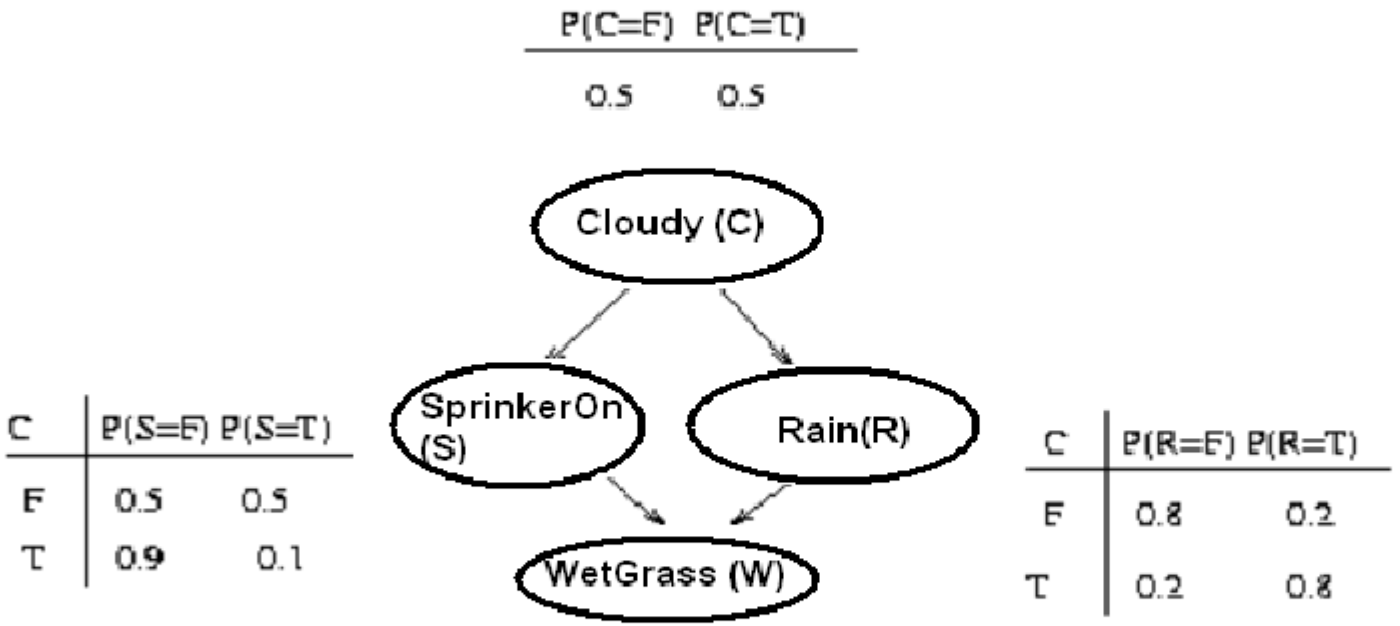
Bayes Nets

D-Separation & Inference

Some slides taken from previous 10701 recitations

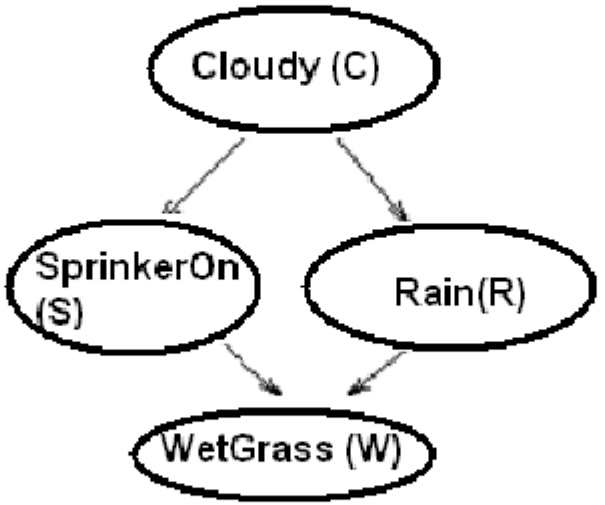
Bayesian Network Inference Example

The most common task we wish to solve using Bayesian networks is probabilistic inference.



BN Inference Example

- Observe that the grass is wet. What is the probability that the Sprinkler was on?



