Constructive Logic
Fall 2000

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Assignment 9

Out: Tuesday Nov 21

Due: Thursday Nov 30

1. Verification of the equivalence of circuits

In the following we present two standard ways to implement boolean functions by digital circuits. In both cases the implemented function is

$$f(x_1, x_2, x_3) \leftrightarrow \text{ exactly one of } \{x_1, x_2, x_3\} \text{ is } 1$$

The funtion f is the *exclusive or* for three arguments. The first implementation will be via a conjunctive normal form and the second will be via a disjunctive normal form of the boolean formula denoted by f. We will show that the resulting circuits are equivalent by showing that their OBDDs are the same. In all OBDDs keep the variable ordering $[x_1, x_2, x_3]$.

A) Conjunctive normal form

A *literal* is a disjunction of variables or their negations. A formula is in *conjunctive normal form* (CNF) if it is a conjunction of literal. In our case the literals are:

$$\begin{aligned} A &= x_1 + x_2 + x_3 \\ B &= \overline{x}_1 + \overline{x}_2 \\ C &= \overline{x}_1 + \overline{x}_3 \\ D &= \overline{x}_2 + \overline{x}_3 \end{aligned}$$

The CNF representation of f is

$$f(x_1, x_2, x_3) = A \cdot B \cdot C \cdot D$$

1. Construct and the OBDDs for A, B, C and D
2. Construct and simplify the OBDDs for
   - $B \cdot C$
   - $(B \cdot C) \cdot D$
   - $A \cdot (B \cdot C) \cdot D$

B) Disjunctive normal form

A *monom* is a conjunction of variables or their negations. A formula is in *disjunctive normal form* (DNF) is it is a disjunction of monoms. In our case the monoms are

$$\begin{aligned} A &= x_1 \cdot \overline{x}_2 \cdot \overline{x}_3 \\ B &= \overline{x}_1 \cdot x_2 \cdot \overline{x}_3 \\ C &= \overline{x}_1 \cdot \overline{x}_2 \cdot x_3 \end{aligned}$$

The DNF representation of f is

$$f(x_1, x_2, x_3) = A + B + C$$

1. Construct the OBDDs for A, B and C
2. Construct and simplify the OBDDs for
   - A + B
   - (A + B) + C

Now compare the resulting OBDD with the one from A)!

2. Optimal OBDDs

Find an ordering on the variables $x_1, \ldots, x_4$ s.th. the OBDD for the following DNF-formula is optimal:

$$A = x_1 x_2 + x_3 x_4 + x_2 \overline{x}_4 + \overline{x}_2 x_4$$

Hint: The solution has 5 variable nodes.

Have a nice thanksgiving!