Induction Variables and

Strength Reduction

I. Overview of optimization

II. Algorithm to find induction variables

[ALSU 9.1.8]

Example

```
FOR i = 0 to 100
   A[i] = 0;
```

Induction variables:
`t1 = 4i`
`t2 = 4i + &A`

```
     i = 0
L2:  IF i>=100 GOTO L1
     t1 = 4 * i
     t2 = &A + t1
     *t2 = 0
     i = i+1

     GOTO L2
L1:
```

```
t1’ = 0
t2’ = &A


~~t1 = t1’~~
t2 = t2’

t1’ = t1’+4
t2’ = t2’+4
```

```
     t2’ = &A
     t3’ = &A + 400
L2:  IF t2’>=t3’ GOTO L1
     *t2’= 0
     t2’ = t2’+ 4
     GOTO L2
L1:
```

Definitions

- A **basic induction variable** is
  - a variable X whose only definitions within the loop are assignments of the form:

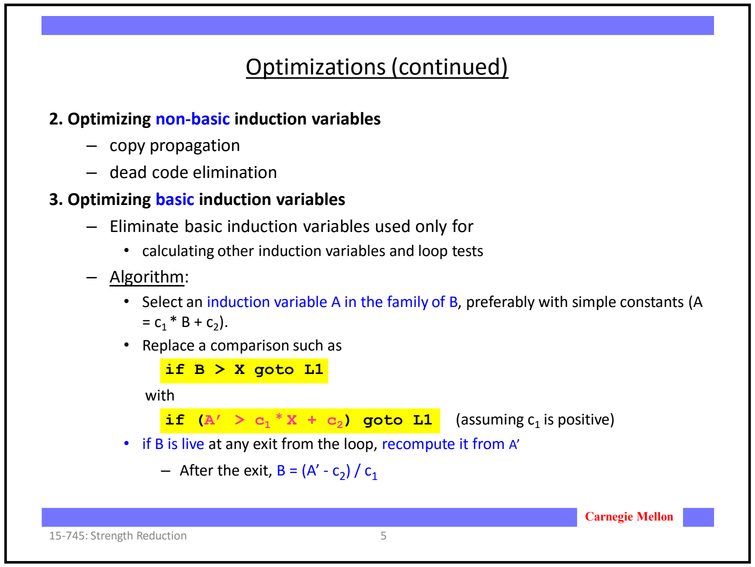
    X = X+c or X = X-c,

    where c is either a constant or a loop-invariant variable. (e.g., `i`)
- An **induction variable** is
  - a basic induction variable, or
  - a variable defined once within the loop, whose value is a linear function of some basic induction variable at the time of the definition:

    $A = c_1 * B + c_2$ (e.g., `t1`, `t2`)
- The **FAMILY of a basic induction variable B** is
  - the set of induction variables A such that each time A is assigned in the loop, the value of A is a linear function of B. (e.g., `t1`, `t2` in family of `i`)

Optimizations

1. **Strength reduction:**
   - A is an induction variable in family of basic induction variable B ($A = c_1 *B + c_2$)
     - Create new variable: A’
     - Initialization in preheader: $A’= c_1 * B + c_2$;
     - Track value of B: add after B=B+x: $A’=A’+x*c_1$;
     - Replace assignment to A: A=A’



Optimizations (continued)

2. Optimizing non-basic induction variables

- copy propagation
- dead code elimination

3. Optimizing basic induction variables

- Eliminate basic induction variables used only for
  - calculating other induction variables and loop tests
- Algorithm:
  - Select an induction variable A in the family of B, preferably with simple constants (A = c_1 * B + c_2).
  - Replace a comparison such as

    if B > X goto L1

    with

    if (A' > c_1 * X + c_2) goto L1 (assuming c_1 is positive)
  - if B is live at any exit from the loop, recompute it from A'
    - After the exit, B = (A' - c_2) / c_1