More Details

$$P(S=1|W=1)=\frac{P(S=1,W=1)}{P(W)}$$

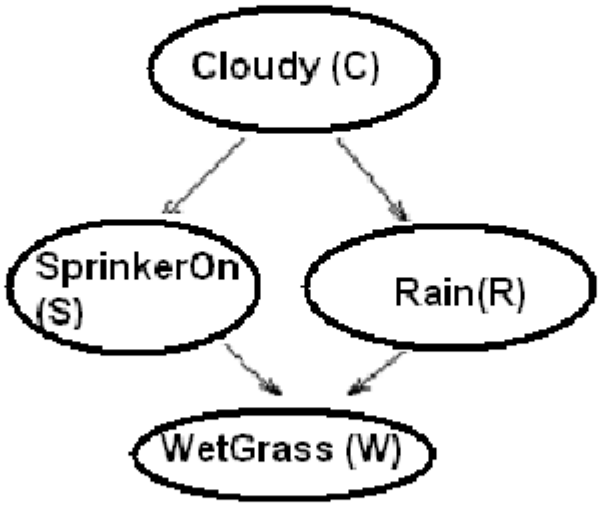
$$=\frac{\sum_{c,r}P(C=c,S=1,R=r,W=1)}{\sum_{c,r,s}P(C=c,S=s,R=r,W=1)}$$

$$=\frac{\sum_{c,r}P(C=c)P(S=1|C=c)P(R=r|C=c)P(W=1|S=1,R=r)}{\sum_{c,r,s}P(C=c)P(S=s|C=c)P(R=r|C=c)P(W=1|S=s,R=r)}$$

$$=\frac{0.2781}{0.6471}=0.43$$

Monte Carlo Sampling

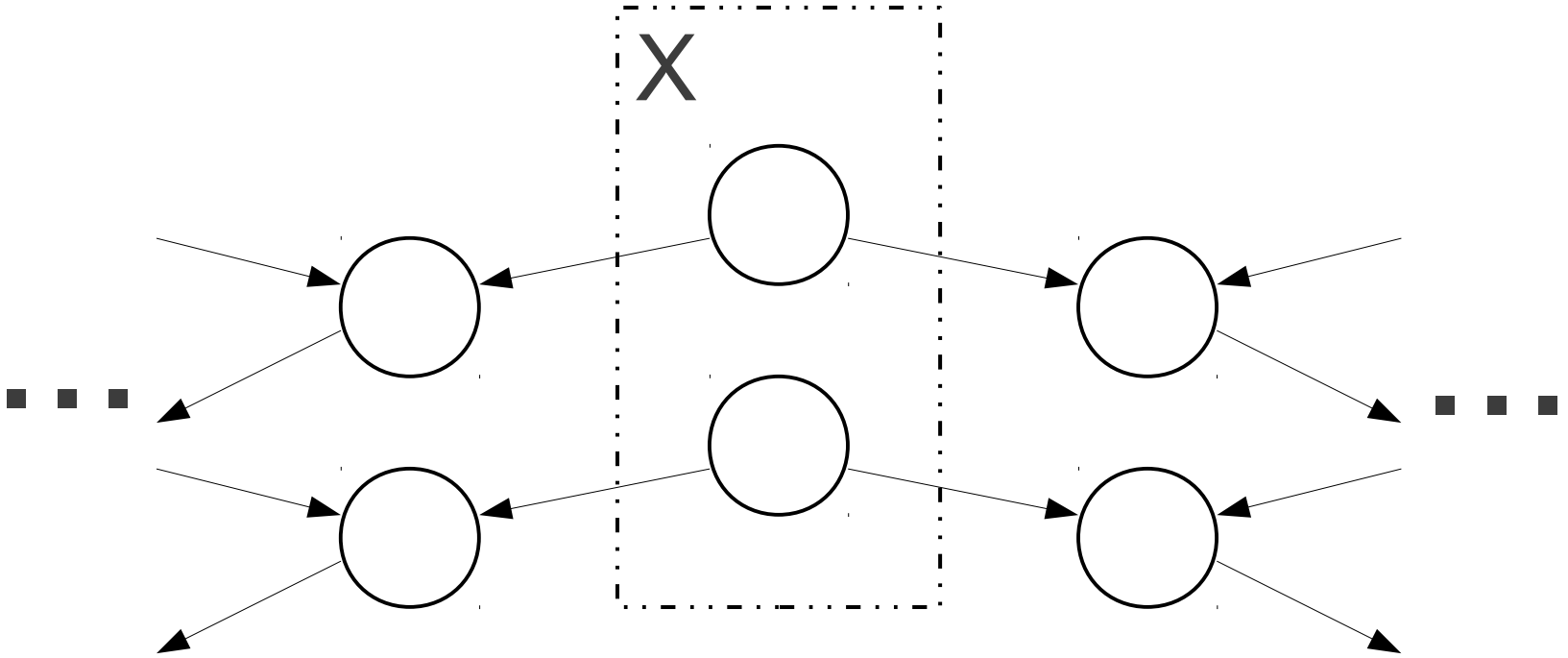
- What is the probability that the sprinkler was on given that the grass is wet?



- Sample C, then S, R, and finally W many times.
- Approximate P(W), P(S,W) via counting.

Why D-Separation?

