Homework 6

Due Thursday, November 3

1. Do problem 1 on page 72. In the language L, use the symbols P, T, S, and c for addition, multiplication, successor ("+1"), and 0 respectively.
2. Do problems 2 and 3 on page 72.
3. Fix a language, L, which has one binary relation symbol, R. Which of the following statements are true and which are false? Justify your answers.
   (a) If φ is any sentence, either $\models \varphi$ or $\models \neg\varphi$.
   (b) If φ is any sentence and $\mathcal{A}$ is any structure, either $\mathcal{A} \models \varphi$ or $\mathcal{A} \models \neg\varphi$.
   (c) If φ is any sentence and Γ is any set of sentences, then either $\Gamma \models \varphi$ or $\Gamma \models \neg\varphi$.
4. Do problems 2 and 4 on page 80.
5. Do problems 14a and 14c on page 81.

⋆ 6. Consider the equivalence

$$\exists x(\varphi(x) \wedge \psi(x)) \leftrightarrow (\exists x\varphi(x) \wedge \exists x\psi(x)).$$

   (a) Show that one direction of this equivalence is valid (i.e. true in every structure). Prove this carefully; you can use Lemma 2.4.5.
   (b) Find examples of φ and ψ where the other direction is not valid (and justify this claim).