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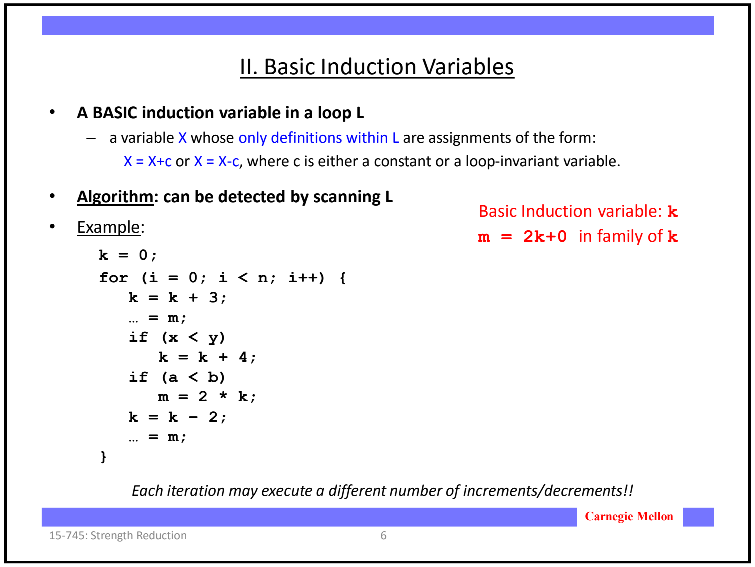

II. Basic Induction Variables

- **A BASIC induction variable in a loop L**
  - a variable X whose only definitions within L are assignments of the form:

    X = X+c or X = X-c, where c is either a constant or a loop-invariant variable.
- **Algorithm: can be detected by scanning L**
- Example:

Basic Induction variable: **k**

m = 2k+0 in family of **k**

```
k = 0;
for (i = 0; i < n; i++) {
   k = k + 3;
   … = m;
   if (x < y)
       k = k + 4;
   if (a < b)
       m = 2 * k;
   k = k – 2;
   … = m;
}
```

Each iteration may execute a different number of increments/decrements!!

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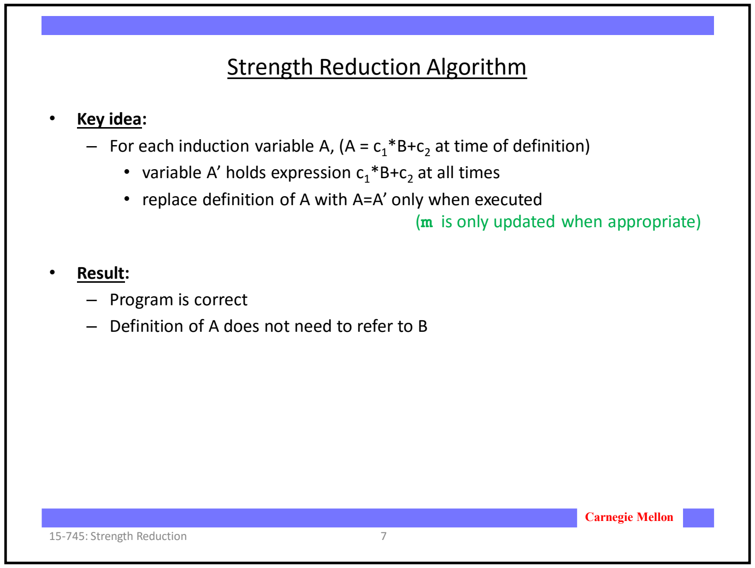

Strength Reduction Algorithm

- **Key idea:**
  - For each induction variable A, (A = c_1*B+c_2 at time of definition)
    - variable A' holds expression c_1*B+c_2 at all times
    - replace definition of A with A=A' only when executed

      (**m** is only updated when appropriate)
- **Result:**
  - Program is correct
  - Definition of A does not need to refer to B