- Helps us understand the dependencies implied by a graph
- Helps us perform inference efficiently



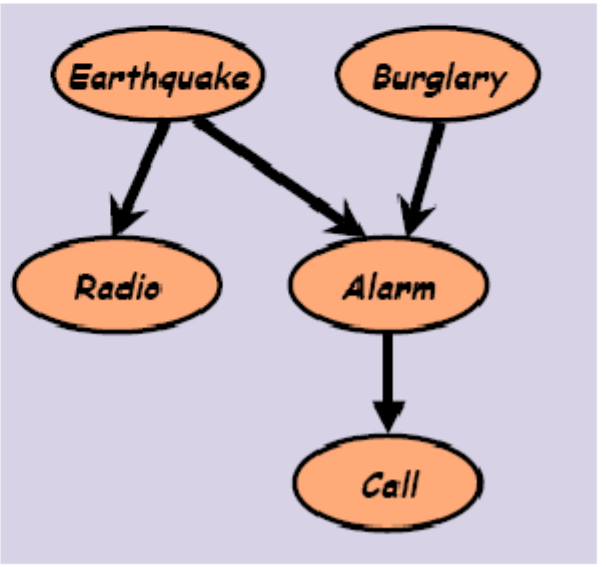
Path

- Intuition: dependency must "flow" along paths in the graph.
- A path is a sequence of neighboring variables.

- Examples:

  $R \leftarrow E \rightarrow A \leftarrow B$

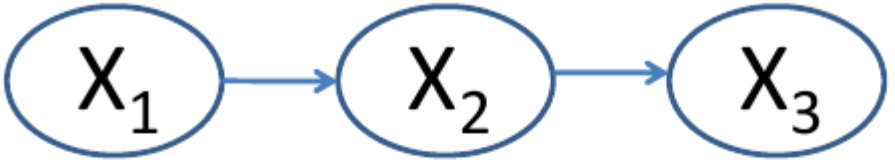
  $C \leftarrow A \leftarrow E \rightarrow R$



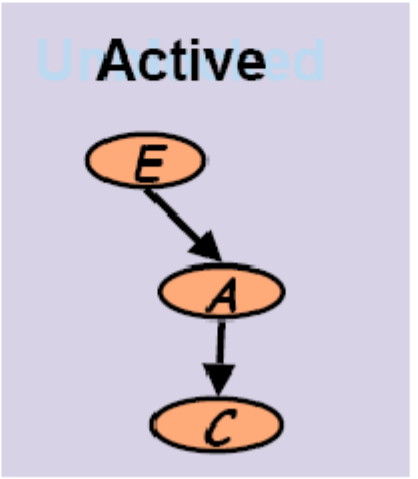
d-separation

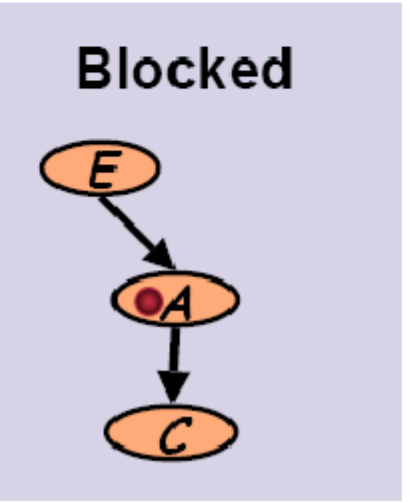
- **Definition: If X1, X2 and X3 are three disjoint** subsets of nodes in a DAG, then **X2** is said to d-separate **X1** from **X3** if **every undirected path** from **X1** to **X3** is blocked by **X2**. A path is blocked if it contains **a node Z** such that:
- (1) Z has one incoming and one outgoing arrow and Z is in ***X2***; or
- (2) Z has two outgoing arrows and Z is in ***X2***; or
- (3) Z has two incoming arrows and neither Z nor any of its descendants is in **X2**.

A serial connection

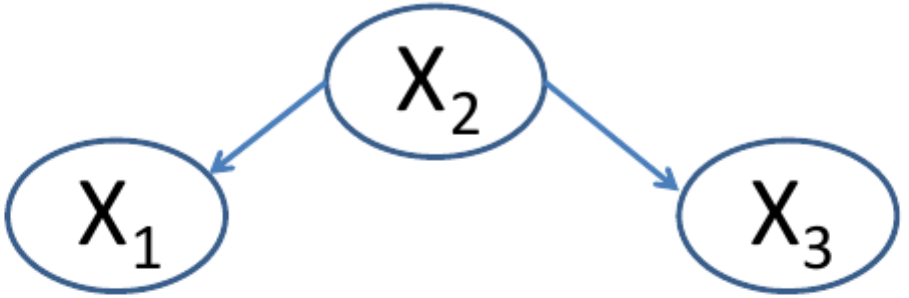


- In a serial connection from *X1 to X3 via X2, evidence from X1 to X3 is* ***blocked*** *only when we have* hard evidence about *X2.*
- Intermediate cause.

