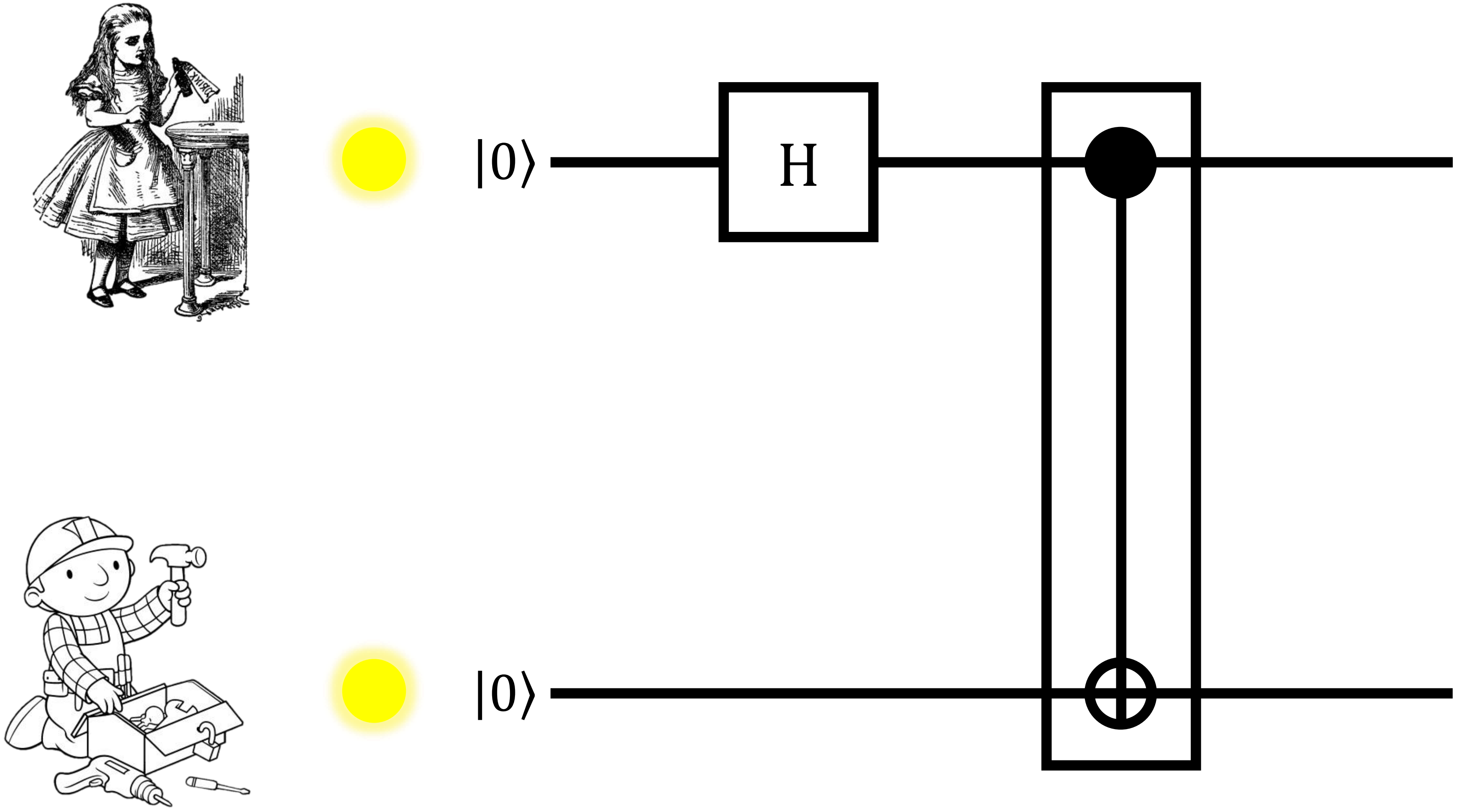
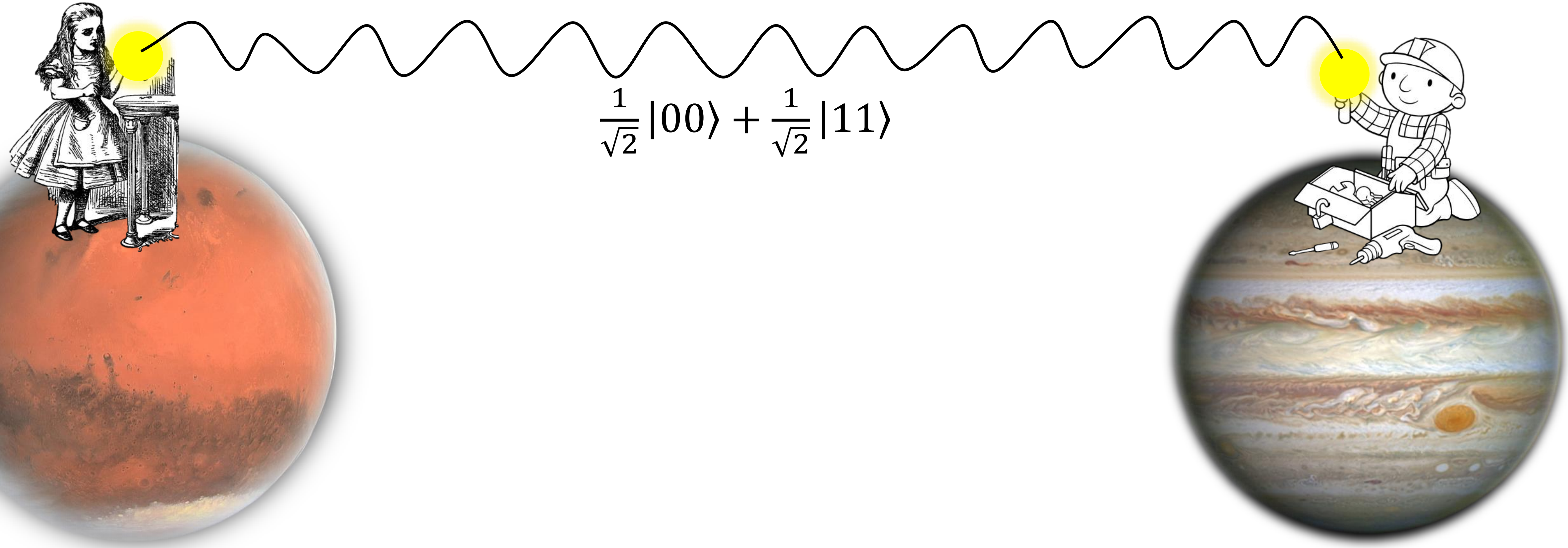
