

Final Exam Solutions

16-311: Introduction to Robotics

2018

Name: _____

Andrew ID: _____

Team Number: _____

- You will have 1 hour and 30 minutes to complete this exam
- There are 10 sections on 28 pages. Make sure you have all of the pages. Write your Andrew ID on all the sections and keep your work in that section (they will be graded separately). There are blank pages throughout the sections.
- When making drawings - be precise. Rounded edges should look rounded, sharp edges should look sharp, sizes should be close to scale. Neatness counts.
- Show your work. Partial credit may apply. Likewise, justify algebraically your work to ensure full credit, where applicable.
- It should be very clear what your final answer is, circle it if necessary.
- You may need to make certain assumptions to answer a problem. State them (e.g. what is optimal).
- You are allowed one handwritten two-sided reference sheet for the exam. No cell phones, laptops, calculators, neighbors, etc. allowed.
- Good luck and you can do it.

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1 Vision

Andrew ID: _____

1. You are working at a packaging company with a robot that picks up cylindrical objects from a conveyor belt. Every second, the conveyor belt stops and a downward facing camera takes a picture of the belt surface. A colleague of yours has already thresholded each image so that pixels that are the color of the objects are 0 and pixels that are the color of the conveyor belt are 1.

Describe (in words or pseudocode) an algorithm that would find the x and y coordinate of the center of the object in the frame below. (Assume that there will only be one object in every image and that the objects will always be in the same orientation.)

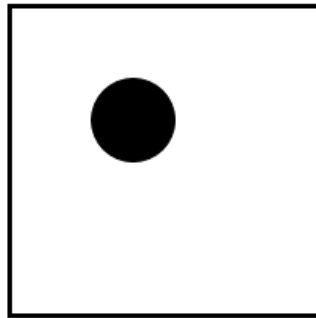


Figure 1: Example frame from the downward-facing camera.

```
Assume origin is the top left. Define xsum=0, ysum=0, n=0.  
for (int x=0; x<cols; x++) (2 points for iterating through)  
for (int y=0; y<rows; y++)  
if (pixel(x,y)==0) (2 points for checking if 0)  
xsum+=x;  
ysum+=y;  
n++;  
return(xsum/n, ysum/n) (6 points for correctly calculating)
```

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2 Control

Andrew ID: _____

1. You have a self-balancing robot with an inertial measurement unit on the top. You want to control this robot using PID control. You control the voltage you send to the motors and you want to make the pitch equal 0 degrees. Through initial testing, you determine that a feedforward term may also be necessary.

Write an equation for V (input voltage) in terms of K_p , K_i , K_d , K_f (feed forward), θ (the measurement of pitch angle from the IMU), $\theta_{previous}$ and $\theta_{accumulated}$.

$$V = \underline{k_f + k_p\theta + k_d(\theta - \theta_p) + k_i(\theta_a + \theta)}$$

(1 point for correct k_f , 1 point for correct k_p , 2 points for correct k_d , 1 point for correct k_i . Signs don't matter. Also accept θ_a alone (they assumed the accumulation was done at the beginning of the time step.))

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3 Path Planning

Andrew ID: _____

1. Draw the configuration space resulting from the following workspace and robot with the given reference point. The robot cannot rotate. The walls are obstacles. The start and goal are just points, not obstacles. Draw directly on the image.

