Homework 3

Due Thursday, September 29

1. Use the semantic definitions to prove or find a counterexample to each of the following:
   (a) For every set of formulas Γ, every formula φ, and every formula ψ, if $\Gamma \models \varphi \wedge \psi$, then $\Gamma \models \varphi$ and $\Gamma \models \psi$.
   (b) For every set of formulas Γ, every formula φ, and every formula ψ, if $\Gamma \models \varphi \vee \psi$, then $\Gamma \models \varphi$ or $\Gamma \models \psi$.
2. (a) Show that $\{\rightarrow, \bot\}$ is a complete set of connectives.
   (b) Show that $\{\rightarrow, \vee, \wedge\}$ is not a complete set of connectives.
   (c) Conclude that $\{\rightarrow, \vee, \wedge, \leftrightarrow, \top\}$ is not a complete set of connectives. (Hint: define the last two in terms of the others.)
3. Do problem 10 on pages 28–29 of van Dalen.
4. Do problem 1 and 2 on page 39 of van Dalen. Remember that we are taking $\varphi \leftrightarrow \psi$ to abbreviate $(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$.

$\star$ 5. Do problem 11 on page 29.

$\star$ 6. Do problem 12 on page 29.