Final Exam

16-311: Introduction to Robotics

Last Updated: 9 May 2017

Name: ____

Andrew ID: _______________________

Team Number: _________

- You will have 1 hour and 15 minutes to complete this exam
- There are 6 sections on 20 pages. Make sure you have all of the pages. Write your Andrew ID on all the sections and keep your work in that section (they will be graded individually). There is a blank page after every section.
- When making drawings - be precise. Rounded edges should look rounded, sharp edges should look sharp, sizes should be close to scale. Neatness counts.
- Show your work. Partial credit may apply. Likewise, justify algebraically your work to ensure full credit, where applicable.
- It should be very clear what your final answer is, circle it if necessary.
- You may need to make certain assumptions to answer a problem. State them (e.g. what is optimal).
- You are allowed one handwritten two-sided reference sheet for the exam. No cell phones, laptops, neighbors, etc. allowed.
- Good luck and you can do it.

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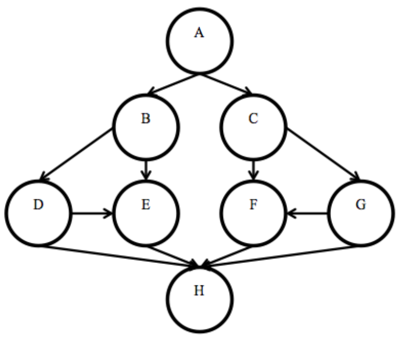
1 Graph Search

Andrew ID: ________________________

1.1 Depth-First Search

List the nodes in the order in which they were visited traveling from A to H. If there is a tie in which node you should pick next, break ties alphabetically.

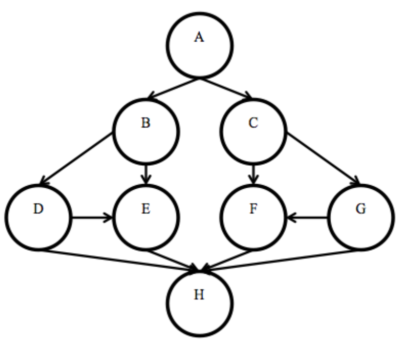


ABDEH 4 points

Figure 1: Graph for DFS.

1.2 Breadth-First Search

List the nodes in the order in which they are visited traveling from A to H. If there is a tie in which node you should pick next, break ties alphabetically.



ABCDEFGH 4 points

Figure 2: Graph for BFS.

1.3 A* Search

List the nodes EXPANDED in the order in which they were expanded traveling from A to H. If there is a tie in which node you should pick next, break ties alphabetically.

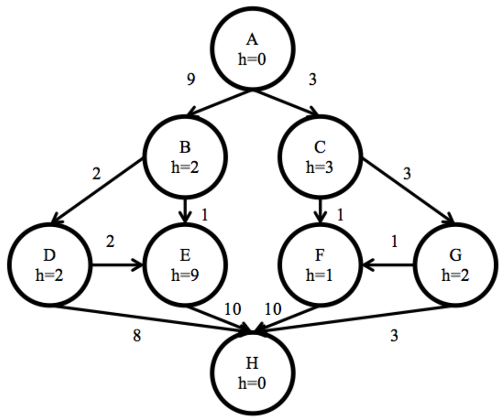


Figure 3: Graph for AStar Search.

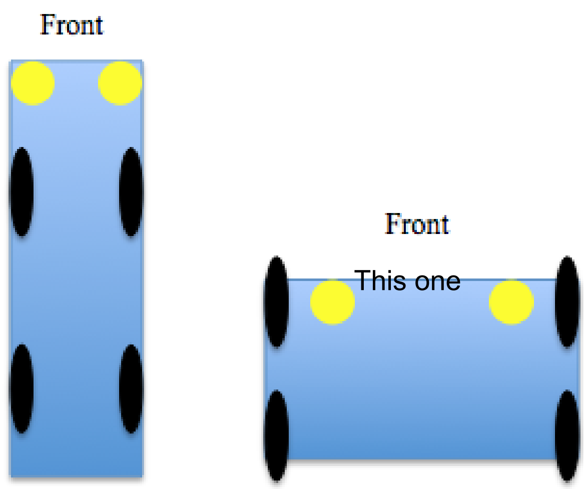
ACFGH

8 points

2 Robot Design

Andrew ID: ________________________

1. Which of the following vehicles would be easier to turn?



2 points

Figure 4: Two vehicles with equal wheels and motors but different wheel bases. There are two motors that are geared to the left and right, respectively

2. State one advantage of wheels and one advantage of tracks.

Wheels: less moving parts, less losses due to friction, etc. 4 points
Tracks: greater surface area, no risk of "bottoming out", etc.

