

0. Exercise. Consider the feasible region K defined by the following constraints:

$$\begin{aligned}-5x_1 + 3x_2 &\leq 5 \\ x_1 - 2x_2 &\leq 4 \\ x_1 + 2x_2 &\leq 12 \\ -x_1 + x_2 &\leq 3 \\ x_1, x_2 &\geq 0\end{aligned}$$

- (a) What are the vertices of this feasible region?
- (b) What is the maximizer for the cost vector $c^T = (3, 1)$? What about $c^T = (1, 3)$?
- (c) Which of the basic feasible solutions are degenerate? (Recall these are solutions $x \in \mathbb{R}^n$ —where $n = 2$ —such that more than n constraints are tight at x .)

Now add in slack variables x_3, x_4, x_5, x_6 to these four constraints to obtain a linear program in equational form.

- (d) What is the basic feasible solution corresponding to $B = \{3, 4, 5, 6\}$? How about $B = \{1, 2, 3, 4\}$ and $B = \{1, 2, 4, 5\}$? How about $B = \{1, 2, 3, 6\}$?
- (e) Which of the basic feasible solutions are degenerate? (Recall these are solutions $x \in \mathbb{R}^n$ which contain more than $n - m$ zeroes, where $n = 6$ and $m = 4$ in this case.)
- (f) Obtain access to either Mathematica or Maple. You should be able to download it from

<http://www.cmu.edu/computing/software/all/index.html>.

If you are registered for the class but still cannot download the software, please let us know. Both can solve LPs either exactly (with rational numbers) or numerically. Using their exact LP-solving capability, solve problem (b). Try also to get them to assist you in solving (a)–(e) as well. (Use Google to find appropriate documentation.)