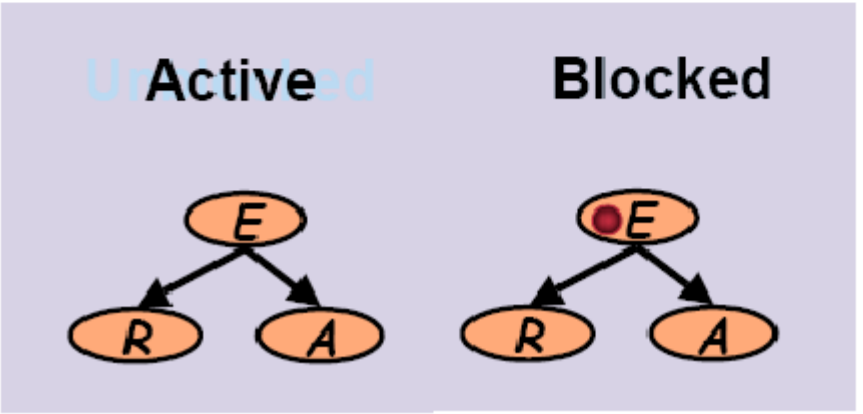




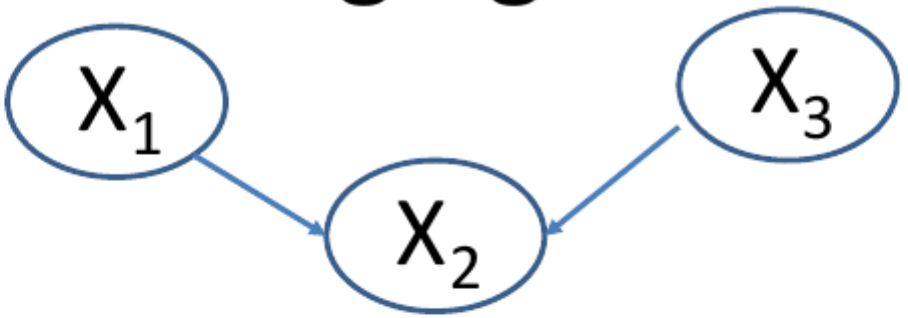
A diverging connection



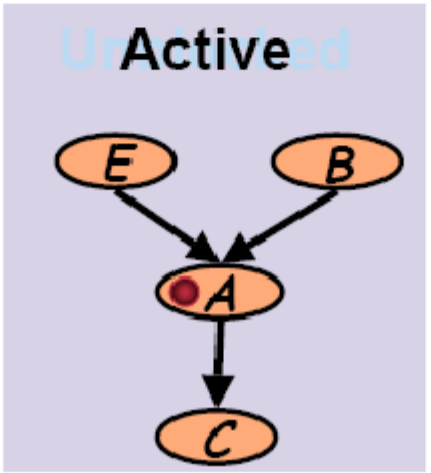
- In a diverging connection where *X1 and X3 have the common parent X2, evidence from X1 to X3 is* blocked only when we have hard evidence about *X2.*
- Common cause.

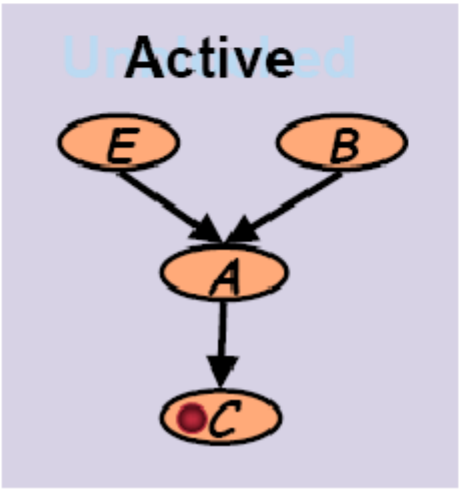


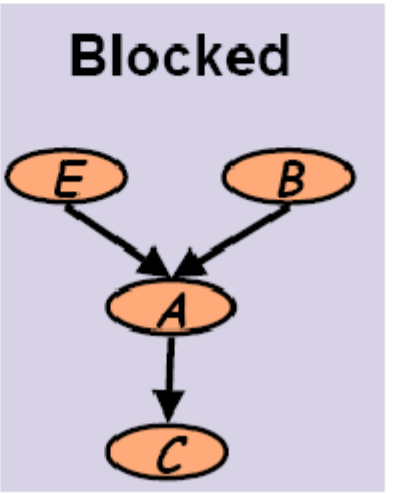
A Converging connection



- In a converging connection where *X2 has parents X1 and X3, any evidence about X2 results in* evidence transmitted between *X1 and X3.*
- Common Effect.