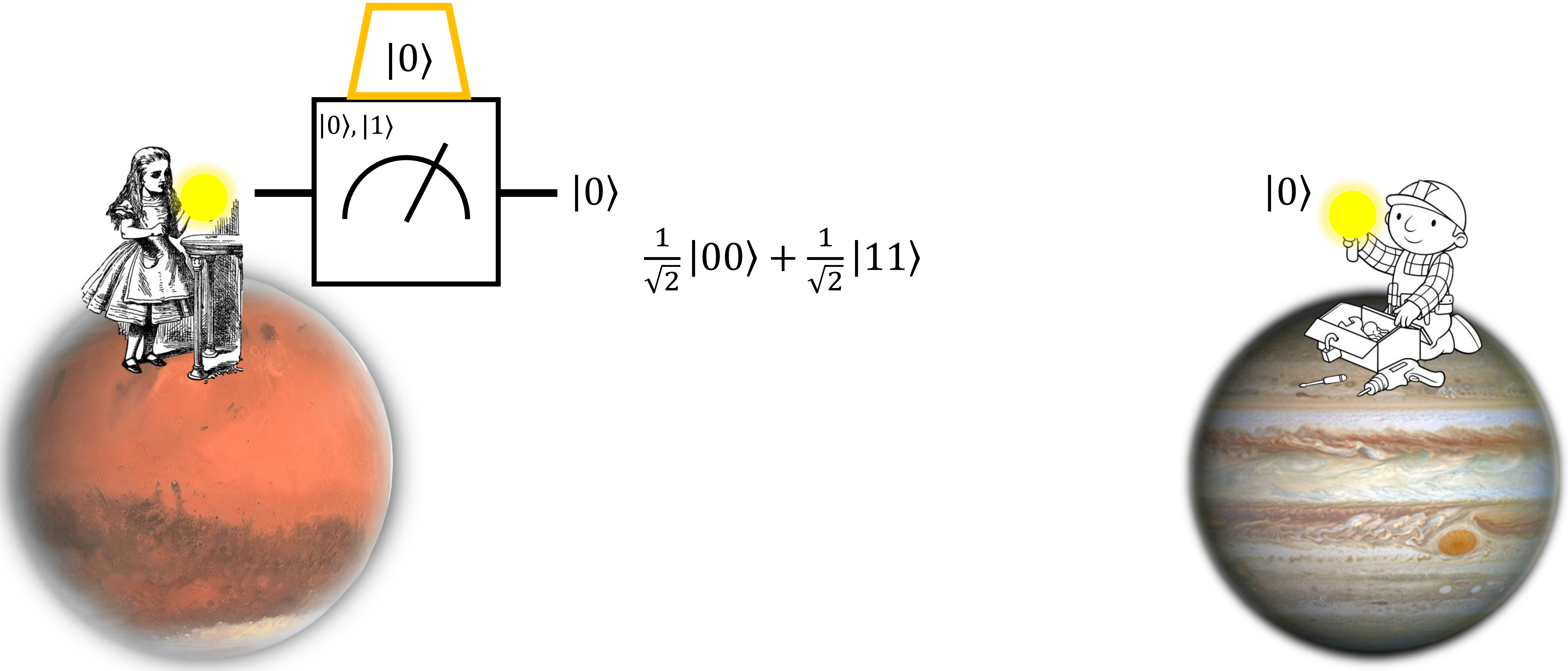
# Lecture 7:
# The CHSH Game