3. Which of the following gear trains would enable the vehicle to move faster? We assume that a motor applies the same speed and torque to the gear labled input for each circumstance and both output gears are attached to axles with wheels of equal diameters.

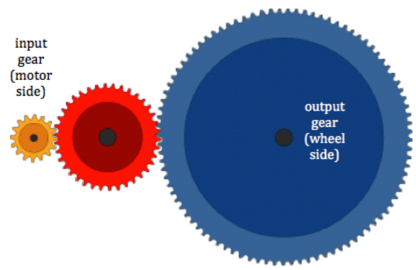
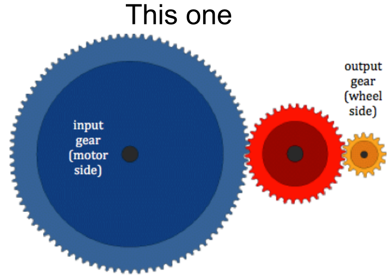


Figure 5: Gear train 1. Image adapted from howstuffworks.com

This one



2 points

Figure 6: Gear train 2. Image adapted from howstuffworks.com

3 Homogeneous Transformations and Forward Kinematics

Andrew ID: ________________________

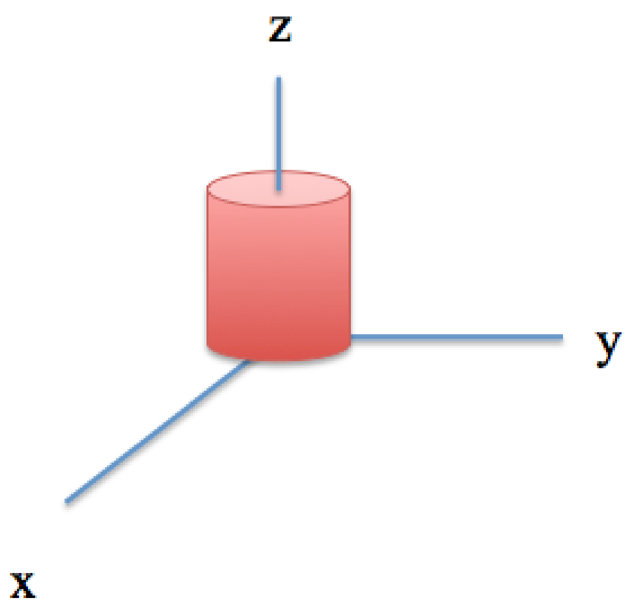
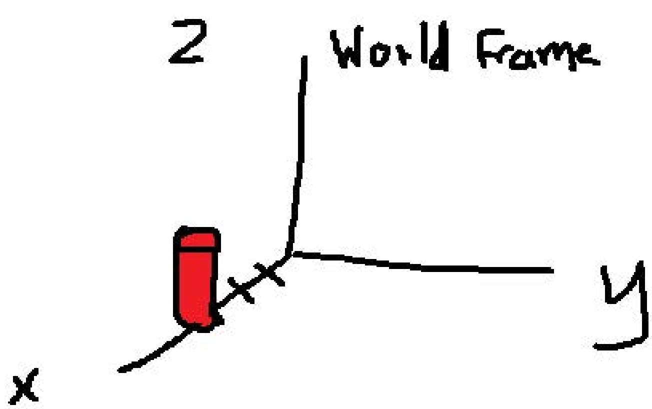


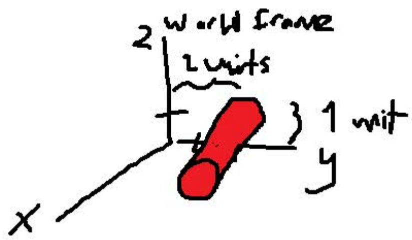
Figure 7: Starting cylinder configuration. Bottom face resting on xy plane. Center of circle on origin.