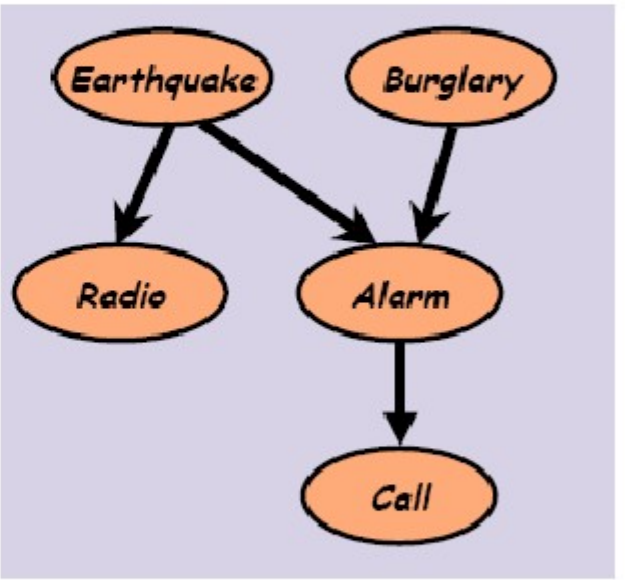






Example 1

- d-sep(R,B)?

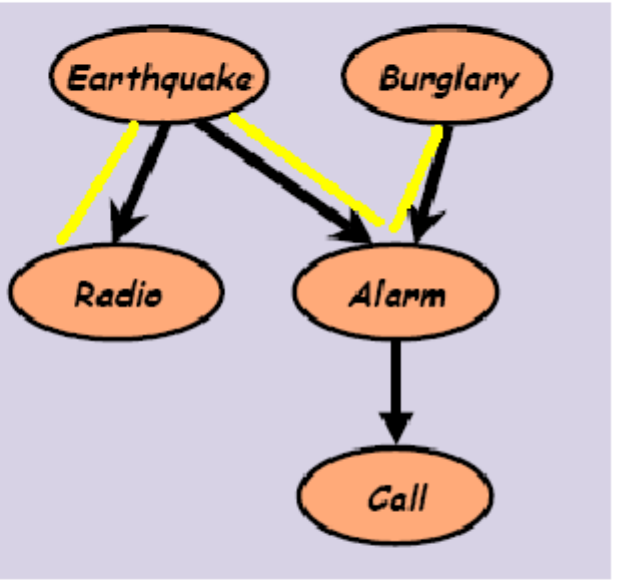


Example 1

- d-sep(X1, X3|X2)
- d-sep(R,B)?
  - X1 = {R}, X3={B}, X2 ={}
  - Find all the path between R, B
  - Check the node:
    - Earthquake.

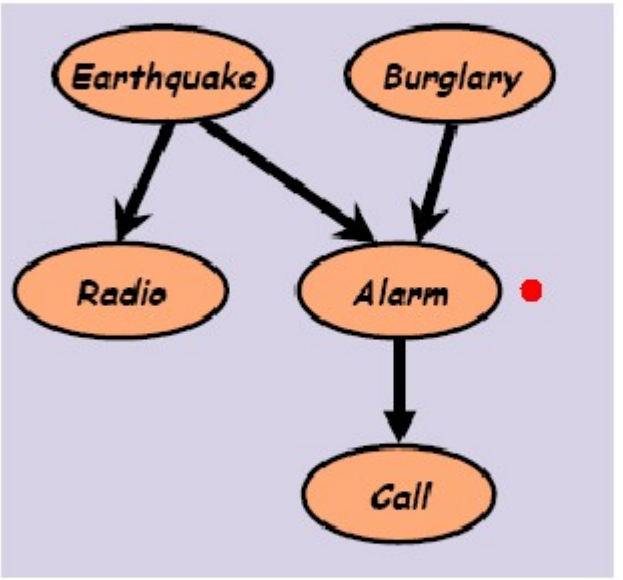
      (diverging, not in X2). Not blocking.
    - Alarm

      (Converging, A or C are not in X2). Block!



Example 2

- d-sep(R,B|A)?

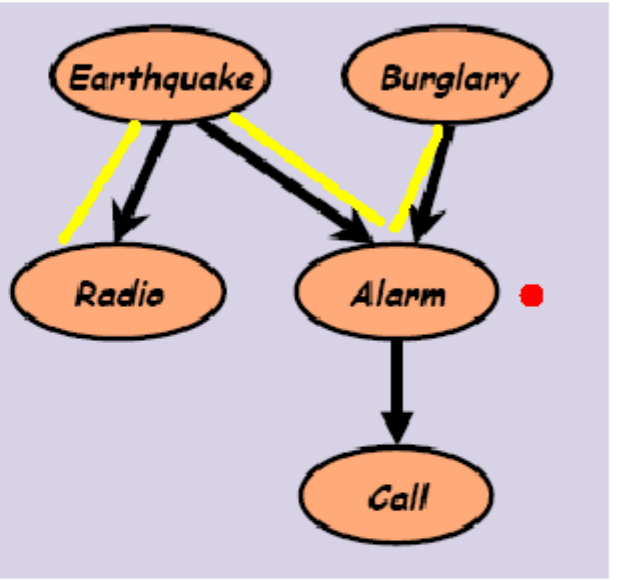


Example 2

- d-sep(X1, X3|X2)
- d-sep(R,B|A)?
  - X1 = {R}, X3={B}, X2 ={A}
  - Find all the path between R, B
  - Check the node:
    - Earthquake.

