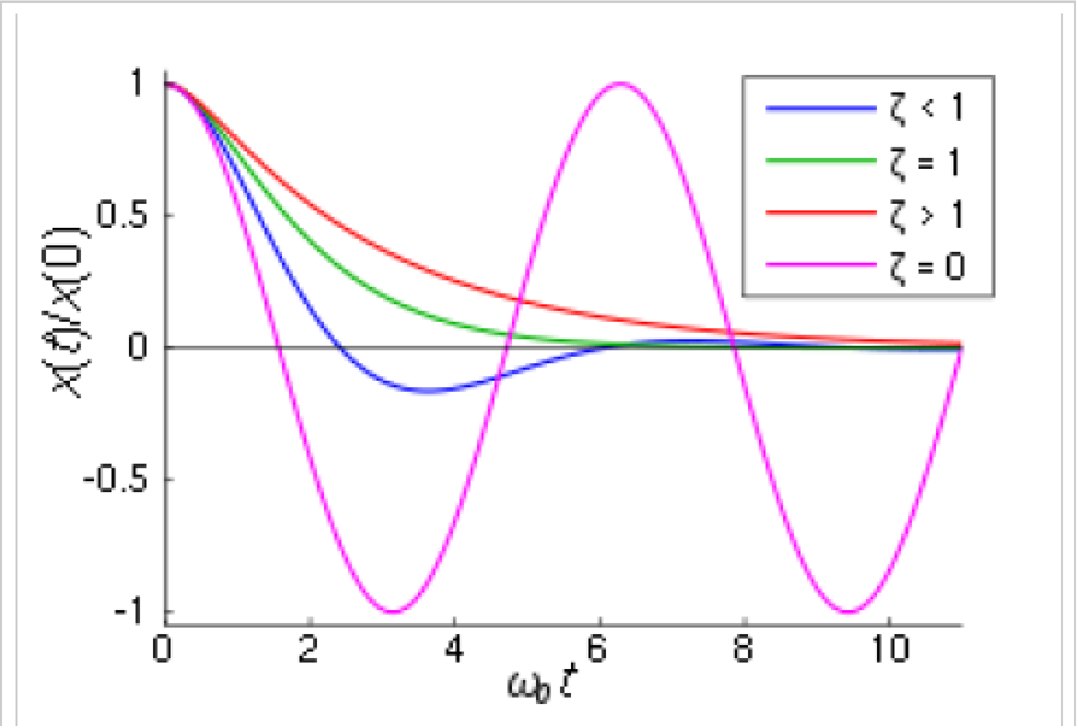
$$B = \dot{x}(0) + \omega_0 x(0)$$

Over-damping ($\zeta > 1$)

$$x(t) = Ae^{\gamma_+ t} + Be^{\gamma_- t}$$

$$A = x(0) + \frac{\gamma_+ x(0) - \dot{x}(0)}{\gamma_- - \gamma_+}$$

$$B = -\frac{\gamma_+ x(0) - \dot{x}(0)}{\gamma_- - \gamma_+}.$$



Under-damped ($0 < \zeta < 1$)

$$x(t) = \overbrace{e^{-\zeta\omega_0 t}}^{\text{decay}}\overbrace{(A\cos(\omega_\mathrm{d}\, t) + B\sin(\omega_\mathrm{d}\, t))}^{\text{Oscillation, damped natural frequency}}$$

Let $\gamma = \omega_0\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$

$$\omega_\mathrm{d} = \omega_0\sqrt{1 - \zeta^2}$$

$$A = x(0)$$

$$B = \frac{1}{\omega_\mathrm{d}}(\zeta\omega_0 x(0) + \dot{x}(0)).$$

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Step Response

$$\omega_\mathrm{d} = \omega_0\sqrt{1-\zeta^2}$$

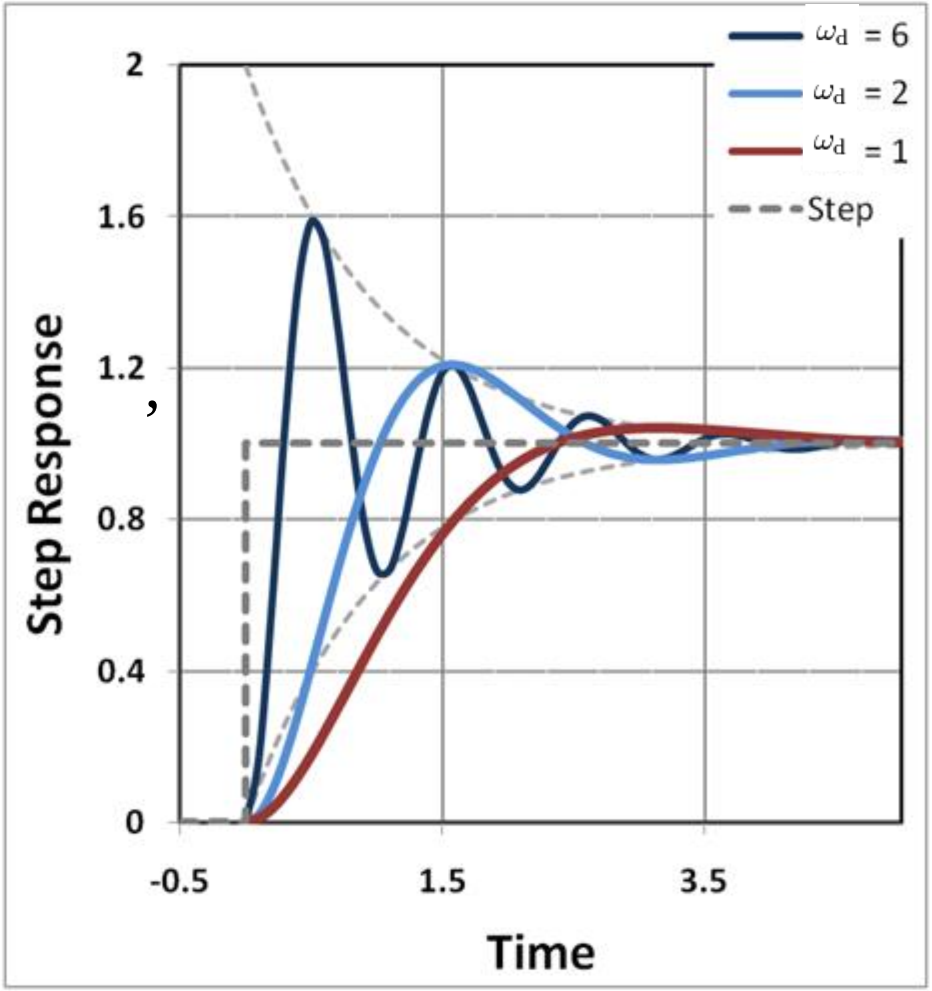
$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\zeta\omega_0\frac{\mathrm{d}x}{\mathrm{d}t} + \omega_0^2 x = \frac{F(t)}{m}$$

