15-381 Homework 6

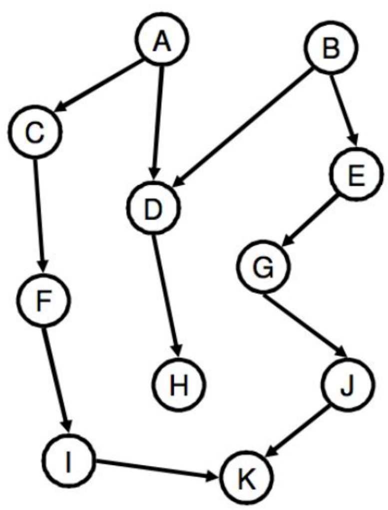
Submitted by: 11/8/2018 11:59PM

1. Hosting Tournament (10 points)

You are hosting a tournament with n players in a **circle**. Each player has a ***biased*** coin. The probability of getting a Head from all their coins are the same. Each round, all player flip their coins simultaneously. For each player, if it is a **Head**, he/she turns to his/her **left** side. If it is a **Tail**, he/she turns to his/her **right** side. After each round, if there are two players facing each other, then the person who got **Tail** will leave the circle. Then, the next round begins. The tournament continues until there is only 1 player left.

What's the probability you want to assign to the coin if you want the tournament to end fast? Explain. (Hint: you want to maximize the expected number of matches).

2. Bayes Net Independence (6 points)

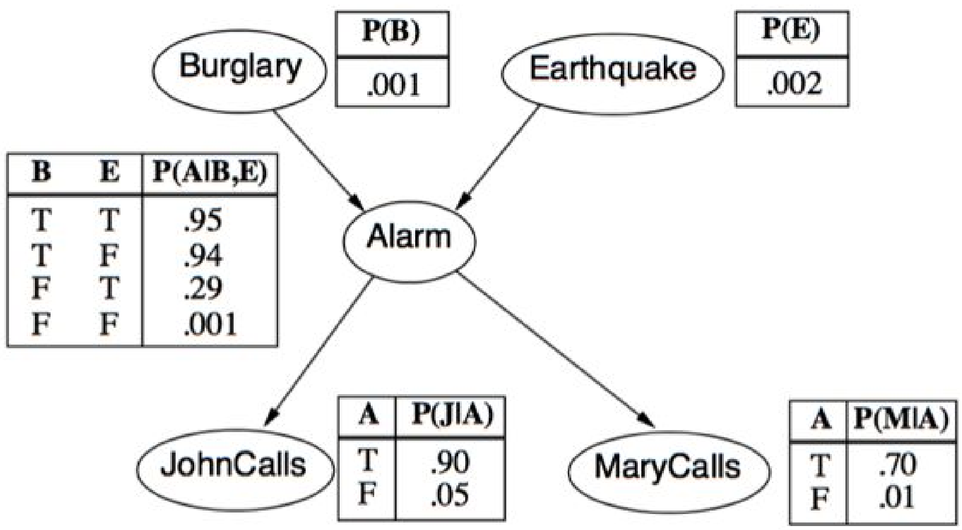


Using the above Bayes net, **state** whether the following statement is True or False.
(You may find this tutorial very helpful: http://bayes.cs.ucla.edu/BOOK-2K/d-sep.html)
1. P(A, F|C, H, K) = P(A|C, H, K)P(F|C, H, K) (2 points)
2. P(E, F|D, I, J) = P(E|D, I, J)P(F|D, I, J) (2 points)
3. P(E, F|B, I, K) = P(E|B, I, K)P(F|B, I, K) (2 points)
(6 Bonus points) **Explain** your answers.

3 The Burglar Scenario (12 points)

Your alarm at home is set off normally when there is a burglary. It might also be set off sometimes when there is an earthquake. You are at work. When the alarm is set off, your neighbors, John and Mary, will normally call you. However, sometimes only one neighbor might call.

In the following graph, P(B) describes the probability that a burglary happens, and P(E) is the probability of an earthquake. P(A|B,E) describes the probability that the alarm set off given whether there is burglary or earthquake. P(J|A) is the probability that John calls when your alarm is set off, and P(M|A) is the probability that Mary calls when your alarm is set off.



When no evidence is given, the Burglary and Earthquake variables are independent. However, if we observe Alarm = true, things change. In particular, Burglary and Earthquake are no longer conditionally independent. Explain why. Justify your answer by computing the appropriate probabilities involved. Hint: you may want to evaluate P(B, E|a), P(B|a), P(E|a).