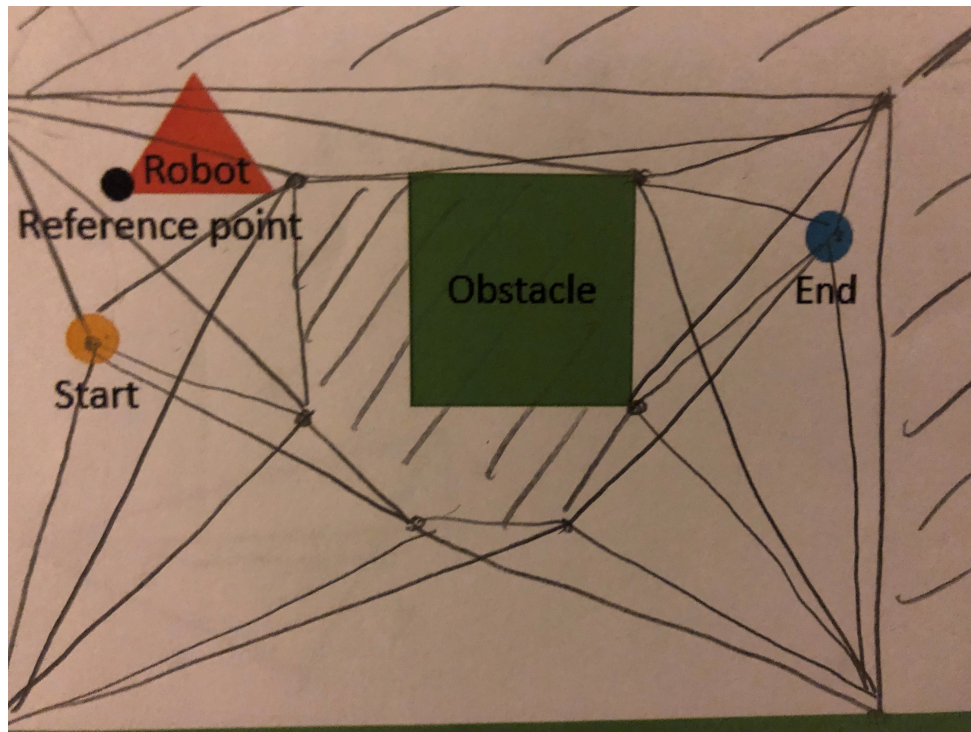


Figure 2: Workspace and robot.

2. On the image above, draw the visibility graph of the configuration space. The walls are still obstacles.
(1 point for walls, 1 point for corners, 1 point for border on the top)
(4 points for all lines or missing one on vis graph, 3 points if forgot 2 to half, if forgot start and end -1)

3. The visibility graph can produce the optimal path with respect to which metric?

L2 distance travelled (2 points)

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4 Graph Search

Andrew ID: _____

The graph below includes heuristic and actual cost values. The numbers inside the circles represent the heuristic guess of the distance to the goal which, for this example, is euclidean distance. The starting node is Node A and the ending node is Node H.

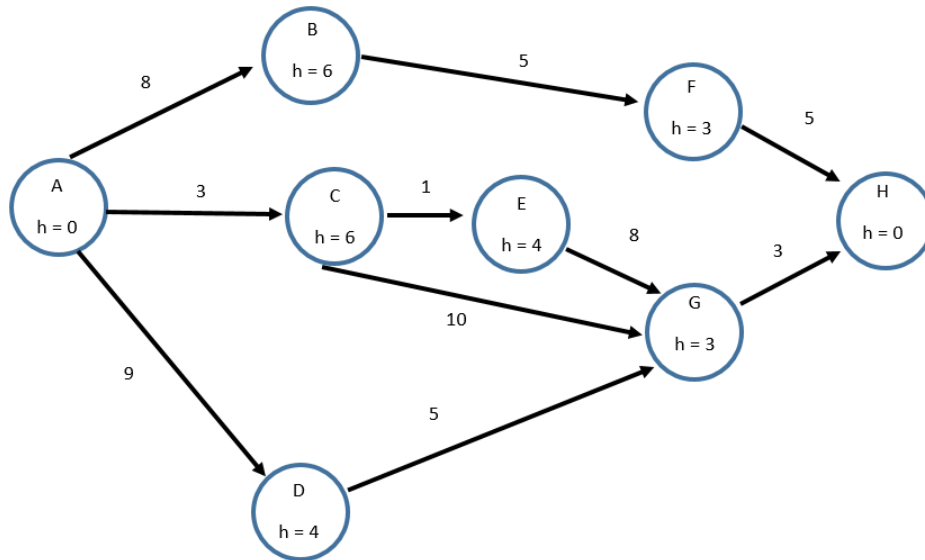


Figure 3: A* graph.

