

HOMEWORK 4
Due Thursday, October 6

1. Do problems 1d, 2d, 3b on page 39 of van Dalen.
2. Do problem 1 and 5 on page 54 and 55 of van Dalen.
3. Do problem 1 on page 47 of van Dalen. If you claim the set is inconsistent, show that you can prove a contradiction from those assumptions. If you claim the set is consistent, demonstrate this by providing a valuation under which all the formulas are true. Explain why this shows that a set of formulas is consistent. (Hint: you may use completeness.)
4. Show that the system of natural deduction is *free from contradiction*, in the sense that not both $\vdash \varphi$ and $\vdash \neg\varphi$ hold, for any propositional formula φ . (Hint: you may use soundness.)
5. Do problem 6 on page 48 of van Dalen. (Note: what is to be shown is that if Γ is a consistent set of formulas such that for every formula φ , either $\varphi \in \Gamma$ or $\neg\varphi \in \Gamma$, then Γ is maximally consistent.)
- ★ 6. Define an *ultrafilter* of formulas to be a set Γ such that:
 - if $\varphi \in \Gamma$ and $\varphi \models \psi$, then $\psi \in \Gamma$,
 - if $\varphi \in \Gamma$ and $\psi \in \Gamma$, then $\varphi \wedge \psi \in \Gamma$,
 - $\neg\varphi \in \Gamma$ iff $\varphi \notin \Gamma$

Show that a maximally consistent set of sentences is the same thing as an ultrafilter.