$$\frac{F(t)}{m} = \begin{cases}\omega_0^2 & t \geq 0 \\ 0 & t < 0\end{cases},$$

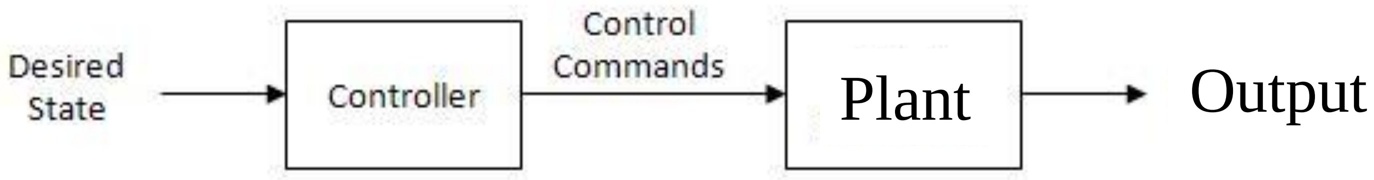
$$x(t) = 1 - \mathrm{e}^{-\zeta\omega_0 t}\frac{\sin\left(\sqrt{1-\zeta^2}\,\omega_0 t + \varphi\right)}{\sin(\varphi)}$$

$$\cos\varphi = \zeta$$

As time goes on, x(t) goes to 1



Open Loop Controller



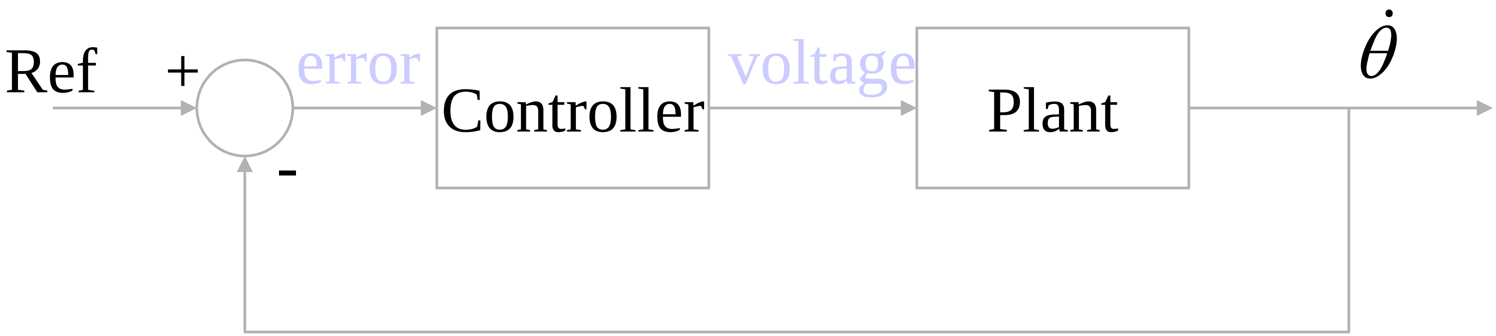
controller tells your system to do something, but doesn't use the results of that action to verify the results or modify the commands to see that the job is done properly

Closed Loop Controller

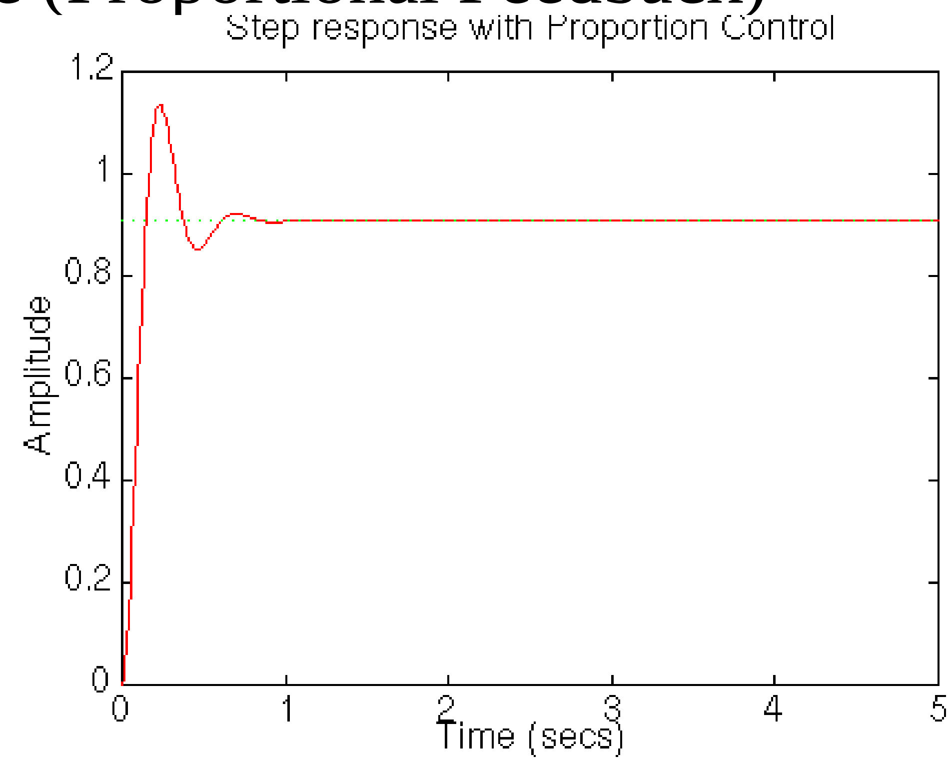
Give it a velocity command
and get a velocity output

<u>Controller Evaluation</u>

Steady State Error
Rise Time (to get to ~90%)
Overshoot
Settling Time (Ring) (time to steady state)
Stability



Closed Loop Response (Proportional Feedback)