1. Draw the outcome of the above cylinder that experiences the following RELATIVE transformation: Translation 3 units in the x direction. Rotation 90 degrees on the z axis. Make sure you keep the world frame in your drawing, also.



2 points

2. Draw the outcome of the above cylinder that experiences the following RELATIVE transformation starting from the initial pictured configuration: Rotation 90 degrees on the y axis, translation 2 units in the y direction, translation 1 unit in the negative x direction. Make sure you keep the world frame in your drawing, also.



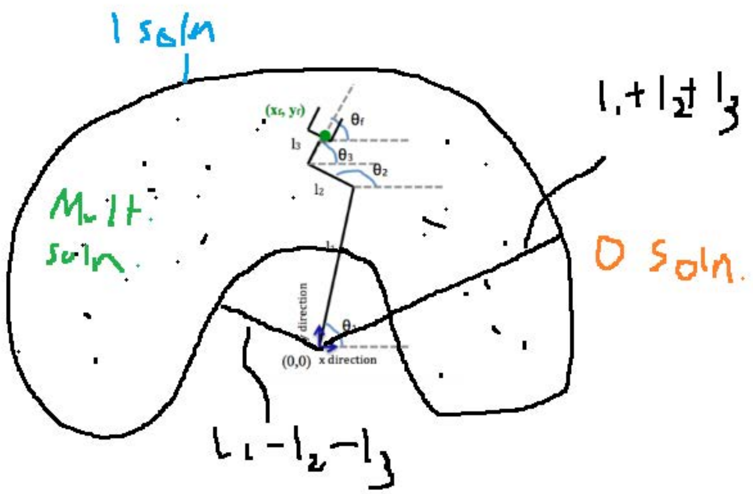
3 points

3. Write the homogeneous matrices representing the previous relative rotation 90 degrees on the y axis, 2 units in the y direction, then 1 unit in the negative x direction.

[0 0 1 0
0 1 0 2
-1 0 0 1
0 0 0 1]

4 points

4. Given the following 2D revolute, revolute, revolute arm, draw the workspace of the end effector (places where the end effector can position itself, do not worry about orientation for this moment). Specifically, indicate regions where there are no sets of angles that can reach it, areas where one set of angles can reach it and areas where there are more than one set of angles that can reach it. Here, $|l_1| > |l_2| > |l_3|$, $|l_1| > |l_2| + |l_3|$ and θ_1 is limited to 0 to 180 degrees.



3 points

Figure 8: RRR Arm in XY plane with base joint located at (0,0).

5. Describe the x_f, y_f and θ_f of the end effector in terms of θ_1, θ_2 and θ_3. NOTE THAT ANGLES ARE DESCRIBED IN RELATION TO THE WORLD X AXIS.

thf = th3
x = l1costh1+l2costh2+l3costh3
y=l1sinth1+l2sinth2+l3sinth3

5 points

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4 Inverse Kinematics

Andrew ID: ________________________

1. Derive equations for θ_1, θ_2 and θ_3 given x_f, y_f and θ_f. NOTE THAT ANGLES ARE DESCRIBED IN RELATION TO THE WORLD X AXIS.

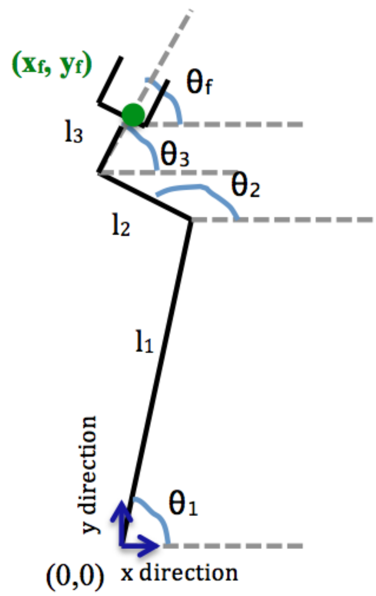

Figure 9: RRR Arm in XY plane with base joint located at (0,0).

th1 = atan2(y2/x2)-asin(l2sinB/sqrt(x2^2+y2^2))
th2 = th1+B
th3 = thf
B = acos(x2^2+y2^2-l1^2-l2^2/2l1l2)
x2 = xf-l3costhf
y2 - yf-l3sinthf