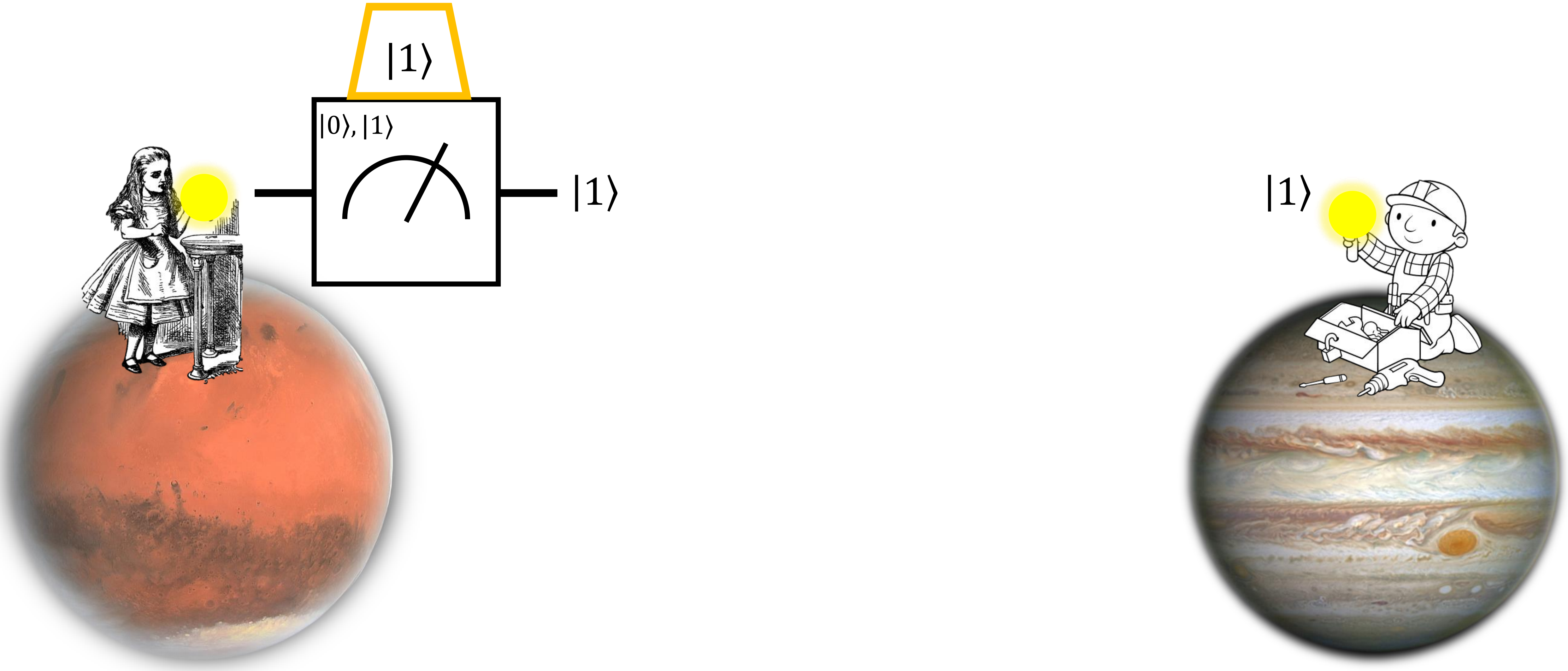

EPR Pair:

