Constructive Logic
Fall 2000

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Assignment 10

Out: Tuesday Dec 5

Due: Tuesday Dec 12

1. Encoding of Hamiltonian Cycles in SAT (30 Points)

We say that a directed graph G contains a Hamiltonian cycle if there is a path through the graph, starting and ending at the same node, such that it contains every node of the graph exactly once. In this problem we reduce the question if a given graph has a Hamiltonian cycle to SAT.
Let $1, 2, \ldots, n$ be the nodes of G. We represent the edges of G by Boolean variables

$$x_{ij} = 1 \quad \text{iff} \quad \text{there is an edge from } i \text{ to } j \text{ in } G$$

We represent a path P of length m in G by Boolean variables

$$p_{ij} = 1 \quad \text{iff} \quad \text{the } i\text{th node of path } P \text{ is } j$$

A path P is a Hamiltonian cycle in G if it has length $n + 1$ and

(a) two consecutive nodes are connected by an edge,

(b) the first and last node of the path coincide, and

(c) each node of G appears in the path.

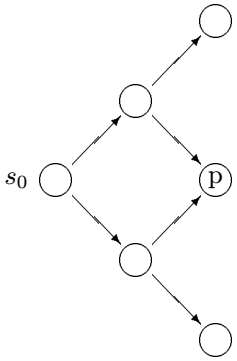
Give three Boolean formulas whose satisfiability is *equivalent* to conditions (a), (b), and (c), respectively. What is the Boolean formula f such that f is satisfiable if and only if the given graph has a Hamiltonian cycle? What is the order of the size of f as a function of n (e.g. linear, quadratic, ...)?

2. CTL (30 Points)

Under the assumption of excluded middle and finiteness, any two propositions are either equivalent, or we can exhibit a system in which one is true in the initial state but not the other. For example, considering

$$\text{AX EX } p \quad \text{and} \quad \text{EX AX } p$$

we find that they are not equivalent because for the system



we have that AX EX p is true at s_0 while EX AX p is false at s_0.
For each of the following pairs of formulas, indicate if they are equivalent or exhibit a system in which one is true but not the other.

(a) EF p and EG p

(b) EF $p \vee$ EF q and EF $(p \vee q)$

(c) AF $p \vee$ AF q and AF $(p \vee q)$

(d) AF $\neg p$ and $\neg$EG p

(e) EF $\neg p$ and $\neg$AF p

(f) A[p_1 U A[p_2 U p_3]] and A[A[p_1 U p_2] U p_3]

(g) $\top$ and AG $p \supset$ EG p

(h) $\top$ and EG $p \supset$ AG p

3. Model Checking (40 Points)

Consider the system with the following states

State	Atomic Labels	Successor States
s_0	p, q	s_1, s_3
s_1	r	s_1, s_2
s_2	p, t	s_0, s_3
s_3	q, r	s_0

Carry out the model-checking algorithm to see in which states the following four formulas hold. You should show the final picture of the algorithm for each formula and also list the states in which the given formula is true.

(a) AF q

(b) AG (EF $(p \vee r)$)

(c) EX (EX r)

(d) AG (AF q)