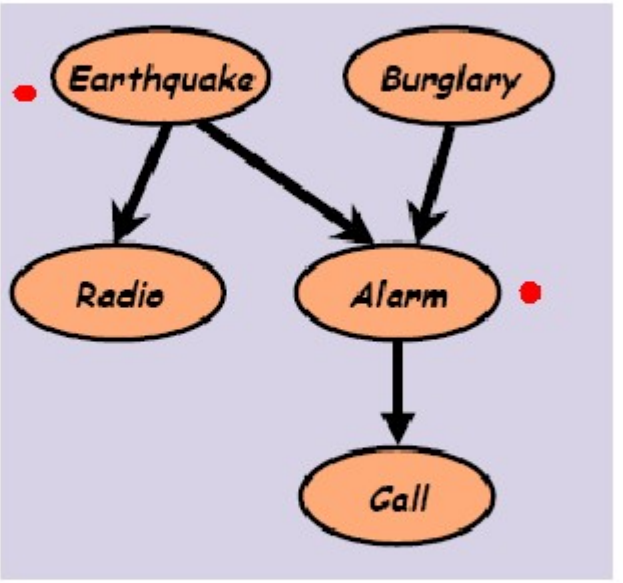
      (diverging, not in X2). Not blocking.
    - Alarm

      (Converging, A or C are **IN** X2). Not blocking!



Example 3

- d-sep(R,B|E,A)?

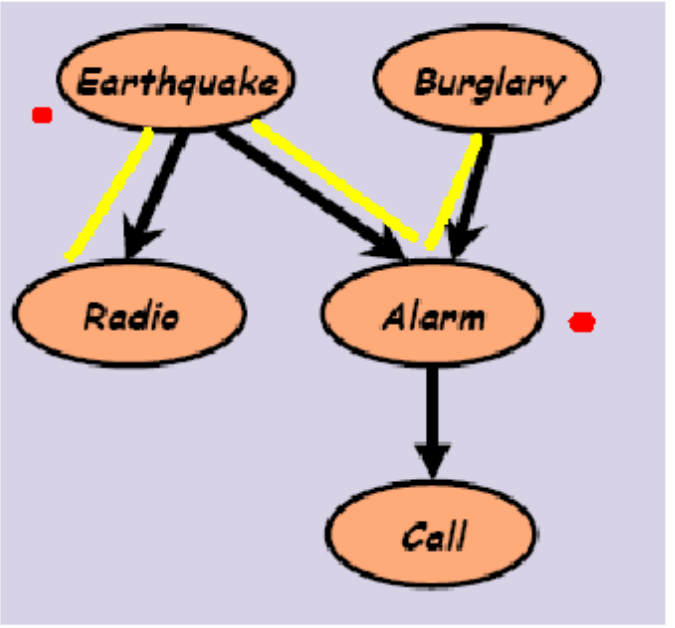


Example 3

- d-sep(X1, X3|X2)
- d-sep(R,B|E,A)?
  - X1 = {R}, X3={B}, X2 ={E,A}
  - Find all the path between R, B
  - Check the node:
    - Earthquake.

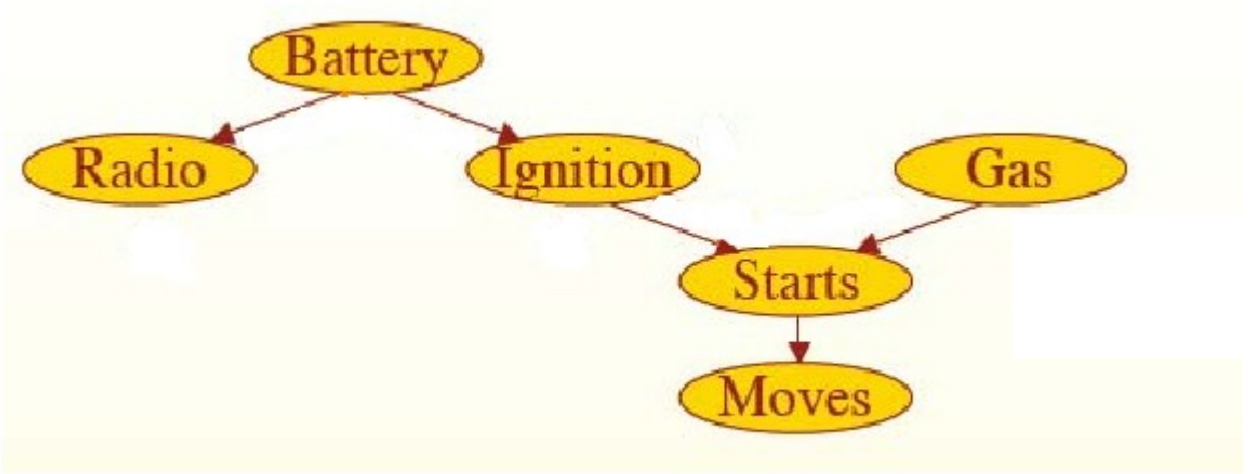
      (diverging, **IN** X2). Blocking!
    - Alarm