1. Using A*, find the path from start to goal. On the line below, list the nodes that are EXPANDED in the A* algorithm above in the order in which they are expanded:

ACEDBGH (3 points)

2. On the line below, list the nodes in the path found through this algorithm and the final cost of this path.

ACEGH (15) (2+1 points)

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5 Localization

Andrew ID: _____

1. We have a vehicle traveling counterclockwise in a circle with an ultrasonic sensor facing outward. The robot starts at position 0 (the robot does not know this) and moves three units to position 3. For the following pattern of walls, which is the most likely set of predictions for where the robot is (the higher the fraction, the more likely the robot predicts it is at that location)? Circle the letter of your choice.

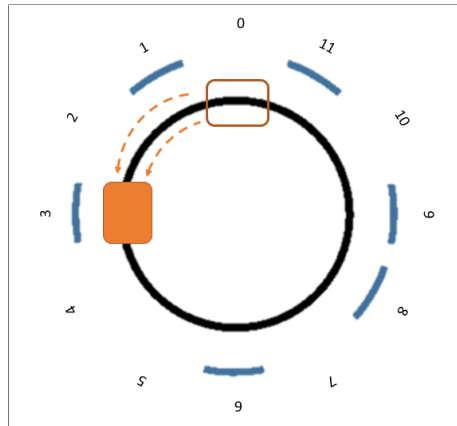


Figure 4: Overhead view of robot and environment.

- A:** [0: 0.05 1: 0.11 2: 0.05 3: 0.11 4: 0.05 5: 0.11
..... 6: 0.11 7: 0.05 8: 0.11 9: 0.11 10: 0.05 11: 0.11]
- B:** [0: 0.083 1: 0.083 2: 0.083 3: 0.083 4: 0.083 5: 0.083
..... 6: 0.083 7: 0.083 8: 0.083 9: 0.083 10: 0.083 11: 0.083]
- C:** [0: 0.05 1: 0.05 2: 0.05 3: 0.45 4: 0.05 5: 0.05
..... 6: 0.05 7: 0.05 8: 0.05 9: 0.05 10: 0.05 11: 0.05]
- D:** [0: 0.05 1: 0.15 2: 0.05 3: 0.15 4: 0.05 5: 0.05
..... 6: 0.05 7: 0.05 8: 0.15 9: 0.05 10: 0.05 11: 0.05]

The correct answer is D. (4 points)

2. Arun Srivatsan came in to discuss a novel probabilistic approach to pose estimation used for image guided surgery. What did he call his contribution to pose estimation that involved grouping measurements?

Probabilistic (1 point)

6 Vehicle Design

Andrew ID: _____

1. The two vehicles below have the same weight and wheels. Which of them is better with respect to tip over angle? Explain your answer in one sentence. You may refer to a diagram that you draw for clarity.

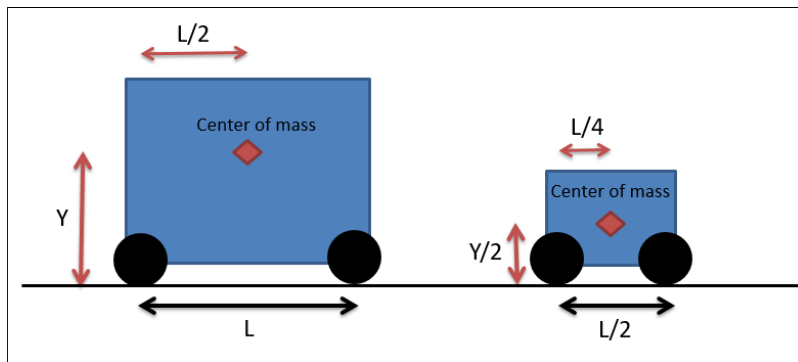


Figure 5: Two vehicle designs.

Tip over occurs at the same angle. The angle between the COG and point of contact is the same. (4 points)

2. Hannah's research focuses on terramechanics and the interaction of wheels and tracks with terrain. What are two characteristics of the vehicle or terrain that can limit a vehicle's drawbar pull (thrust minus resistances)? Answer in one to two sentences.

