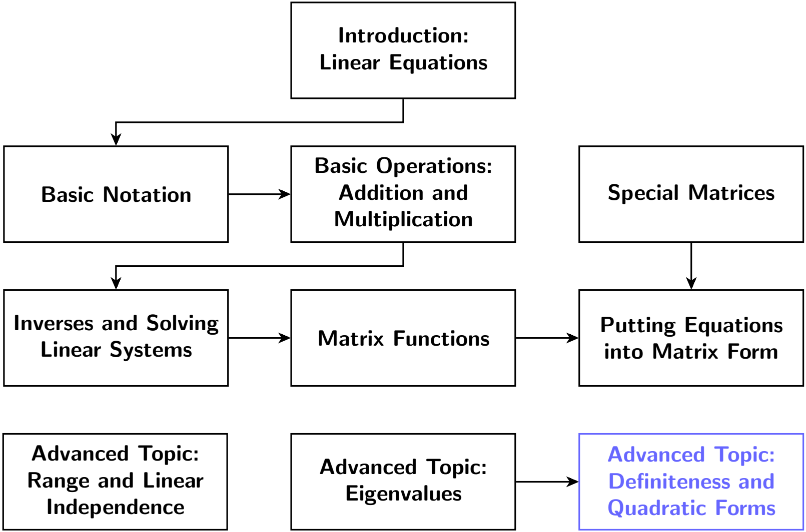
Linear Algebra Review



Quadratic Forms

- A quadratic form is a function $f : \mathbb{R}^n \to \mathbb{R}$

$$f(x) = x^T A x$$

  for some $A \in \mathbb{R}^{n \times n}$

- Can take A to be symmetric, since

$$x^T A x = (x^T A x)^T = x^T A^T x = x^T \frac{1}{2}(A + A^T) x$$

- $A \in \mathbb{R}^{n \times n}$ is positive definite (positive semidefinite) if $x^T A x > 0$ ($x^T A x \geq 0$) for all $x \in \mathbb{R}^n \neq 0$

- $A \in \mathbb{R}^{n \times n}$ is negative definite (negative semidefinite) if $x^T A x < 0$ ($x^T A x \leq 0$) for all $x \in \mathbb{R}^n \neq 0$

- A is indefinite if neither positive nor negative semidefinite

- Definiteness is characterized by eigenvalues of A
  - A positive definite $\Leftrightarrow \lambda_i > 0, \ \forall i$
  - A positive semidefinite $\Leftrightarrow \lambda_i \geq 0, \ \forall i$
  - A negative definite $\Leftrightarrow \lambda_i < 0, \ \forall i$
  - A negative semidefinite $\Leftrightarrow \lambda_i \leq 0, \ \forall i$