      (Converging, A or C are **IN** X2). Not blocking.



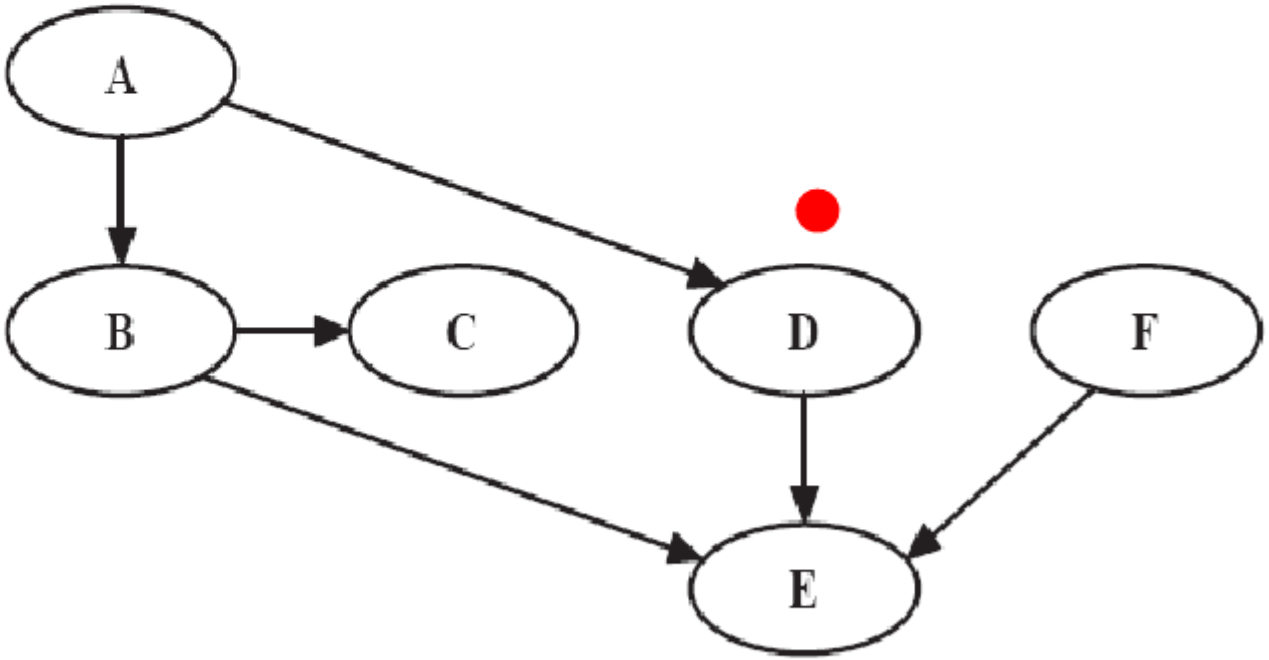
Example 4

- d-sep(Radio,Gas|Moves)?