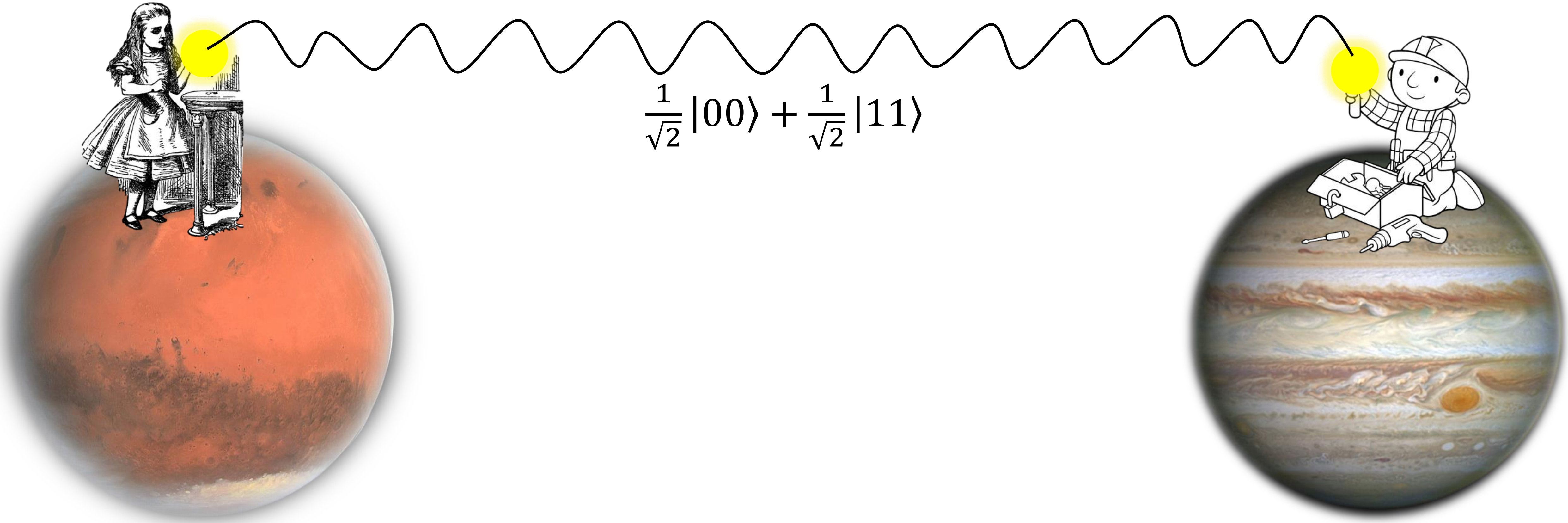
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$