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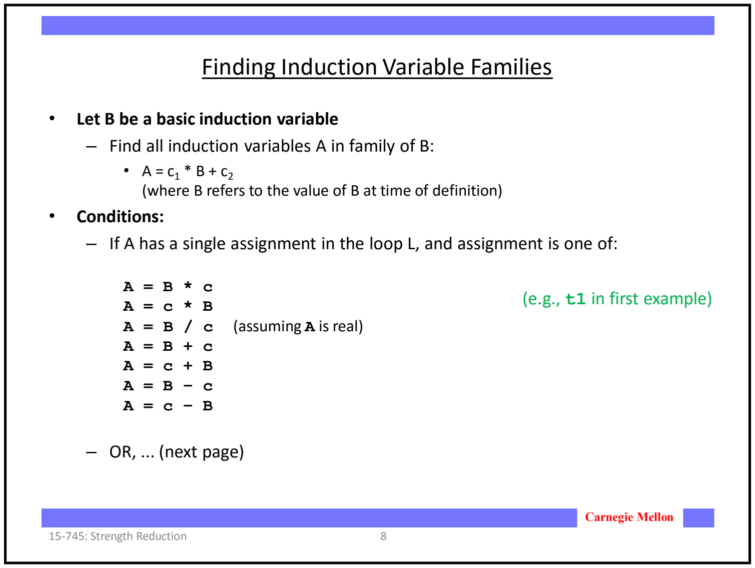

Finding Induction Variable Families

- **Let B be a basic induction variable**
  - Find all induction variables A in family of B:
    - A = c_1 * B + c_2

      (where B refers to the value of B at time of definition)
- **Conditions:**
  - If A has a single assignment in the loop L, and assignment is one of:

```
A = B * c
A = c * B
A = B / c     (assuming A is real)
A = B + c
A = c + B
A = B – c
A = c – B
```

(e.g., **t1** in first example)

  - OR, ... (next page)

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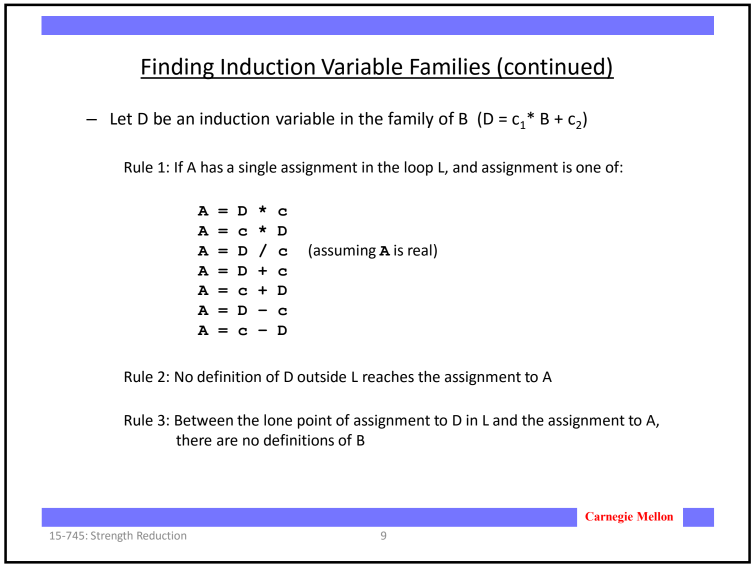


Finding Induction Variable Families (continued)

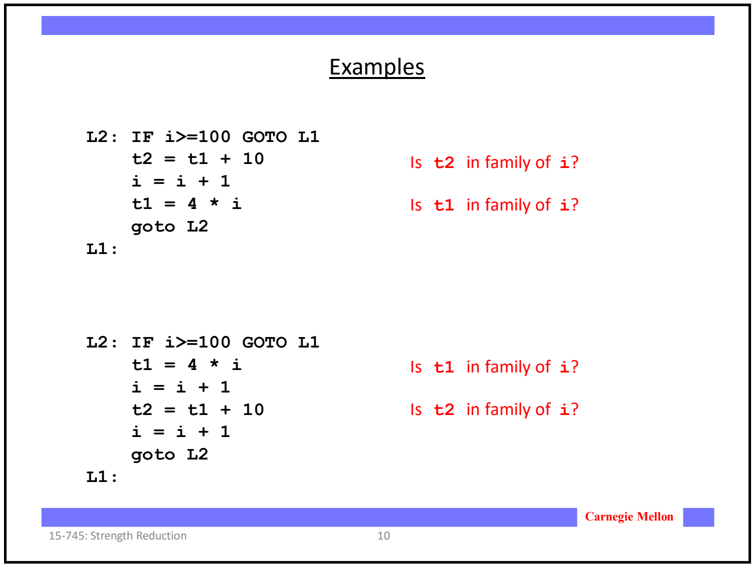
– Let D be an induction variable in the family of B (D = c_1* B + c_2)