4. Hidden Markov Model (16 points)

Rachel's course workload got pretty heavy recently, which has made her stay up for several nights. Her roommate, Jenny, cares a lot about whether Rachel gets any sleep each night. She observed that Rachel is more likely to drink coffee in the morning of day *i+1* if she does not sleep on day i. The table shows her probability of having coffee with/without sleep.

	Have Coffee on day i+1	No Coffee on day i+1
Sleep on day i	0.1	0.9
No Sleep on day i	0.7	0.3

Also, Jenny observed that if Rachel does not sleep on day i, with probability 0.3, she will not sleep on day i+1. However, if she sleeps on day i, then the probability that she will not sleep on day i+1 is 0.5.

We let random variable $S_i = True$ to represent that she sleeps on day i, and $False$ if she does not sleep on day i. We let $C_i = True$ to represent that she drinks coffee on day i+1, and $False$ otherwise. (This might be a little confusing. Whether or not Rachel has coffee on day i+1 is based on her sleep on day i, so we use C_i instead of C_{i+1} to represent it.)
Let's assume that given S_i, C_i and S_{i+1} are conditionally independent.

1). Which are the hidden states and observations? Please provide both the notations and some descriptions. Which are the transition probabilities and emission probabilities? Please provide both the notations and the numbers. (4 points)
2). Draw an HMM diagram to model the scenario. Please clearly label the transition and emission probabilities with arrows. (5 points)
3). On Day 0, the probability that Rachel does not sleep is **0.5**. What is the most probable sleep pattern for the following three days (Day 1, 2 and 3) if $C_1 = True$, $C_2 = False$ and $C_3 = True$ (she only had coffee on Day 2 and Day 4)? (An example sleep pattern is "Sleep, No sleep, Sleep".) Please show all the calculation steps. You can include a diagram if that helps to illustrate your answer. You do not need to normalize the probabilities. (7 points)

5. Scalar Kalman Filters (4 points)

In this question, we will talk about a simplified version of the Kalman filter.

Imagine you want to predict temperature on day m x^m .

First, we have a rough estimate on how temperature changes from the previous day m-1:

$x^m = ax^{m-1} + w^{m-1}$ -- (1)

where *a* is a non-negative scale factor, and *w* is the noise in prediction. $w \sim N(0, Q)$

Based on this estimate, we derive the following a priori estimate (Note: The derivation is straightforward. We are not asking to derive them. Just assume they are correct.):

mean: $\widehat{x}^m = ax^{m-1}$ -- (2)

covariance: $\widehat{p}^m = a^2\widehat{p}^{m-1}$ + Q -- (3)

At the same time, we also have a crude thermometer reading of the temperature ***on day m*** z^m .

z^m is not accurate, and it relates to x^m by this equation:

$z^m = h\,x^m + v^m$ -- (4)

where h is a non-negative scale factor and

v is the noise/inaccuracy in measurement: $v \sim N(0, R)$

Note that *w* and *v* are assumed to be independent (of each other).

Based on the equation of Kalman filter, we can gain a more accurate a posteriori prediction of temperature ***on day m*** $\tilde{x}^m$ by the following equation (Note: Again, the derivation is straightforward. We are not asking to derive them. Just assume they are correct.):a

$$k_m = \frac{h\widehat{p}^m}{h^2\hat{p}^m + R} \qquad \text{--- (5)}$$

$$\tilde{x}^m = \hat{x}^m + k_m(z^m - h\hat{x}^m) \qquad \text{--- (6)}$$

1. When the thermometer reading is always accurate (R = 0), show that $\tilde{x}^m = z^m/h$. (3 points)
2. Assume $\hat{p}^0 = a = h = 1$, When R >> Q (R is significantly larger than Q), show that k_3 will be very small. (3 points)

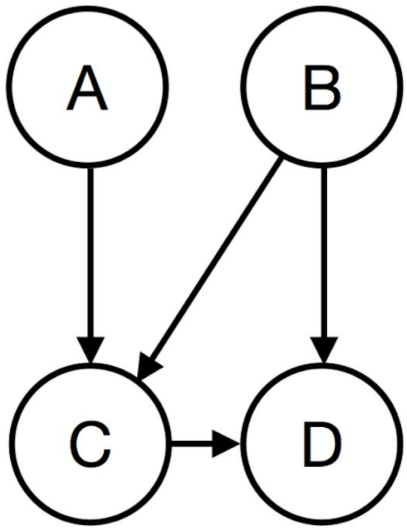
6. Programming Question (50 points):