$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

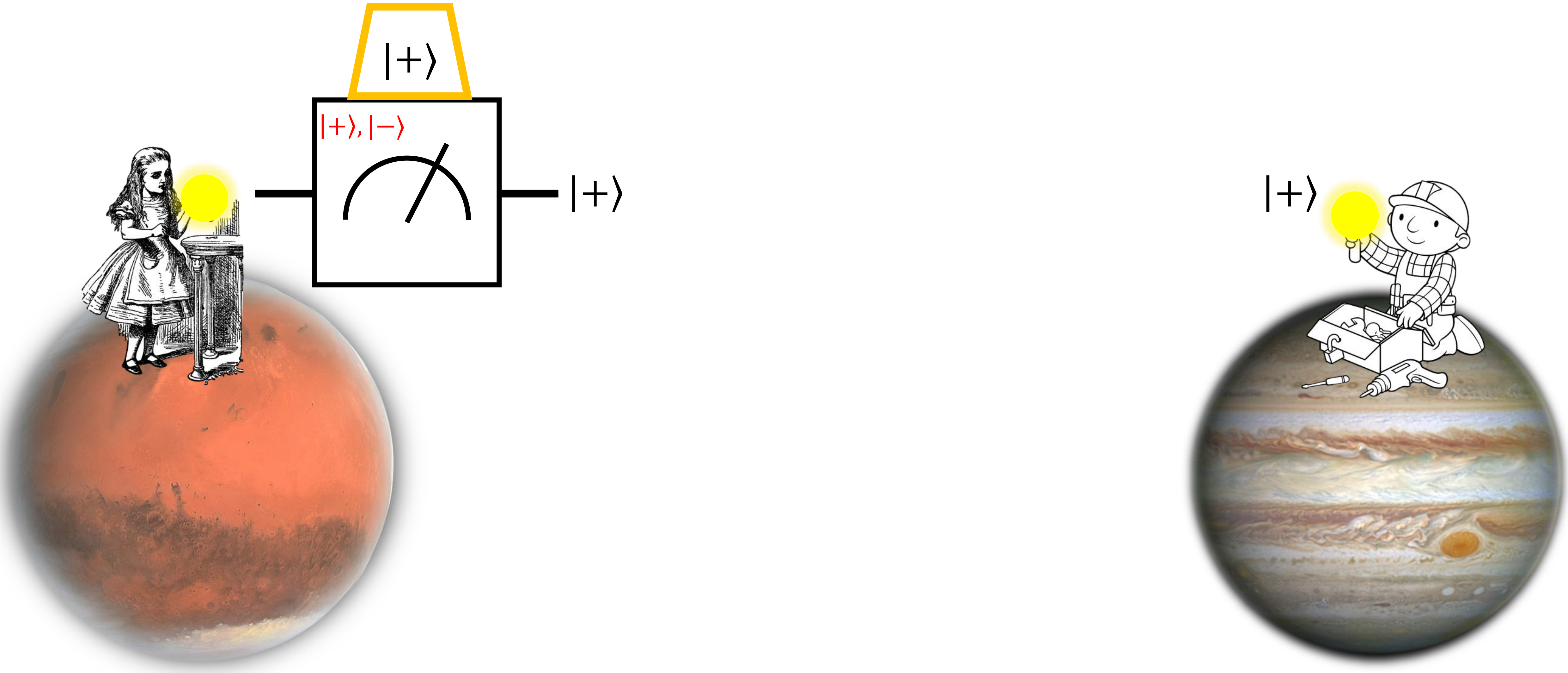


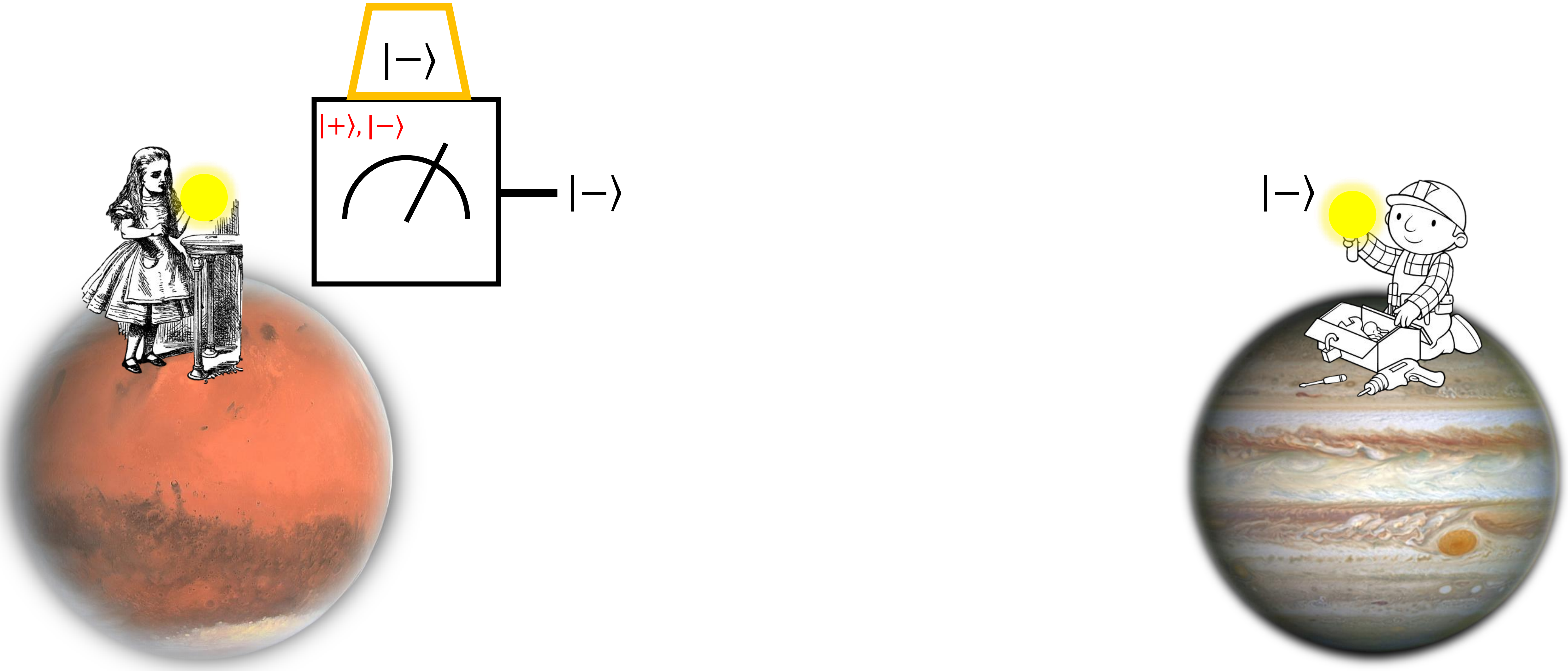
$|0\rangle$
$|0\rangle, |1\rangle$
$|0\rangle$
$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
$|0\rangle$


