Homework 5

Due Thursday, October 27

1. Consider a first-order language, with relation symbols $<$ and $=$, and constant symbol 0. The intended interpretation is the natural numbers, with "less-than" and "equality." Formalize the following statements:
   (a) "x is less than or equal to y"
   (b) "0 is the smallest number"
   (c) "there is a smallest number"
   (d) "there is no largest number"
   (e) "every number has an immediate successor" (in other words, for every number, there is another one that is the "next largest")
   (f) "every number is greater than some (other) number"
   (g) "there is some number that every (other) number is greater than"
   (h) "3 is greater than 2"

2. Consider a first-order language, with predicate symbols M and G, and constant symbol s. The intended interpretation is all people, living or dead, with $M(x)$ meaning "x is mortal", $G(x)$ meaning "x is Greek", and s meaning Socrates. Formalize the following statements:
   (a) "if x is Greek, then x is mortal"
   (b) "all Greeks are mortal"
   (c) "some Greeks are mortal"
   (d) "no Greeks are mortal"
   (e) "no Greeks are immortal"
   (f) "Socrates is a mortal Greek, but there are some who are immortal"
   (g) "if anyone is mortal, Socrates is"
   (h) "if all Greeks are mortal, and Socrates is Greek, then Socrates is mortal"
   (i) "if some mortals are Greeks, then some Greeks are not Mortal"

3. Consider a first-order language, with a constant symbol a, a predicate symbol P, and a binary relation symbol R. Consider the two different interpretations:

   $\mathcal{A}$: $|\mathcal{A}|$ = all natural numbers, $a^{\mathcal{A}} = 0$, $P^{\mathcal{A}}$ = the even numbers, $R^{\mathcal{A}}$ = the "less than" relation.

   $\mathcal{B}$: $|\mathcal{B}|$ = all people, $a^{\mathcal{B}}$ = Socrates, $P^{\mathcal{B}}$ = Greeks, $R^{\mathcal{B}}$ = the "teacher of" relation.

   Find simple formulae which are: (i) true under both interpretations, (ii) under neither, (iii) under $\mathcal{A}$ only, (iv) under $\mathcal{B}$ only.

4. Do problem 1 on page 60 of van Dalen.

5. Do problem 4 on page 68 of van Dalen. In each case, just indicate whether the term is "free" or "not free" for the specified variable in the specified term, and carry out the substitution either way.

⋆ 6. Do problem 2 on page 60.