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5 Non-Holonomic Constraints

Andrew ID: ______________________

Our goal is to identify an additional allowable motion for the following vehicle:

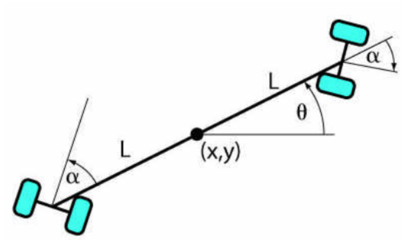


Figure 10: Coupled four-wheeled vehicle.

For this vehicle, when you move the front axle, the back axle also moves an equal amount (α), but in the opposite direction. Assume that the wheels cannot slip perpendicular to their typical direction of travel.

Here are some trig identities: $tan = sin/cos$, $sec = 1/cos$, $csc = 1/sin$, $cot = cos/sin$, $cos^2 + sin^2 = 1$, $sin(a + b) = sin(a)cos(b) + cos(a)sin(b)$, $cos(a + b) = cos(a)cos(b) - sin(a)sin(b), d/dt(cos(t)) = -sin(t)$, $d/dt(sin(t)) = cos(t)$, $d/dt(tan(t)) = sec^2(t)$, $d/dt(cot(t)) = -csc^2(t)$.

For the purposes of this problem, we will define our state as:

$$q = \begin{bmatrix} x \\ y \\ \theta \\ \alpha \end{bmatrix} \tag{1}$$

1. What are two constraints (w_1 and w_2)?

[-sin(th+alpha)
cos(th+alpha)
-Lcos(alpa)
0]

[-sin(th-alpha)
cos(th-alpha)
Lcos(alpha)
0]

6 points

2. What is the allowable motion that describes a rotation of the axles in place (g_1)?

[0
0
0
1]

2 points

3. Of the following choices, which one is an allowable motion?

$$g_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{2}$$

3 points

$$g_2 = \begin{bmatrix} -Lcos(\theta)cot(\alpha) \\ -Lsin(\theta)cot(\alpha) \\ 1 \\ 0 \end{bmatrix} \quad \text{This one} \tag{3}$$

$$g_2 = \begin{bmatrix} cos(\theta) \\ sin(\theta) \\ \frac{1}{L}tan(\alpha) \\ 0 \end{bmatrix} \tag{4}$$

4. Why did you choose this vector?

Perpendicular to constraints

5. Perform the Lie bracket on g_1 and g_2.

[-Lcosth(d/dx cotalpha)
-Lsinth(d/dx cotalpha)
0
0]

6 points

No double jeopardy here

6. Did you achieve another motion through this Lie Brakcet? How do you know this?

Yes, linearly independent of g1 and g2

2 points

6 Short Answer

Andrew ID: ________________________

1. In one to two sentences, how can you turn a grayscale image into a meaningful black and white image for your application?

   Make histogram, select threshold, set all above one color, all below a different color

   2 points

2. We take a single image from a known distance (d, in inches) to an object of known width (w, in inches). Knowing our number of pixels across the width the picture on the image plane (n) and our number of pixels per inch (p), write an expression for the focal length of the camera in inches.

   f=nd/pw

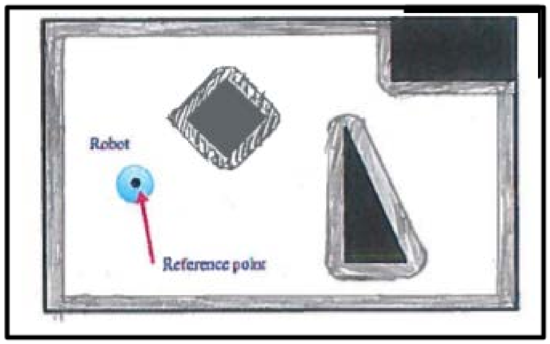
   4 points

3. We want to make a motor spin at a desired rate (ω_d). We can only control the voltage sent to the motor (v). We can measure the speed of the motor direction (ω_m). We decide to apply a PID controller to this system. Write an equation for v that uses a K_p, K_d and K_i term.