$|1\rangle$
$|0\rangle, |1\rangle$
$|1\rangle$
$|1\rangle$

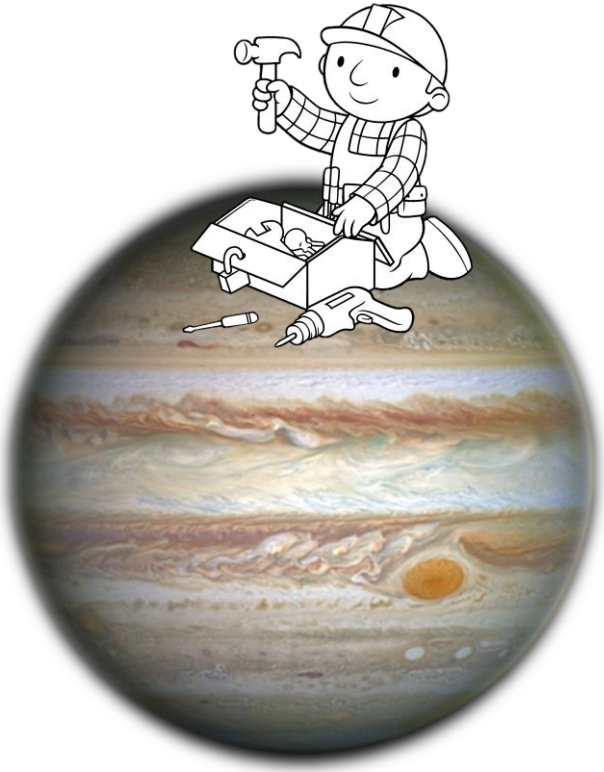


$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$



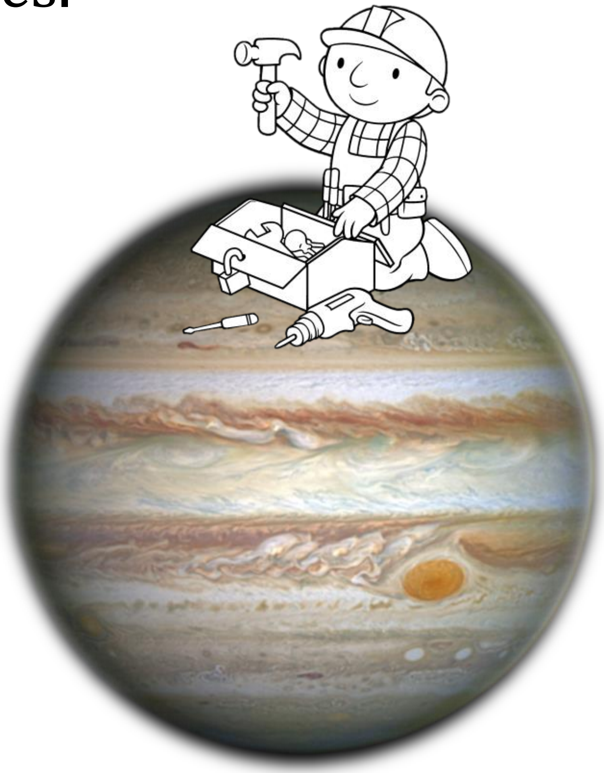


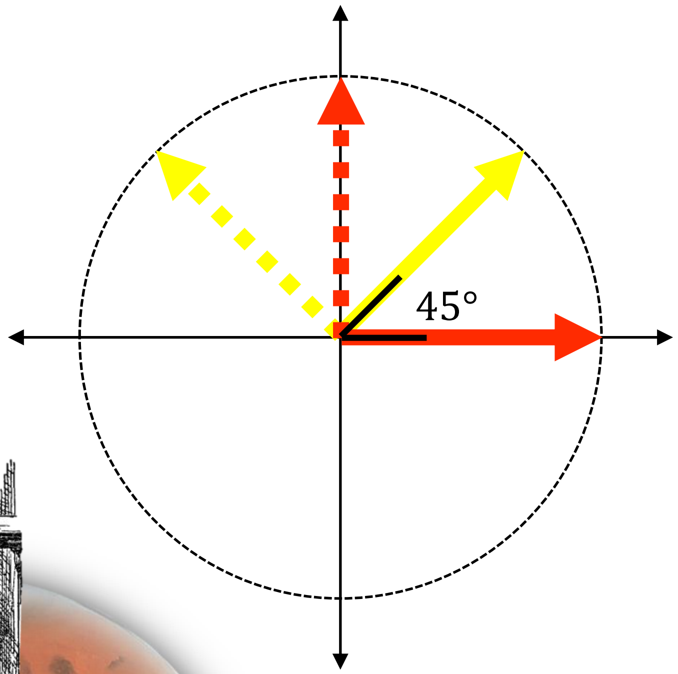
$|-\rangle$
$|+\rangle, |-\rangle$
$|-\rangle$
$|-\rangle$

