## Measurement \& Performance

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Topics:

- Timers
- Performance measures
- Relating performance measures
-system perfomance measures
- latency and throughput
- Amdahl's law


## The Nature of Time


$=$ System Time: time spent executing instructions in the kernel on behalf of the user process


Unless otherwise specified, "time" often refers to "user time".

## Anatomy of a Timer



A counter value ( T ) is updated upon discrete ticks

- a tick occurs once every $\Delta$ time units
- upon a tick, the counter value is incremented by $\Delta$ time units Some Terminology:
- timer period $=\Delta$ seconds $/$ tick
- timer resolution = $1 / \Delta$ ticks $/$ second


## Using Timers



Estimating elapsed time:

- based on discrete timer values before $\left(\mathrm{T}_{\mathrm{s}}\right)$ and after $\left(\mathrm{T}_{\mathrm{f}}\right)$ the event

How close is $T_{\text {observed }}$ to $T_{\text {actual }}$ ?

## Timer Error: Example \#1



$$
\begin{aligned}
& \mathrm{T}_{\text {actual }}: \sim 2 \Delta \\
& \mathrm{~T}_{\text {observed }}: \Delta
\end{aligned}
$$

Absolute measurement error: ~ $\Delta$
Relative measurement error: $\sim \Delta / 2 \Delta=\sim 50 \%$

## Timer Error: Example \#2



$$
\begin{aligned}
& \mathrm{T}_{\text {actual }}: \varepsilon(\sim \text { zero }) \\
& \mathrm{T}_{\text {observed }}: \Delta
\end{aligned}
$$

Absolute measurement error: ~ $\Delta$
Relative measurement error: $\sim \Delta / \varepsilon=\sim$ infinite

## Timer Error: Example \#3


$T_{\text {actual }}: X$
$T_{\text {observed }}: 0$

Absolute measurement error: $\mathbf{X}$
Relative measurement error: $X$ / $X=100 \%$

## Timer Error: Summary



Absolute measurement error: +/- $\Delta$
Key point: need a large number of ticks to hide error

- can compute $\mathrm{T}_{\text {threshold }}$ as a function of $\Delta$ and $E$
- $\mathrm{T}_{\text {threshold }}=$ minimum observed time to guarantee relative error bound
- $E$ = maximum acceptable relative measurement error


## Homework 1 Timer Package

## Unix interval countdown timer

- decrements timer value by $\Delta$ every $\Delta$ seconds
- setitimer(): initialize timer value
- getitimer(): sample timer value
- measures user time
"etime" package:
- based on Unix interval timers
- set_etime(): initializes timer
- get_etime(): returns elapsed time in seconds since last call to set_etime()


## Performance expressed as a time

## Absolute time measures

- difference between start and finish of an operation
- synonyms: running time, elapsed time, response time, latency, completion time, execution time
- most straightforward performance measure

Relative (normalized) time measures

- running time normalized to some reference time
- (e.g. time/reference time)

Guiding principle: Choose performance measures that track running time.

## Performance expressed as a rate

## Rates are performance measures expressed in units of work per unit time.

## Examples:

- millions of instructions / sec (MIPS)
- millions of floating point instructions / sec (MFLOPS)
- millions of bytes / sec (MBytes/sec)
- millions of bits / sec (Mbits/sec)
- images / sec
- samples / sec
- transactions / sec (TPS)


## Performance expressed as a rate(cont)

Key idea: Report rates that track execution time.

Example: Suppose we are measuring a program that convolves a stream of images from a video camera.

Bad performance measure: MFLOPS

- number of floating point operations depends on the particular convolution algorithm: n^2 matix-vector product vs nlogn fast Fourier transform. An FFT with a bad MFLOPS rate may run faster than a matrix-vector product with a good MFLOPS rate.

Good performance measure: images/sec

- a program that runs faster will convolve more images per second.


## Performance expressed as a rate(cont)

Fallacy: Peak rates track running time.
Example: the i860 is advertised as having a peak rate of 80 MFLOPS ( 40 MHz with 2 flops per cycle).

However, the measured performance of some compiled linear algebra kernels (icc -O2) tells a different story:

| Kernel | 1d fft | sasum | saxpy | sdot | sgemm | sgemv | spvma |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| MFLOPS | 8.5 | 3.2 | 6.1 | 10.3 | 6.2 | 15.0 | 8.1 |
| \%peak | $11 \%$ | $4 \%$ | $7 \%$ | $13 \%$ | $8 \%$ | $19 \%$ | $10 \%$ |

## Relating time to system measures

Suppose that for some program we have:

- T seconds running time (the ultimate performance measure)
- C clock ticks, I instructions, P seconds/tick (performance measures of interest to the system designer)

T secs = C ticks $x P$ secs/tick
$=(I$ inst/l inst) $\times C$ ticks $\times P$ secs/tick
T secs = I inst $\times$ (C ticks/l inst) $x P$ secs/tick


## Pipeline latency and throughput



Latency (L): time to process an individual image.
Throughput (R): images processed per unit time
One image can be processed by the system at any point in time

## Video system performance

$L=3$ secs/image.
$R=1 / L=1 / 3$ images/sec.
$T=L+(N-1) 1 / R$
$=3 N$


## Pipelining the video system



One image can be in each stage at any point in time.
$L_{i}=$ latency of stage $i$
$\mathbf{R}_{\mathrm{i}}=$ throughput of stage i
$L=L_{1}+L_{2}+L_{3}$
$R=\min \left(R_{1}, R_{2}, R_{3}\right)$