Any two of the following: weight distribution, wheels, tracks, dimensions, soil shear, slope, obstacle (4 points)

3. Show where the piston is for the following positions of the crank.

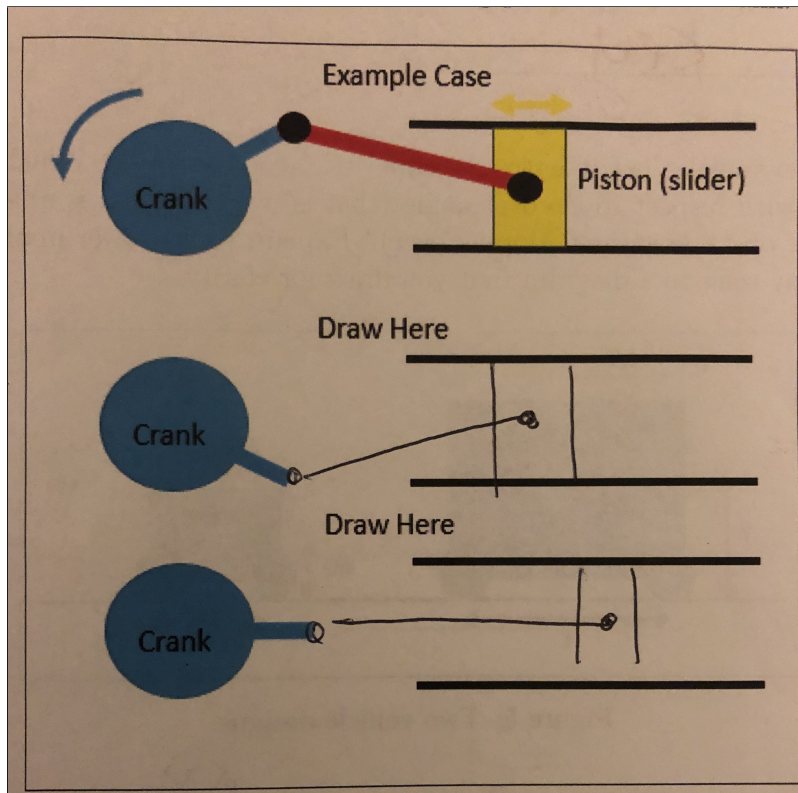


Figure 6: Slider-crank mechanism.

(1 point per picture)

7 Human Robot Interaction

Andrew ID: _____

1. Professor Hollis discussed the shortcomings of traditional home assistive robots. What is one advantage of ballbot compared to these standard configurations? Answer in one sentence.

Any one of the following: thinner, smaller, non-holonomic, compliant. (2 points)

2. Professor Forlizzi conducted a series of experiments with an autonomous delivery medical robot in a local hospital. This robot had great reviews from the nurses in the postpartum ward (baby-delivery area) but terrible reviews from the nurses in the oncology ward (cancer area). What is the hypothesis for the difference in feedback from health professionals at the same hospital? Answer in one to two sentences.

The hypothesis is that the oncology ward is more stressful, so any interference with work was viewed negatively and potentially dangerous. (3 points)

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8 Forward Kinematics

Andrew ID: _____

1. We have an RP arm as shown below. The first joint is the revolute joint at the origin. The first link is 10 cm long and moves along the front plane. The angle of this joint (θ) is limited from 0 to 180° . The second joint is the prismatic joint and its movement is OUT from the front plane (parallel to the top and right frames). The length of the prismatic link (s) is limited from 0 to 5 cm.

