0. Exercise: old-school. A lumber mill saws both finish-grade and construction-grade boards from the logs that it receives. Suppose that it takes 2 hr to rough-saw each 1000 board feet of the finish-grade boards and 5 hr to plane each 1000 board feet of these boards. Suppose also that it takes 2 hr to rough-saw each 1000 board feet of the construction-grade boards, but it takes only 3 hr to plane each 1000 board feet of these boards. The saw is available 8 hr per day, and the plane is available 15 hr per day. If the profit on each 1000 board feet of finish-grade boards is \$120 and the profit on each 1000 board feet of construction-grade boards is \$100, how many board feet of each type of lumber should be sawed to maximize the profit? Solve using the Simplex method, showing your work.[1]

0. Exercise. Simplex suffering: Unboundedness, Degeneracy, Cycling.

(a) Draw a diagram for the following LP:

$$\begin{aligned} \text{maximize} \quad & x_1 \\ \text{subject to} \quad & x_1 - x_2 \le 1 \\ & -x_1 + x_2 \le 2 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Show the execution of the Simplex method on it. At termination, explain how to read from the tableau a half-infinite *ray* of feasible points on which the objective function goes to infinity.

(b) Solve the following LP using the Simplex method (starting from the origin):

$$\begin{aligned} \text{maximize} \quad & x_2 \\ \text{subject to} \quad & -x_1 + x_2 \le 0 \\ & x_1 \le 2 \\ & x_1, x_2 \ge 0 \end{aligned}$$

(Be sure to persevere in the first step.) Illustrate with a picture, and state all intermediate basic feasible solutions and objective values.

(c) Show the execution of the Simplex method on the below LP. (Suggestion: use Mathematica/Maple to help you through the boring computations.) Use Dantzig's pivot rule: choose the improving variable with largest coefficient, pivot to increase its value as much as possible, break ties in favor of the equation (with negative coefficient for the improving variable) whose basic variable has least index. When something goes wrong, stop and identify what it is.

$$\begin{aligned} \text{maximize} \quad & 10x_1 - 57x_2 - 9x_3 - 24x_4 \\ \text{subject to} \quad & .5x_1 - 5.5x_2 - 2.5x_3 + 9x_4 \le 0 \\ & .5x_1 - 1.5x_2 - .5x_3 + x_4 \le 0 \\ & x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

[1] Thanks to Kolman and Beck for the problem.