## Pipelined video system performance

| Suppose: |
| :--- |
| $L_{1}=L_{2}=L_{3}=1$ |
| Then: |
| $L=3$ secs/image. |
| $R=1$ image/sec. |
| $T=L+(N-1) 1 / R$ |
| $=N+2$ |

Stage 1 Stage 2 Stage 3


## Relating time to latency and thruput

In general:

- $T=L+(N-1) / R$

The impact of latency and throughput on running time depends on N :

- $(N=1)=>(T=L)$
- $(N \gg 1)=>(T=N-1 / R)$

To maximize throughput, we should try to maximize the minimum throughput over all stages (i.e., we strive for all stages to have equal throughput).

## Amdahl's law

You plan to visit a friend in Normandy France and must decide whether it is worth it to take the Concorde SST $(\$ 3,100)$ or a $747(\$ 1,021)$ from NY to Paris, assuming it will take 4 hours Pgh to NY and 4 hours Paris to Normandy.

|  | time NY->Paris | total trip time | speedup over 747 |
| :--- | :--- | :--- | :--- |
| 747 | 8.5 hours | 16.5 hours | 1 |
| SST | 3.75 hours | 11.75 hours | 1.4 |

Taking the SST (which is 2.2 times faster) speeds up the overall trip by only a factor of 1.4 !

## Amdahl's law (cont)

Old program (unenhanced)

| $\mathrm{T}_{1}$ | $\mathrm{~T}_{2}$ |
| :---: | :---: |

Old time: $\mathrm{T}=\mathrm{T}_{1}+\mathrm{T}_{2}$

New program (enhanced)

$$
\begin{array}{l|l}
\hline \mathrm{T}_{1}{ }^{\prime}=\mathrm{T}_{1} & \mathrm{~T}_{2}{ }^{\prime}<=\mathrm{T}_{2} \\
\hline
\end{array}
$$

New time: $\mathrm{T}^{\prime}=\mathrm{T}_{1}{ }^{\prime}+\mathrm{T}_{2}{ }^{\prime}$

Speedup: $\mathrm{S}_{\text {overall }}=\mathbf{T} / \mathrm{T}^{\prime}$
$\mathrm{T}_{1}=$ time that can NOT be enhanced.
$T_{2}=$ time that can be enhanced.
$\mathrm{T}_{2}{ }^{\prime}=$ time after the enhancement.

## Amdahl's law (cont)

## Two key parameters:

$$
\begin{array}{ll}
F_{\text {enhanced }}=T_{2} / T & \text { (fraction of original time that can be improved) } \\
S_{\text {enhanced }}=T_{2} / T_{2}^{\prime} & \text { (speedup of enhanced part) } \\
\left.\begin{array}{rl}
T^{\prime} & =T_{1}^{\prime}+T_{2}^{\prime}=T_{1}+T_{2}^{\prime}=T\left(1-F_{\text {enhanced }}\right)+T_{2}^{\prime} \\
& =T\left(1-F_{\text {enhanced }}\right)+\left(T_{2} / S_{\text {enhanced }}\right) \\
& =T\left(1-F_{\text {enhanced }}\right)+T\left(F_{\text {enhanced }} / S_{\text {enhanced }}\right) \\
& =T\left(\left(1-F_{\text {enhanced }}\right)+F_{\text {enhanced }} / S_{\text {enhanced }}\right)
\end{array} \quad \text { [by def of } S_{\text {enhanced }}\right]
\end{array}
$$

## Amdahl's Law:

$$
S_{\text {overall }}=T / T^{\prime}=1 /\left(\left(1-F_{\text {enhanced }}\right)+F_{\text {enhanced }} / S_{\text {enhanced }}\right)
$$

Key idea: Amdahl's law quantifies the general notion of diminishing returns. It applies to any activity, not just computer programs.

## Amdahl's law (cont)

Trip example: Suppose that for the New York to Paris leg, we now consider the possibility of taking a rocket ship ( 15 minutes) or a handy rip in the fabric of space-time ( 0 minutes):

|  | time NY->Paris | total trip time | speedup over 747 |
| :--- | :--- | :--- | :--- |
| 747 | 8.5 hours | 16.5 hours | 1 |
| SST | 3.75 hours | 11.75 hours | 1.4 |
| rocket | 0.25 hours | 8.25 hours | 2.0 |
| rip | 0.0 hours | 8 hours | 2.1 |

## Amdahl's law (cont)

## Useful corollary to Amdahl's law:

- $1<=S_{\text {overall }}<=1 /\left(1-F_{\text {enhanced }}\right)$

| $F_{\text {enhanced }}$ | Max $S_{\text {overall }}$ | $F_{\text {enhanced }}$ | Max $S_{\text {overall }}$ |
| :--- | :--- | :--- | :--- |
| 0.0 | 1 | 0.9375 | 16 |
| 0.5 | 2 | 0.96875 | 32 |
| 0.75 | 4 | 0.984375 | 64 |
| 0.875 | 8 | 0.9921875 | 128 |

Moral: It is hard to speed up a program.
Moral++ : It is easy to make premature optimizations.