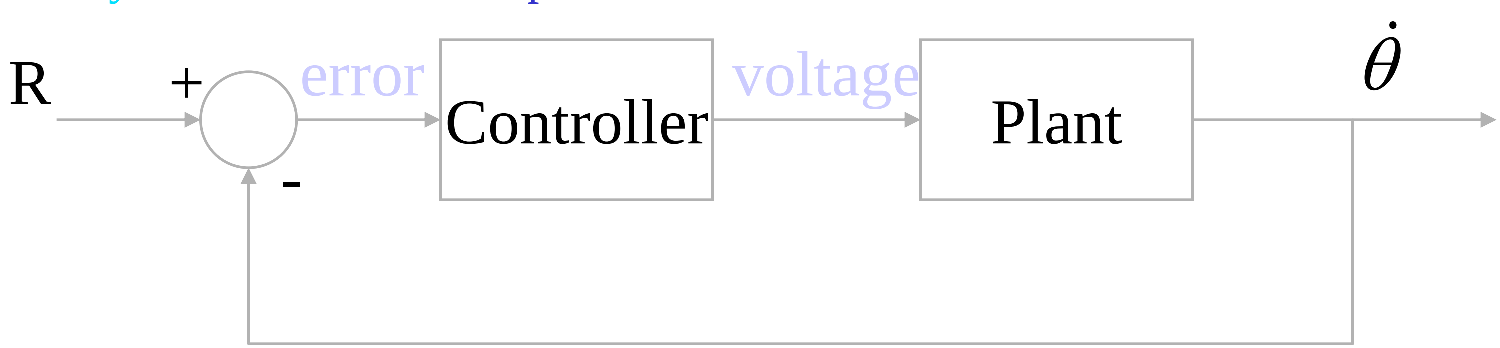


Proportional Control K_p

Easy to implement
Input/Output units agree
Improved rise time

Steady State Error (true)

↑P: ↓Rise Time vs. ↑ Overshoot*
↑P: ↓Rise Time vs. ↓ Settling time*
↑P: ↓Steady state error vs. other problems



Voltage = K_p error

*In some other systems, not mass-spring

Closed Loop Response (PI Feedback)