6.1 The Problem

Your task here is to implement three types of sampling techniques to perform approximate inference on any given Bayes Net: Rejection Sampling, Likelihood Sampling, and Gibbs Sampling. An example Bayes Net is shown below:

P(A)	
+A	0.0
-A	1.0

P(B)	
+B	0.9
-B	0.1



P(C | A,B)			
+A	+B	+C	0.2
+A	+B	-C	0.8
+A	-B	+C	0.6
+A	-B	-C	0.4
-A	+B	+C	0.5
-A	+B	-C	0.5
-A	-B	+C	0.0
-A	-B	-C	1.0

P(D | B,C)			
+B	+C	+D	0.75
+B	+C	-D	0.25
+B	-C	+D	0.1
+B	-C	-D	0.9
-B	+C	+D	0.5
-B	+C	-D	0.5
-B	-C	+D	0.2
-B	-C	-D	0.8

You will develop codes to compute conditional probabilities, such as P(d), P(d|c), P(b|c), or P(d| −a, b), where d = (D=T) = (+D) in our convention. Note that we only have one query variable, and an arbitrary number of evidence variables.

6.2 Format

Please use Python 2.7 to complete this assignment, in a file named sampling.py. We have given you some starter code (in the Autolab handout) that provides a working solution for direct sampling (ignores evidence variables). In this file, please fill in the three methods, one for each sampling technique. The specifics of what each line of the input and output should be is described below.

6.2.1 Input

The format of the input file is as follows:

1. The first line of every file contains the names of all nodes in the graph, in topological (& lexicographical) order.

2. The lines that follow until the first blank line contain conditional probabilities to describe the local tables of the Bayes Net. These will always be given to describe the distribution in topological (lexicographical) order. Each line contains zero or more conditions (denoted by a +/- followed by a variable name, then a space), the dependent variable (denoted with just the variable name and a space), then the probability as a float.

3. Finally, after a single empty line, the file will contain a single query, which asks the probability of a particular variable, given the evidence that follows.

For example, the input file for the above example would look like the following:

```
A B C D
A 0.0
B 0.9
+A +B C 0.2
+A -B C 0.6
-A +B C 0.5
-A -B C 0.0
+B +C D 0.75
+B -C D 0.1
-B +C D 0.5
-B -C D 0.2

D +C
```

Note that the query corresponds to P(D|C). The sign of the variable that you have to compute is always positive.

6.2.2 Output

Please print your output to stdout. For each test case, print a ***single floating point*** number. This number should represent the value of the corresponding test case.

6.3 Method

Please refer to the textbook or lectures for implementation details with regards to each sampling technique. If needed, you can compute the conditional probability of a variable given its Markov blanket using the following equation:

$$P(x_i'|mb(X_i)) = \alpha P(x_i'|parents(X_i)) \times \prod_{Y_j \in Children(X_i)} P(y_j|parents(Y_j))$$

In each sampling method, make sure to follow the topological variable order. A specific `random.seed(10000)` has been provided in the starter code, please keep this seed when you are submitting to autolab. Additionally, you should only generate random assignments using the sample function provided; this is to help ensure the random samples generated will be identical to that of the autograder.

For Gibb's sampling, please initialize all non-evidence variables to ***True***.

6.4 Testing

To run your code, you should run python sampling.py . trials refers to the number of samples you want to generate, and sampling type can be 0,1,2, or 3 depending on which sampling method you want to use.

When you submit to Autolab, we will compare your results with the results generated from the reference solution on several different Bayes Nets and queries. Because we have provided the specific random seed, we expect the output to be nearly identical to the reference output (we will check up to 5 decimal places). If you find that your code seems to be working correctly (i.e. correct answers from section 1.4) but the autograder is not passing, be sure that you are looping through the variables in topological order. If you think it is some precision error in Python, please post on Piazza.

The Autolab tests will generate your final score. There are currently no restrictions on the number of Autolab submissions.

6.5 Submission

Please submit to Autolab a tar file that contains the file named sampling.py and written.pdf. The written.pdf includes your solution to the questions 1-5. To create a tar file, you may run the following command on the same directory as your sampling.py and written.pdf files:

```
tar cvzf handin.tar sampling.py written.pdf
```