Rule 1: If A has a single assignment in the loop L, and assignment is one of:

```
A = D * c
A = c * D
A = D / c   (assuming A is real)
A = D + c
A = c + D
A = D – c
A = c – D
```

Rule 2: No definition of D outside L reaches the assignment to A

Rule 3: Between the lone point of assignment to D in L and the assignment to A, there are no definitions of B



Examples

```
L2: IF i>=100 GOTO L1
    t2 = t1 + 10
    i = i + 1
    t1 = 4 * i
    goto L2
L1:
```

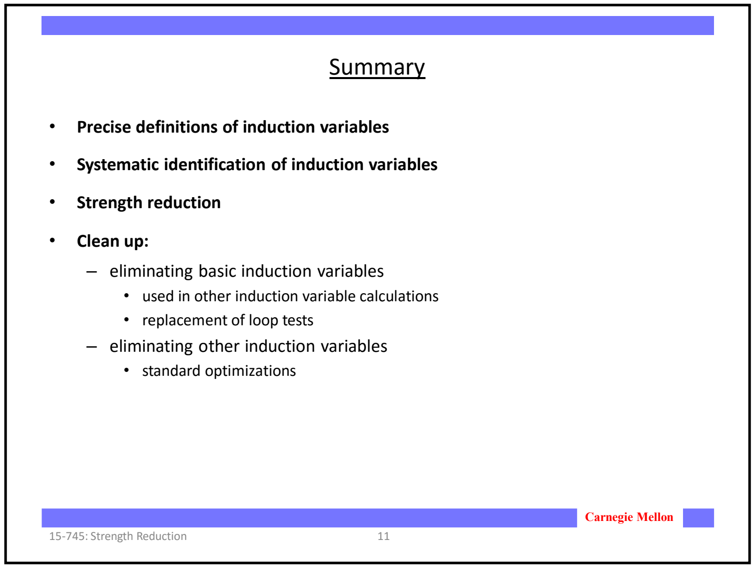
Is t2 in family of i?

Is t1 in family of i?

```
L2: IF i>=100 GOTO L1
    t1 = 4 * i
    i = i + 1
    t2 = t1 + 10
    i = i + 1
    goto L2
L1:
```

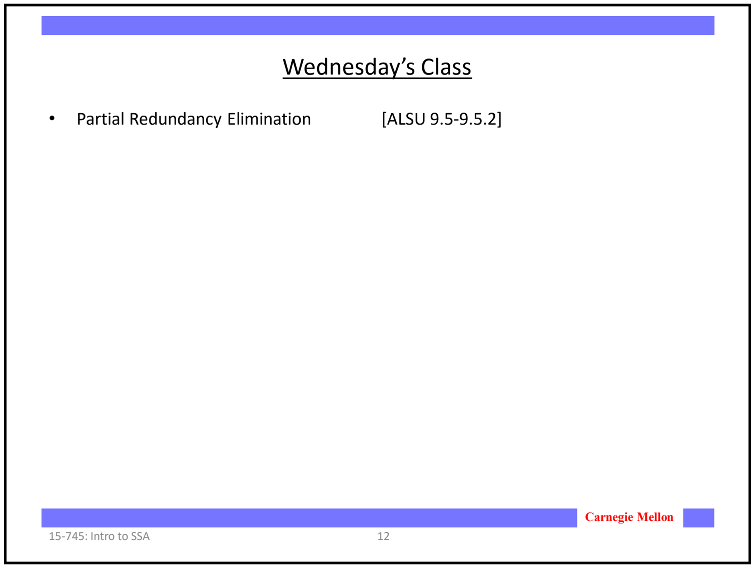
Is t1 in family of i?

Is t2 in family of i?



Summary

- **Precise definitions of induction variables**
- **Systematic identification of induction variables**
- **Strength reduction**
- **Clean up:**
  - eliminating basic induction variables
    - used in other induction variable calculations
    - replacement of loop tests
  - eliminating other induction variables
    - standard optimizations



Wednesday's Class

- Partial Redundancy Elimination [ALSU 9.5-9.5.2]