Proportional/Integral Control

$$K_p + \frac{1}{s} K_I$$

No Steady State Error

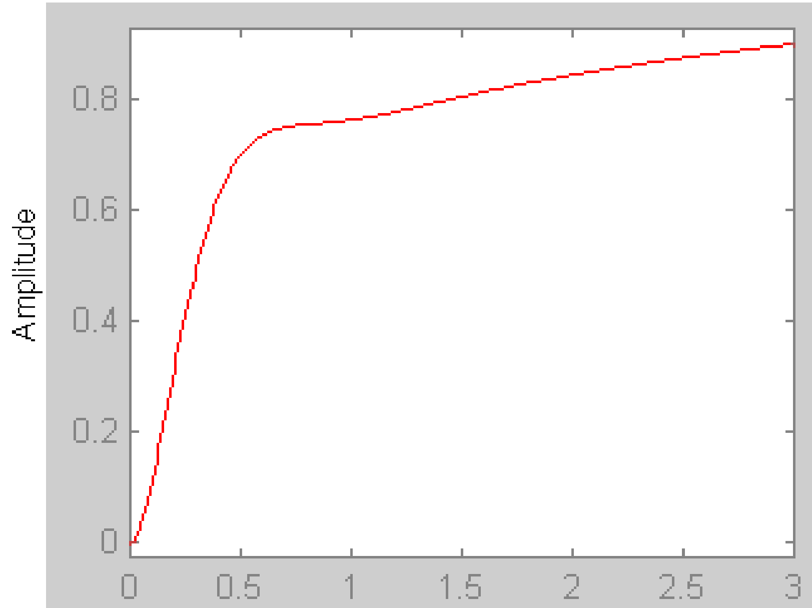
Bigger Overshoot and Settling

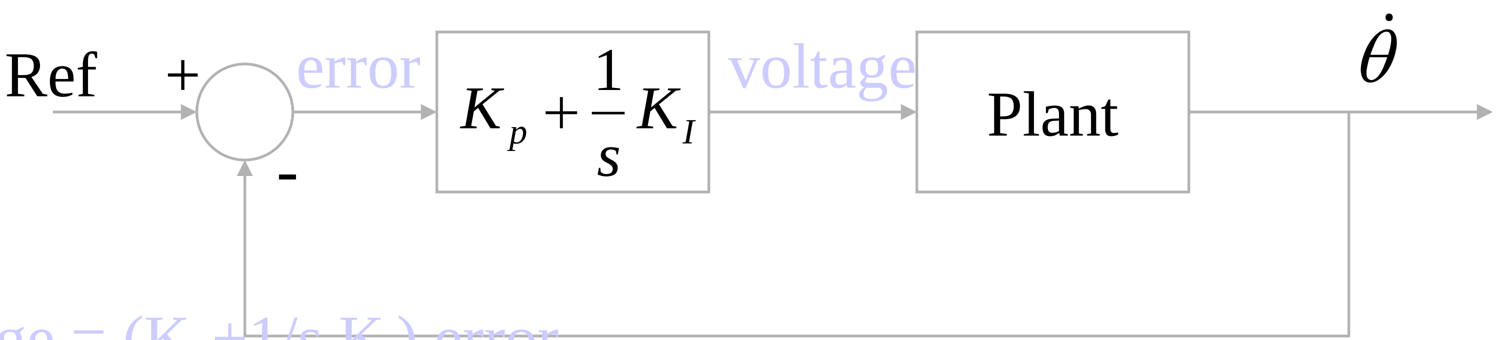
Saturate counters/op-amps

↑P: ↓Rise Time vs. ↑ Overshoot

↑P: ↓Rise Time vs. ↓ Settling time

↑I: ↓Steady State Error vs. ↑Overshoot





Voltage = $(K_p + 1/s\ K_i)$ error

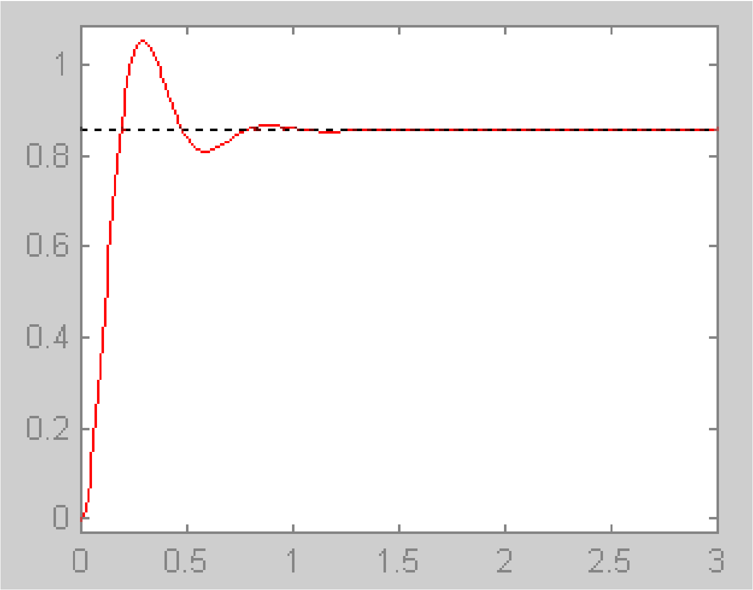
Closed Loop Response (PID Feedback)

Proportional/Integral/Differential

Quick response
Reduced Overshoot

$$K_p + \frac{1}{s}K_I + sK_D$$

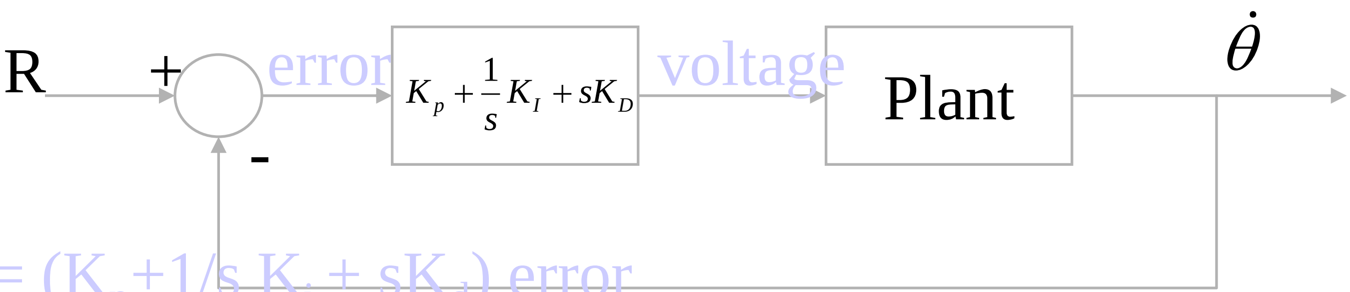
Sensitive to high frequency noise
Hard to tune



$\uparrow$P: $\downarrow$Rise Time vs. $\uparrow$ Overshoot
$\uparrow$P: $\downarrow$Rise Time vs. $\downarrow$ Settling time

$\uparrow$I: $\downarrow$Steady State Error vs. $\uparrow$Overshoot

$\uparrow$D: $\downarrow$Overshoot vs. $\uparrow$Steady State Error



Voltage = (K_p+1/s K_i + sK_d) error

Quick and Dirty Tuning

- Tune P to get the rise time you want
- Tune D to get the setting time you want
- Tune I to get rid of steady state error
- Repeat

- More rigorous methods – Ziegler Nichols, Self-tuning,
- Scary thing happen when you introduce the I term
  - Wind up (example with brick wall)
  - Instability around set point

Feed Forward

Volt

Decouples Damping from PID

To compute K_b

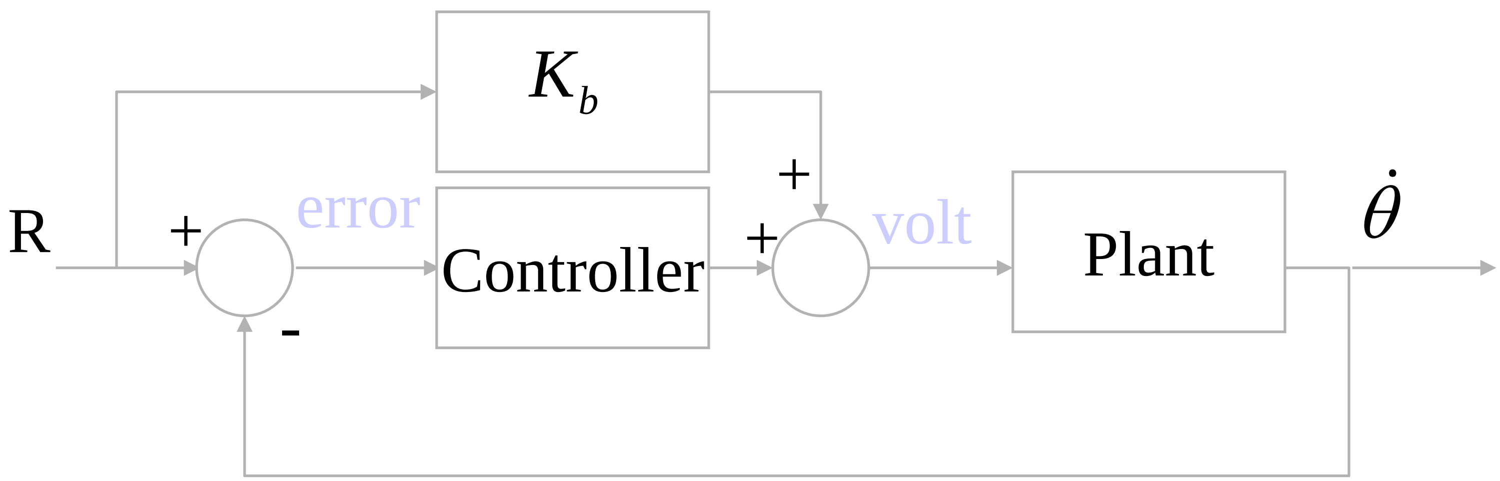
Try different *open* loop inputs and measure output velocities

For each trial i,

Tweak from there.

$$K_b^i = \frac{u_i}{\dot{\theta}_i}, \qquad K_b = \operatorname{avg} K_b^i$$

K



Mobile Robot

- planar workspace

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- position of robot and goal are known

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$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