   V = (wd-wm)kp+(sumOfWmWd)ki+((wd-wm)-(prevwd-prevwm))kd, also accept if they wrote integral/derivative

   4 points

4. Draw the configuration space of this mobile robot:



3 points

Figure 11: Mobile robot workspace with unmodified obstacles.

5. How far away are these houses by the L1 metric and the L2 metric (you can leave your answer unsimplified)?

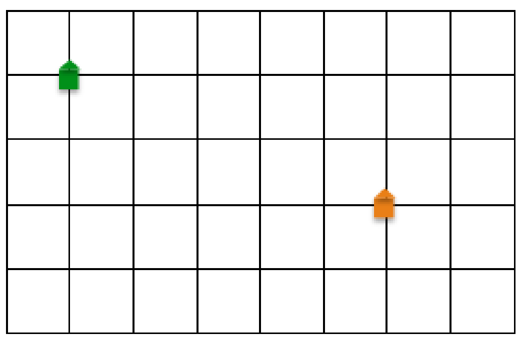
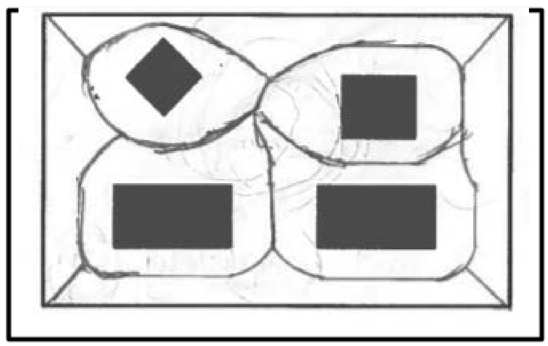


Figure 12: Grid with two houses.

2 points

L1: 7 units
L2: sqrt(29) units

6. Draw the Voronoi Diagram for this space with respect to the L2 metric:



3 points

Figure 13: Mobile robot workspace with unmodified obstacles.

7. What is a benefit of using a Voronoi diagram to plan a path? (What is good about a path produced from this algorithm?)

2 points

Stay as far away from the obstacles as possible.

8. We run a two-motor LEGO robot 1000 times with the same motor speed and duration. We mark the place where the robot ends (shown below). Explain why these dots are not completely on top of each other?

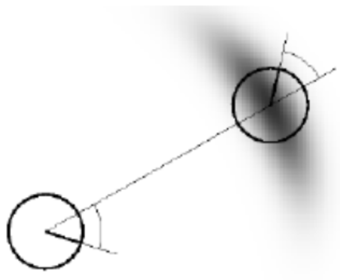


Figure 14: Results of running robot at given theta for constant duration and motor power.

slip, battery power, dust, etc.

2 points

9. Name at least two ways to improve our results so that these 1000 iterations result in a tighter resulting bunch.

PID, add another sensor, use a more accurate encoder, etc.

2 points

10. Choose one of the following to answer:

- Dr. Jodi Forlizzi came in to discuss her research on Human Robot Interaction. What type of relationship (cooperative, augmentation, none) do you think humans SHOULD have with autonomous cars? Explain your reasoning. any thoughtful response
- Dr. Khalid Jawed came in to discuss his research on rod mechanics. What is one application of this research? Explain how basic science research can influence fields like robotics. video games

11. Which of these robots is optimal?

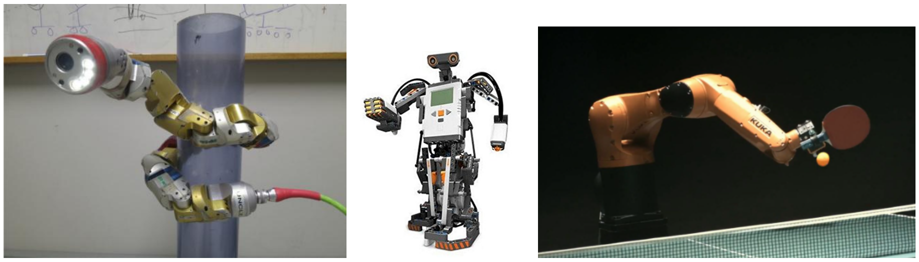


Figure 15: Candidate optimal robots.

Optimal with respect to what?

2 points

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This is the end of the test.