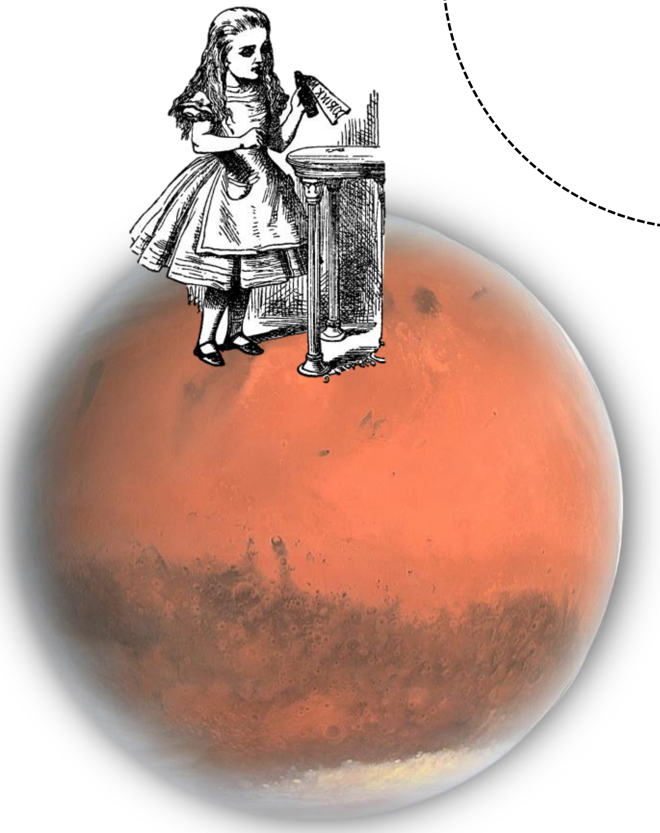


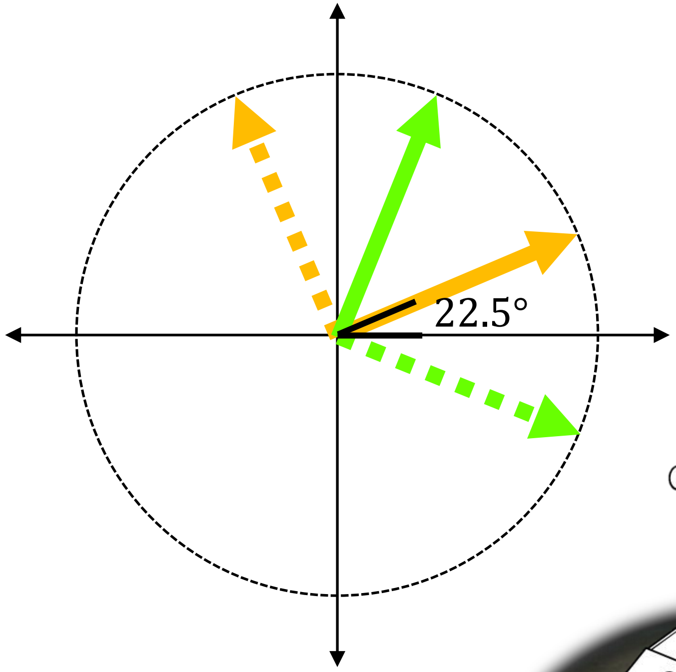
Alice is going to measure in one of two bases.

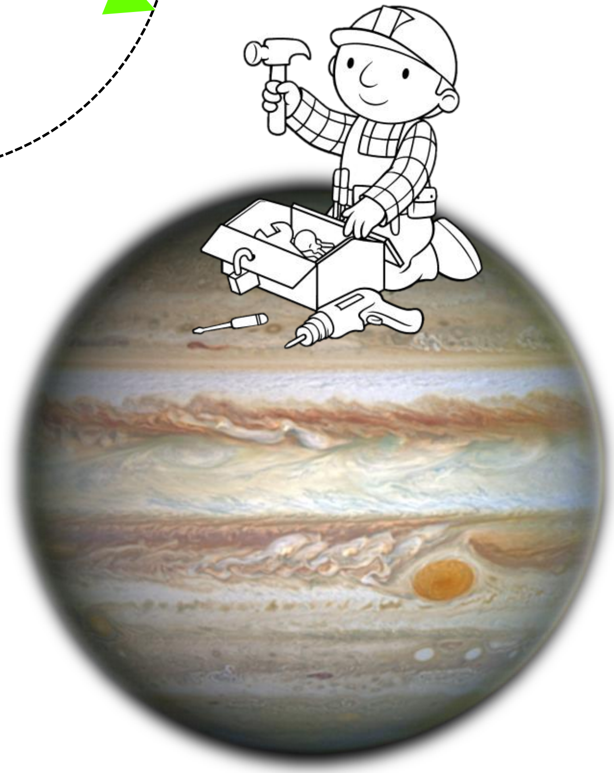
Bob is going to measure in one of two bases.