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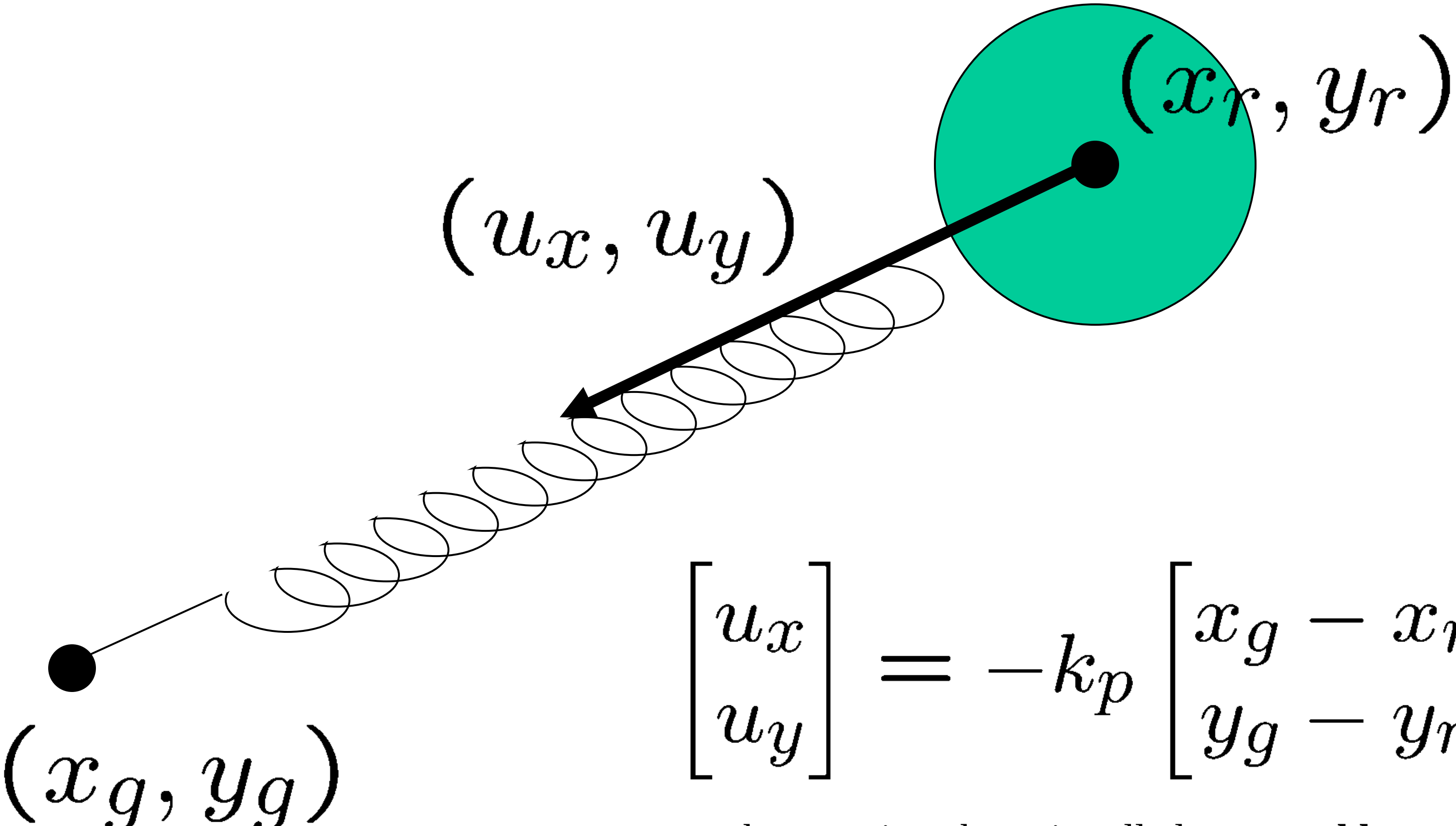
Mobile Robot

- planar workspace
- position of robot and goal are known
- omni-directional robot
- control input is velocity:

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

(boldface lie, we'll relax this later, too)

Proportional (P) Control:

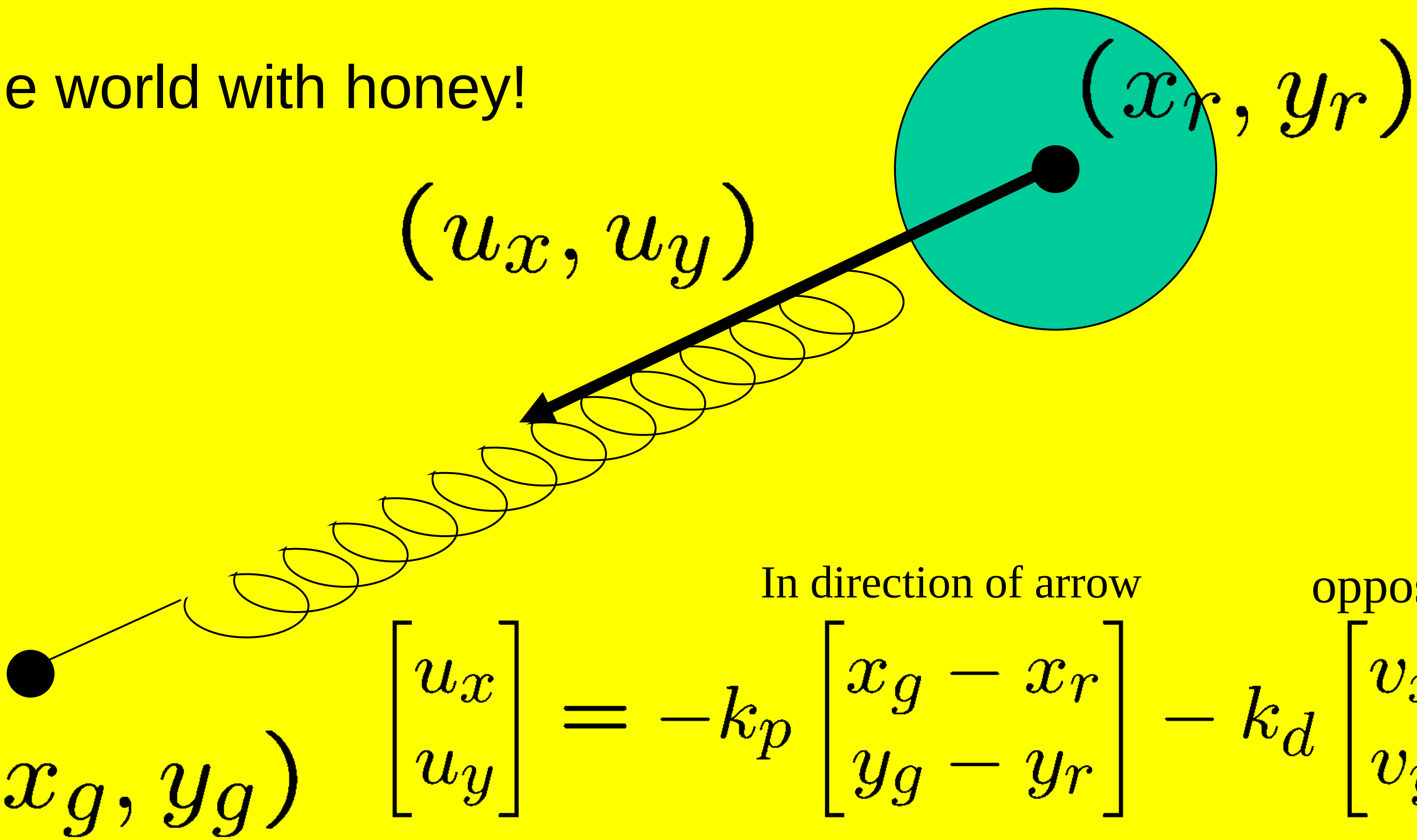


$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = -k_p \begin{bmatrix} x_g - x_r \\ y_g - y_r \end{bmatrix}$$

- the equation above is called a **control law**
- k_p is called the **proportional gain**
- k_p is a tunable parameter
- physically, k_p is the stiffness of the spring

Proportional-Derivative (PD) control:

Fill the world with honey!



In direction of arrow — opposite

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = -k_p \begin{bmatrix} x_g - x_r \\ y_g - y_r \end{bmatrix} - k_d \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

- k_d is called the **derivative gain**
- k_p and k_d are tunable parameters
- physically, k_d is the damping term
- all of the stuff about P control still applies

Robot Inputs

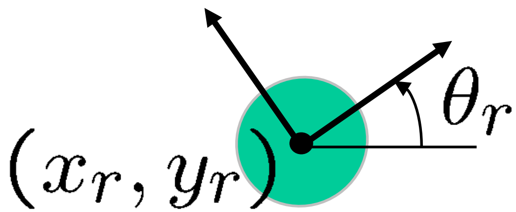
So far we've assumed something like

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$

But really, we control the velocities of the left and right wheels, which can easily be mapped to forward and turning velocities:

$$\begin{bmatrix} v_l \\ v_r \end{bmatrix} \Rightarrow \begin{bmatrix} v_f \\ \omega \end{bmatrix}$$

Nonholonomic Constraints

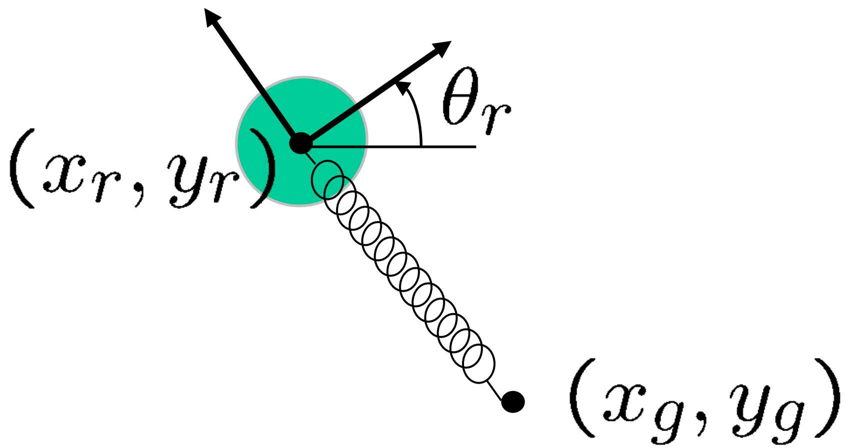


The equations of motion using these controls are:

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} v_f \cos\theta \\ v_f \sin\theta \\ \omega \end{bmatrix}$$

The fact that the robot can't move sideways is a **nonholonomic constraint** (we will see this again).

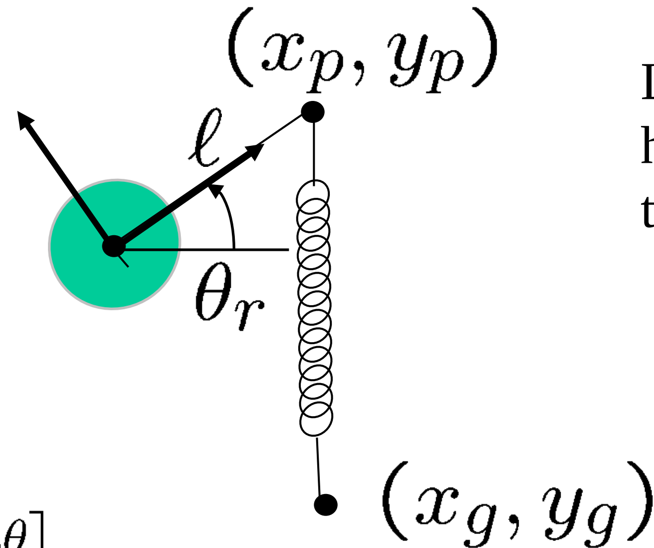
The Problem:



P or PD control won’t work.

No smooth control law will!

A Simple Solution:

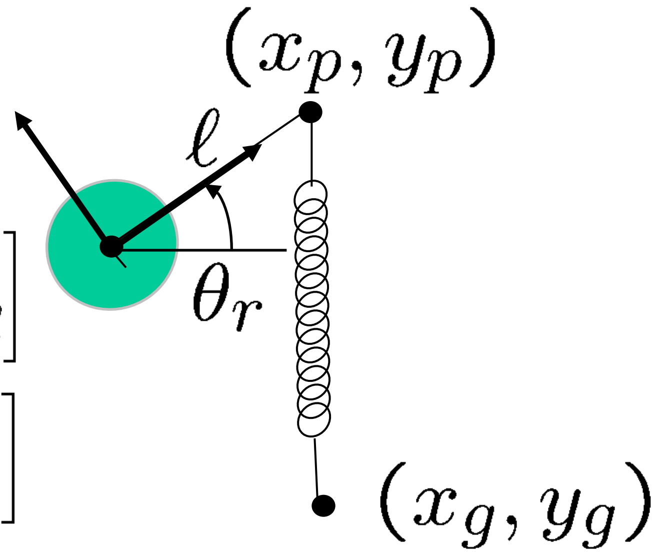


Like a rigid trailor hitch (not driving to point)

$$\begin{bmatrix} x_p \\ y_p \\ \theta_r \end{bmatrix} = \begin{bmatrix} x_r + \ell\cos\theta \\ y_r + \ell\sin\theta \\ \theta_r \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \dot{x}_r - \ell\dot{\theta}_r\sin\theta_r \\ \dot{y}_r + \ell\dot{\theta}_r\cos\theta_r \\ \omega_r \end{bmatrix} = \begin{bmatrix} v_f\cos\theta_r - \omega\ell\sin\theta_r \\ v_f\sin\theta_r + \omega\ell\cos\theta_r \\ \omega \end{bmatrix}$$