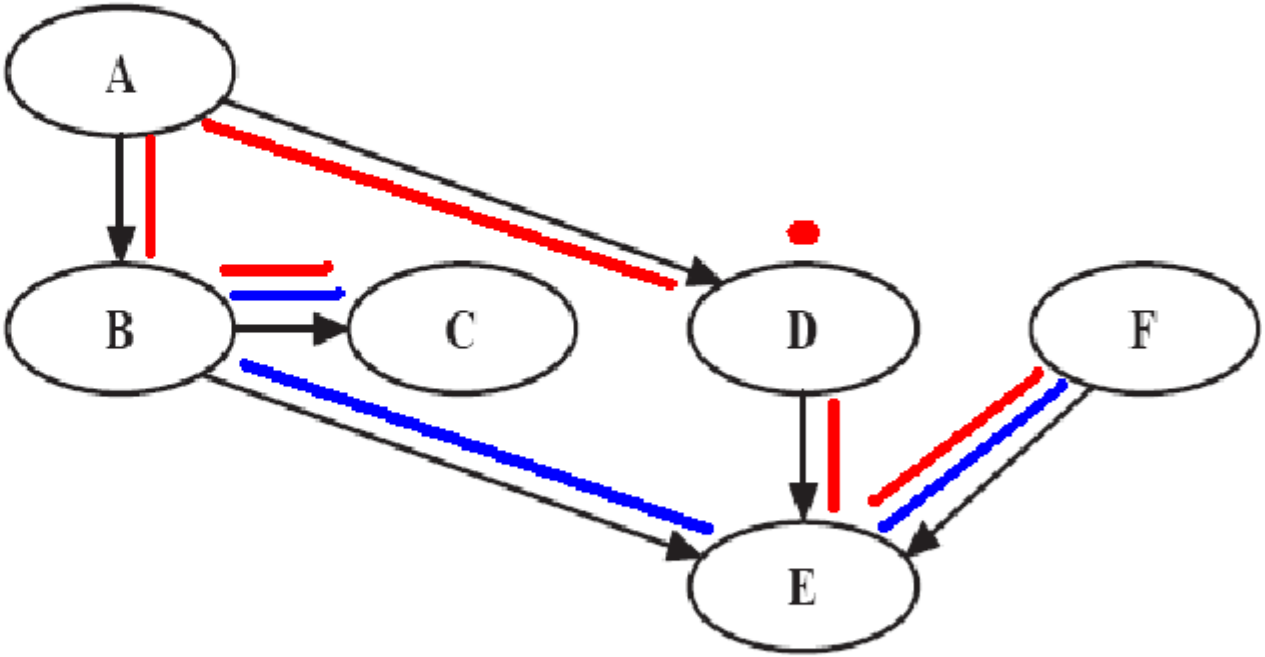
D-seperation: Multiple Paths

- d-sep({C}, {F}|{D})?



D-seperation: Multiple Paths

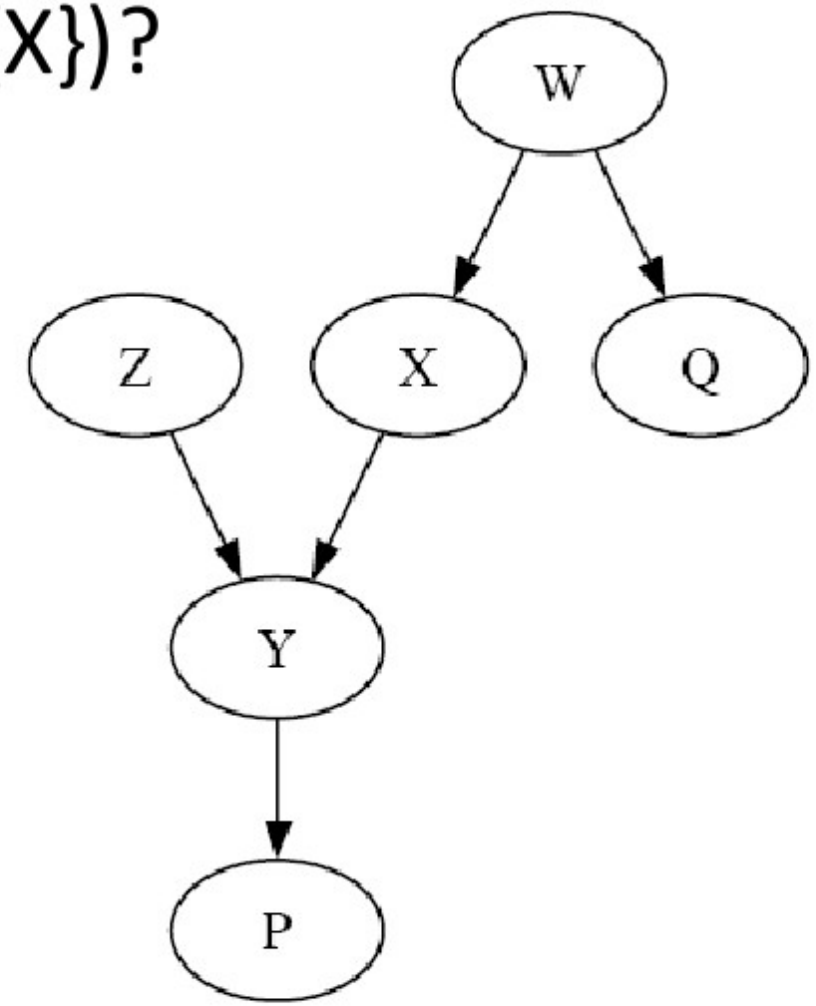
- d-sep({C}, {F}|{D})?



Red path is blocked
by D.
Blue path is blocked
by E not in evidence.

D-seperation on Sets

- d-sep({Z, Y, P}, {W, Q}|{X})?



D-seperation on Sets

- d-sep({Z, Y, P}, {W, Q}|{X})? YES

Blue path is a closed sequential path since we condition on X.