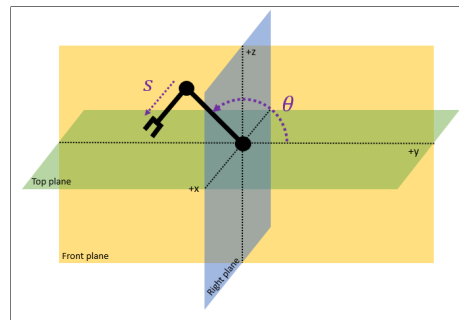
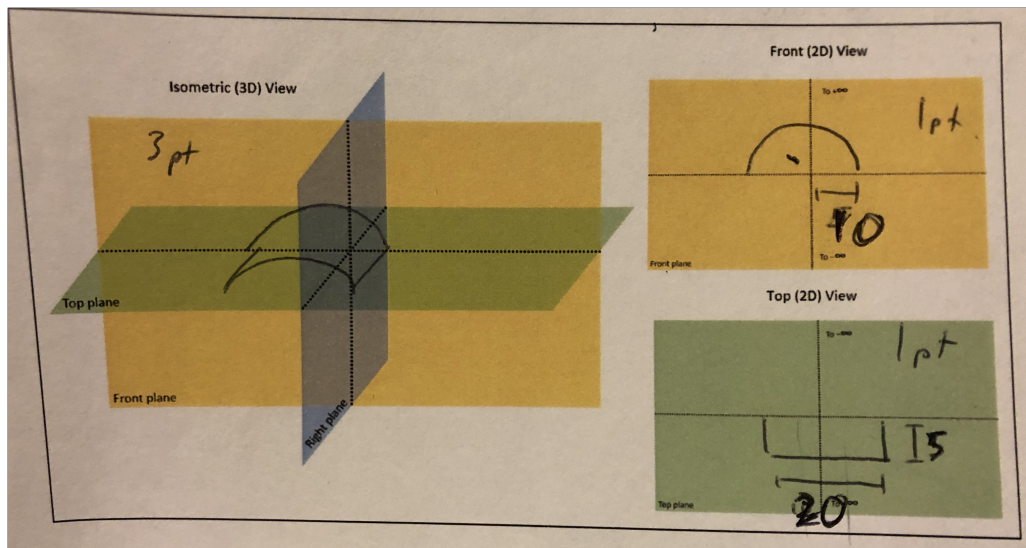


Figure 7: Revolute-prismatic arm at ($\theta = 135^\circ$, $s = 5$ cm).

Draw the shape of the x , y , z coordinates that the end effector can reach. Draw the shape on all three axes for clarity.



(3 points for 3D, 1 point for front, 1 point for top)

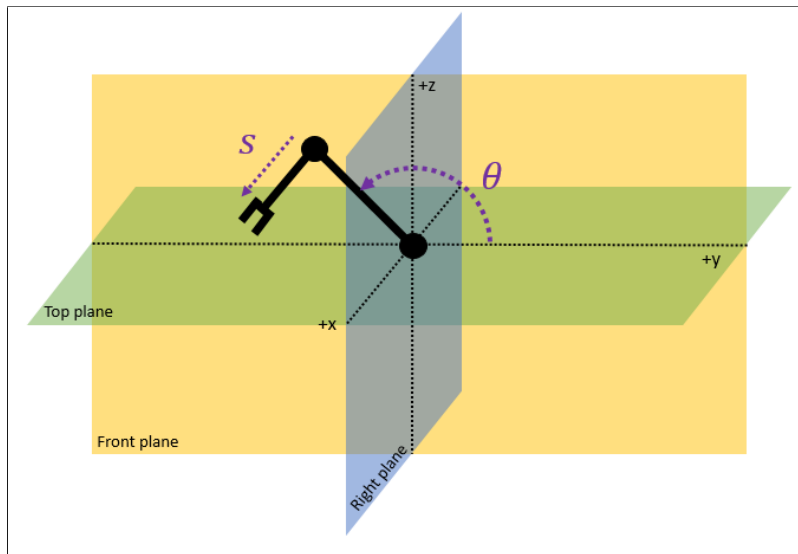


Figure 8: arm.

2. In terms of θ and s , write an equation for the x , y and z coordinates of this arm.

$$x = \underline{\quad}$$

$$y = \underline{10\cos(\theta)}$$

$$z = \underline{10\sin(\theta)}$$

(1 point each)

3. In the image below, which is a possible matrix to describe the rotation? Let theta be positive for counter clockwise rotation. Circle your choice.