A Simple Solution (cont.):



$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix}$$

If we ignore orientation:

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} v_f \\ \omega\ell \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_f \\ \omega\ell \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix}$$

so we can implement the PD control law as:

$$\begin{bmatrix} v_f \\ \omega\ell \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \left(-k_p \begin{bmatrix} x_g - x_p \\ y_g - y_p \end{bmatrix} - k_d \begin{bmatrix} v_x \\ v_y \end{bmatrix} \right)$$

Did not get rid of nh constraint, but moved it to something we don't care about (theta, angular and linear velocities) - trailor hitch story

Follow a straight line with differential drive or at least get to a point

Error can be difference in wheel velocities or accrued distances

Make both wheels spin the same speed

- asynchronous – false start
- wheels can have slight differences (radius, etc)

Make sure both wheels spin the same amount and speed

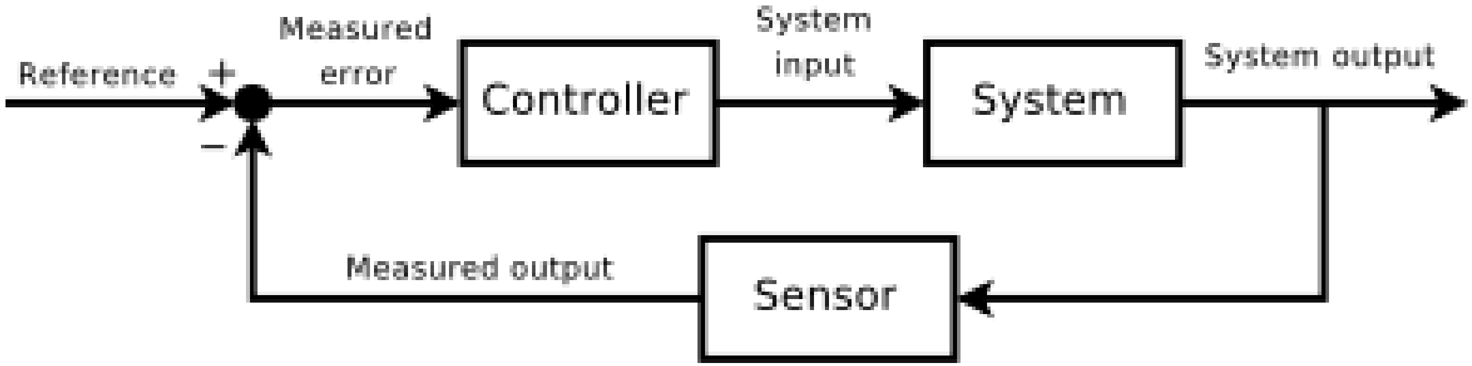
- false start

Line following

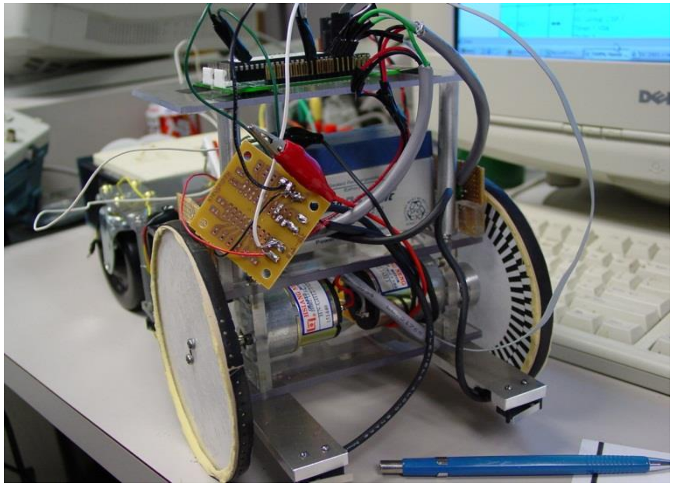
More complicated control laws – track orientation

m1vref = vref + K1 * thetaerror + K2 * offset error
m2vref = vref - K1 * thetaerror - K2 * offset error

