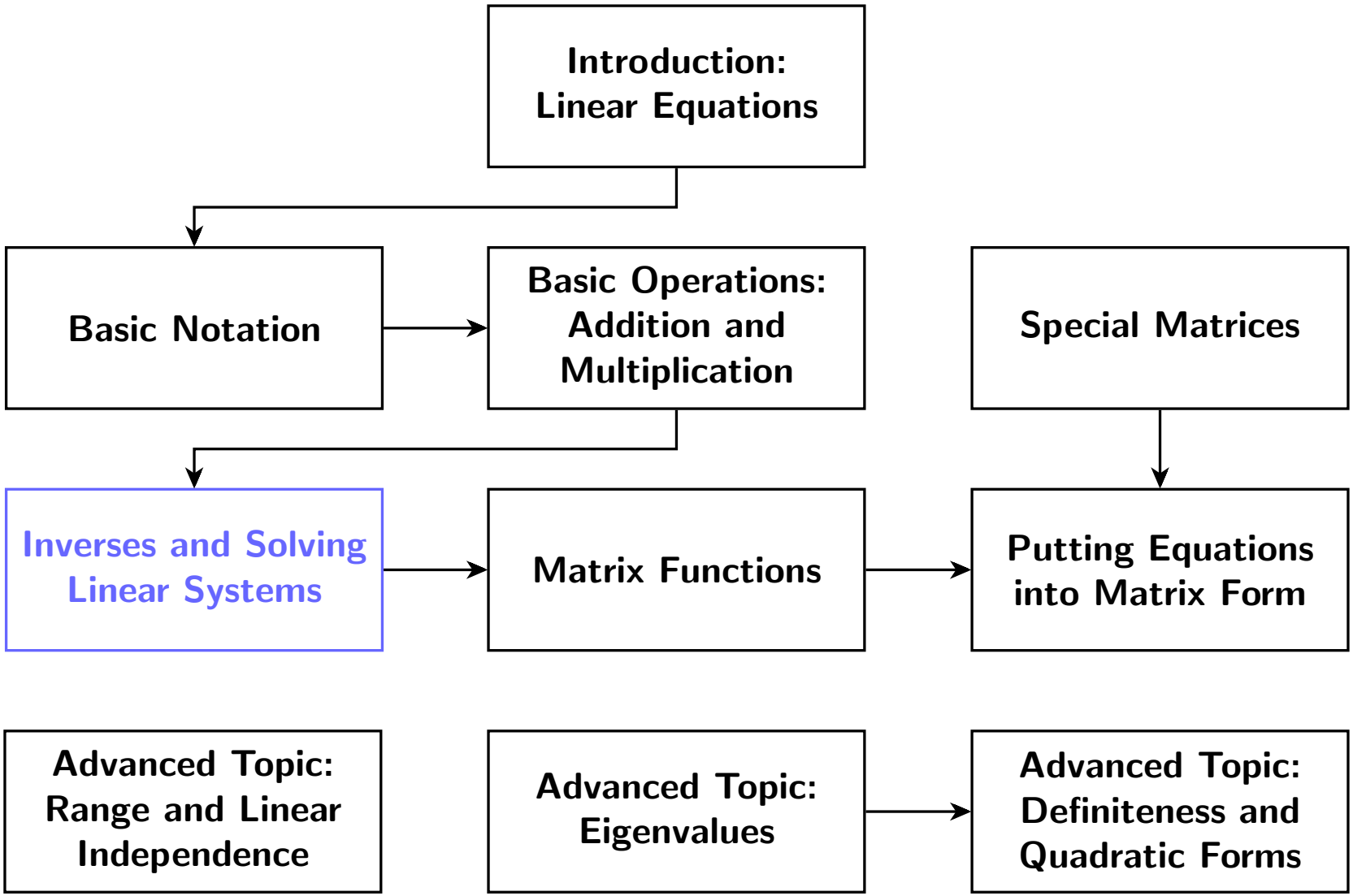
Linear Algebra Review



The Matrix Inverse

- Inverse of a square matrix $A \in \mathbb{R}^{n \times n}$ denoted A^{-1}

$$AA^{-1} = I = A^{-1}A$$

- May not exist (*non-singular* matrix has inverse, *singular* matrix does not)

$$A^{-1} \text{ exists } \iff Ax \neq 0 \text{ for all } x \neq 0$$

- Some important properties for $A, B \in \mathbb{R}^{n \times n}$ non-singular
  - $(A^{-1})^{-1} = A$
  - $(AB)^{-1} = B^{-1}A^{-1}$
  - $(A^T)^{-1} = (A^{-1})^T$

Solving Linear Equations

- Two linear equations

$$\begin{aligned} 4x_1 - 5x_2 &= -13 \\ -2x_1 + 3x_2 &= 9 \end{aligned}$$

- In vector form, $Ax = b$, with

$$A = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad b = \begin{bmatrix} -13 \\ 9 \end{bmatrix}$$

- Solution using inverse

$$
\begin{aligned}
Ax &= b \\
A^{-1}Ax &= A^{-1}b \\
x &= A^{-1}b
\end{aligned}
$$

- Won't worry here about how to compute inverse, but it's very similar to the standard method for solving linear equations