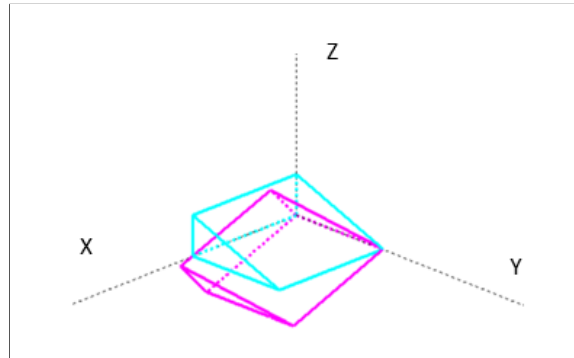


Figure 9: arm.

a)
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9 & -0.3 & 0 \\ 0 & 0.3 & 0.9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b)
$$\begin{bmatrix} 0.9 & 0 & 0.3 & 0 \\ 0 & 1 & 0 & 0 \\ -0.3 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c)
$$\begin{bmatrix} 0 & 0.9 & -0.3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.3 & 0.9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d)
$$\begin{bmatrix} 0 & 0.9 & 0.3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -0.3 & 0.9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B is correct (2 points)

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9 Inverse Kinematics

Andrew ID: _____

We will use the same RP arm for this question. For your convenience, the description and image is copied below:

The first joint is the revolute joint and it is at the origin. The first link is 10 cm long and moves along the front plane. The angle of this joint (θ) is limited from 0 to 180 degrees. The second joint is the prismatic joint and its movement is out from the front plane (parallel to the top and right frames). The length of the prismatic link (s) is limited from 0 to 5 cm. For reference, the sample image below shows the arm at ($\theta = 135^\circ$, $s = 5$ cm).

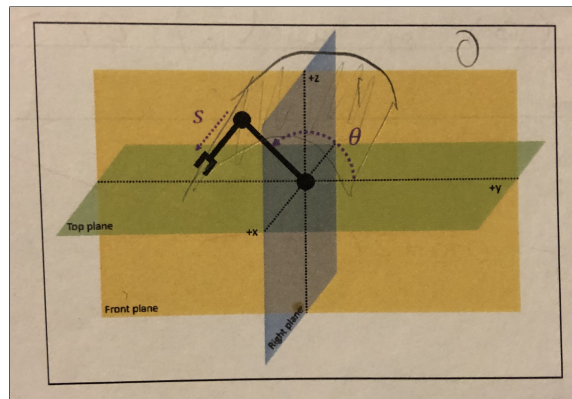


Figure 10: Revolute-prismatic arm.

1. For any (x, y, z) coordinate, how many solutions for θ and s are there? Specify where there are 0, 1 and/or multiple solutions. You may refer to the image you drew in the previous section.

1 on the dome, 0 otherwise (3 points)

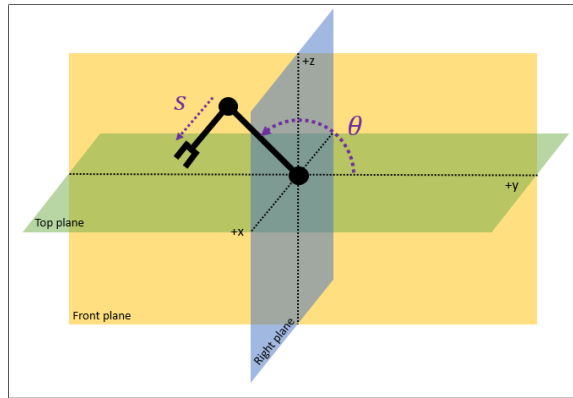


Figure 11: Revolute-prismatic arm.

2. In terms of x , y and z , write an equation for θ and s .

$$\theta = \arccos(y/10) \text{ or } \arcsin(z/10) \text{ (2 points)}$$

$$s = \sqrt{x^2 + y^2 + z^2} \text{ (1 point)}$$

3. Now we add another link ($l_2 = 10$ cm), so our arm is a revolute, revolute, prismatic arm. The two revolute arms pivot in the same plane. The second link's angle is with reference to the first link's angle, where counterclockwise is still positive.