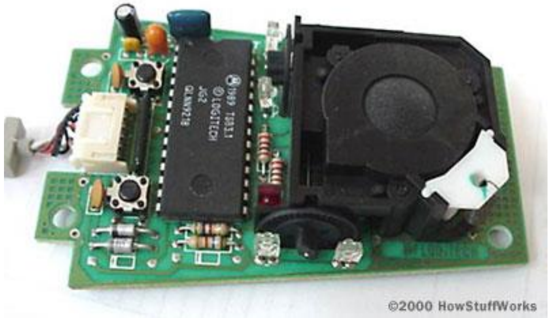
Really, there is a sensor



Encoders

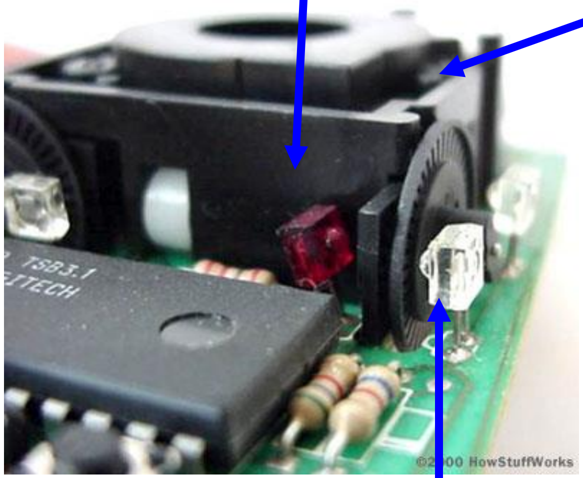


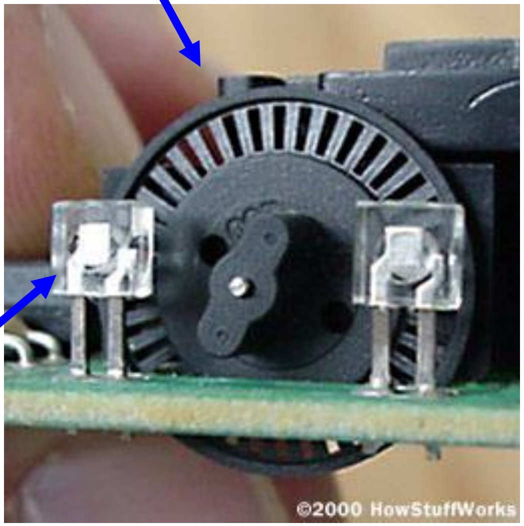
Encoders – Incremental



Photodetector

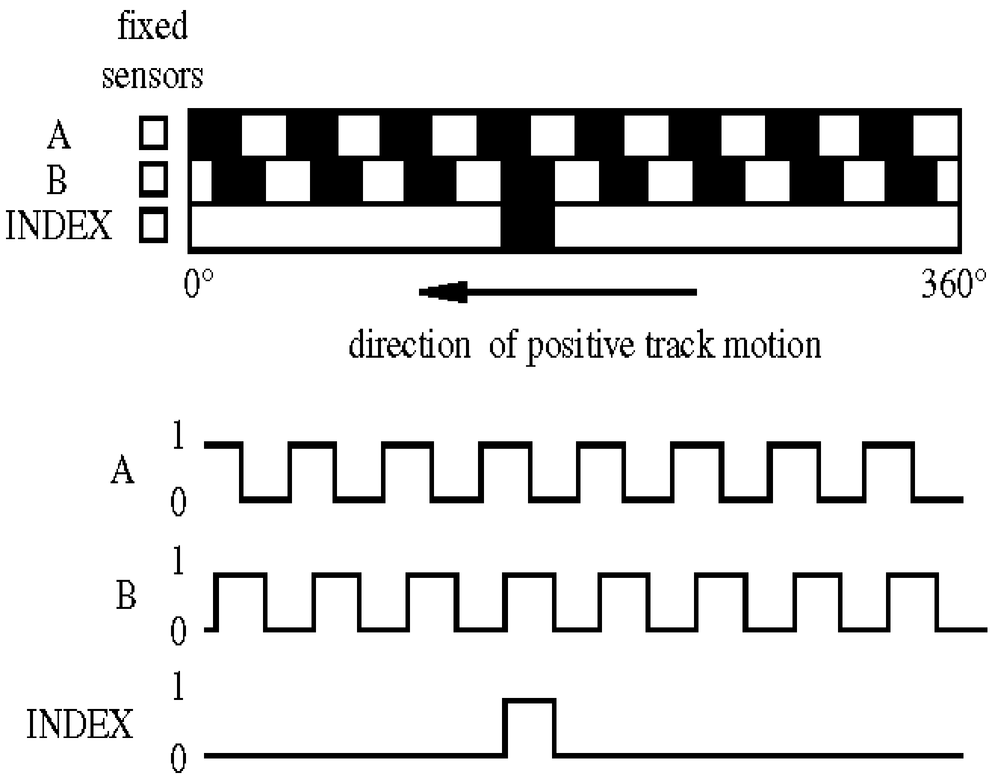
Encoder disk

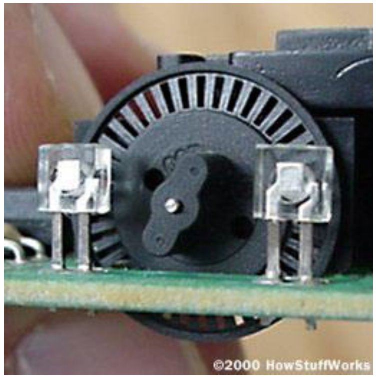




LED Photoemitter

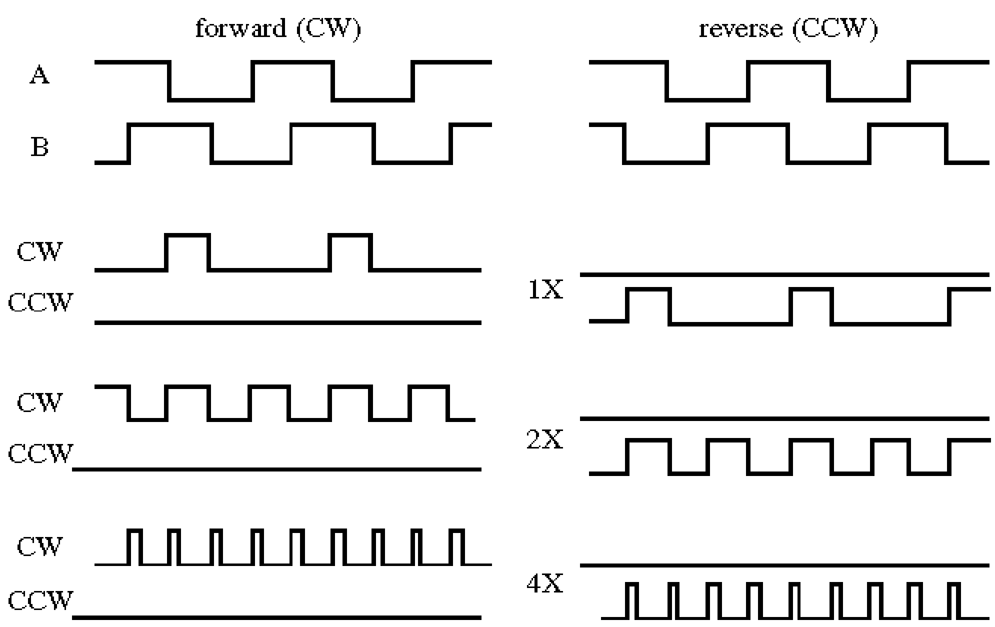
Encoders - Incremental





Encoders - Incremental

- Quadrature (resolution enhancing)



To be continued

- Maps
- Bayesian Localization