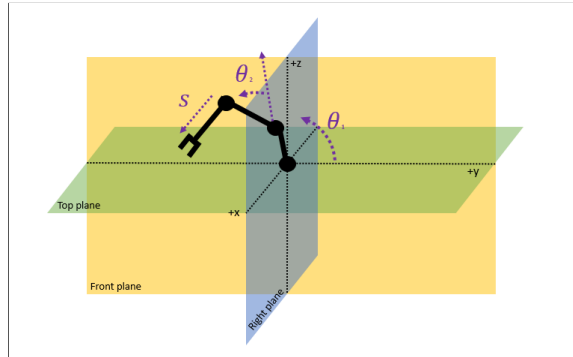


Figure 12: Revolute-revolute-prismatic arm.

4. In terms of x , y and z , write an equation for θ_1 , θ_2 and s . State if there are multiple solutions with your equations. You do not need to simplify your equations, but they should be written as below. If your equations refer to other unknowns, those other unknowns should be solvable from the equations you provide (e.g. if $s = 7\theta$, you should have an equation that allows you to solve directly for θ).

Assume no joint limits, there are multiple solutions for θ_1 and θ_2

$$\theta_1 = \underline{\text{atan2}(z, y) - \text{asin}(10\sin(\theta_2)/(\sqrt{z^2 + y^2}))} \text{ (4 points)}$$

$$\theta_2 = \underline{\text{acos}((z^2 + y^2 - 10^2 - 10^2)/(z * 10 * 10))} \text{ (3 points)}$$

$$s = \underline{x} \text{ (1 point)}$$

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10 Nonholonomic Constraints

Andrew ID: _____

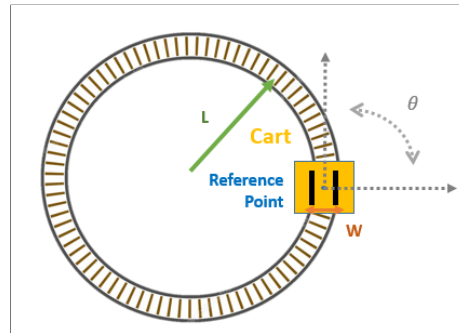


Figure 13: Cart at position $[L;0;90]$.

1. For a two-wheeled cart on a circular train track, what is the constraint on the vehicle's velocity? Use $q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$ as the state vector.

$$w_1 = [-\sin(\theta) \cos(\theta) 0] \text{ (2 points)}$$

2. Is this constraint a holonomic constraint or a nonholonomic constraint? Explain in one sentence.

Holonomic because we can never move in this direction even with a series of motions.

(1.5 points for answer, 1.5 for explanation)

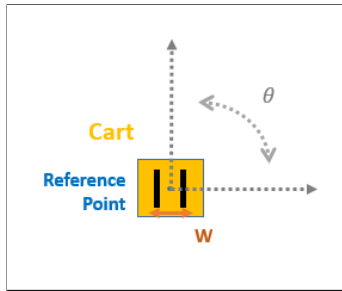


Figure 14: Cart at position $[0;0;90]$.

3. For a two-wheeled cart without a track, what is the constraint on the vehicle's velocity? Use $q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$ as the state vector.

$$w_1 = \underline{[-\sin(\theta) \cos(\theta) 0]} \text{ (2 points)}$$

4. What are the two allowable motions for our vehicle?

$$g_1 = \underline{[0; 0; 1]}$$

$$g_2 = \underline{[\cos(\theta); \sin(\theta); 0]}$$

(4 points)

5. Take the Lie Bracket of your two allowable motions. Show your work, not just your final answer.

$$\underline{[0 \ 0 \ -\sin(\theta); 0 \ 0 \ \cos(\theta); 0 \ 0 \ 0]^* [0; 0; 1] - [0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0]^* [\cos(\theta); \sin(\theta); 0]} = \underline{[-\sin(\theta); \cos(\theta); 0]}$$

(6 points total, 2 points if only Lie bracket formula is known, 4 points for correct set up, 2 for final answer)

6. What does the result of this Lie bracket mean? Explain in one sentence.

We can position anywhere. We can get to a place perpendicular to the allowable motion, though not instantaneously. (3 points)

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END OF EXAM.