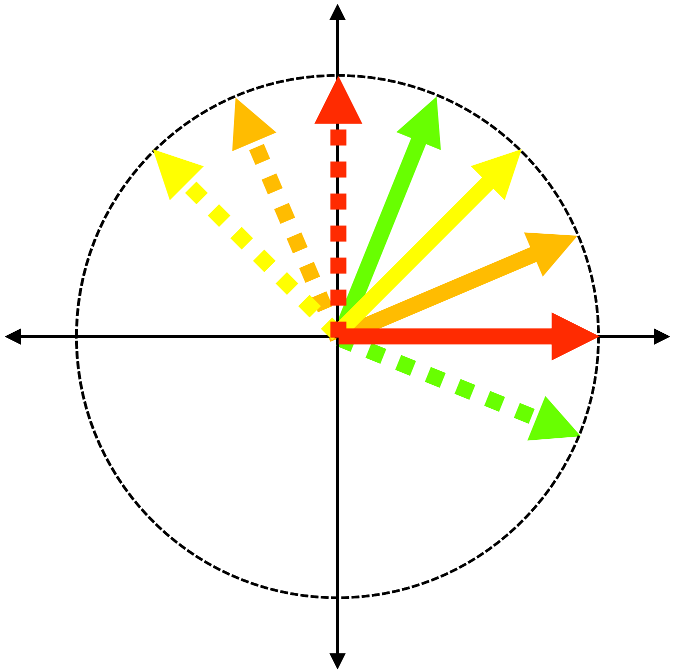

45°

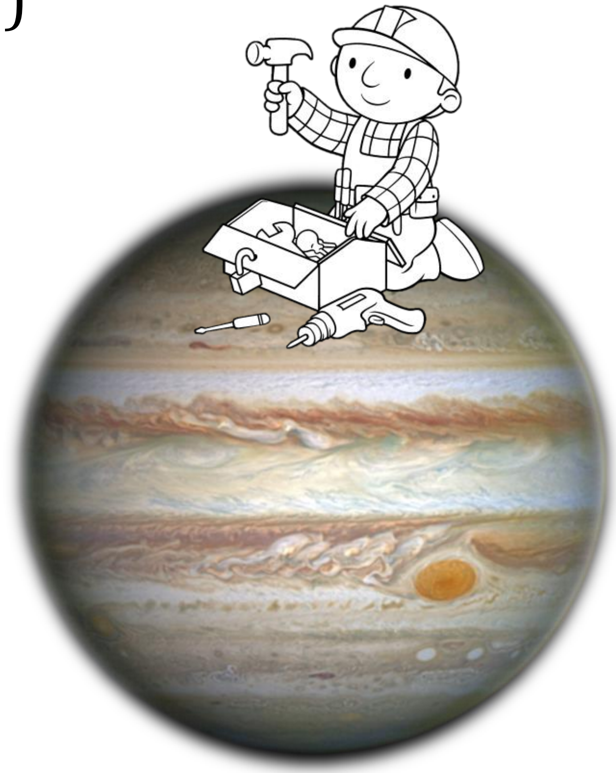


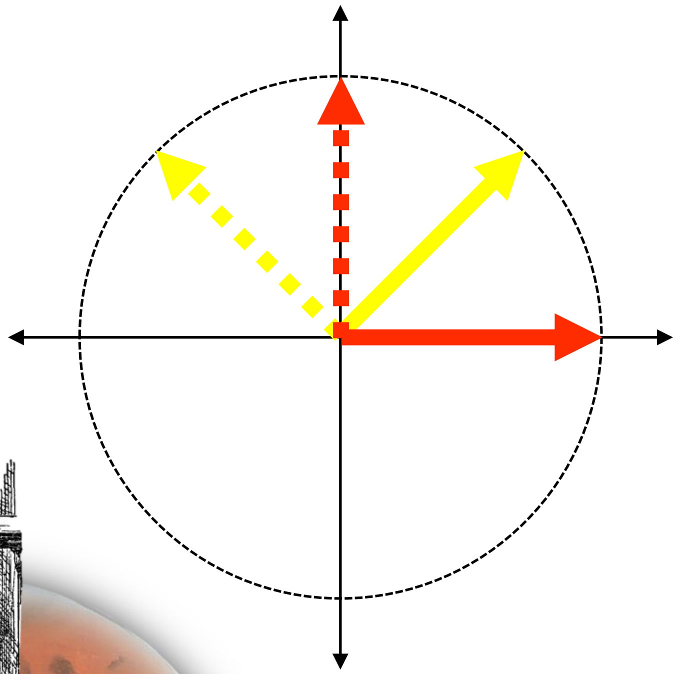

22.5°

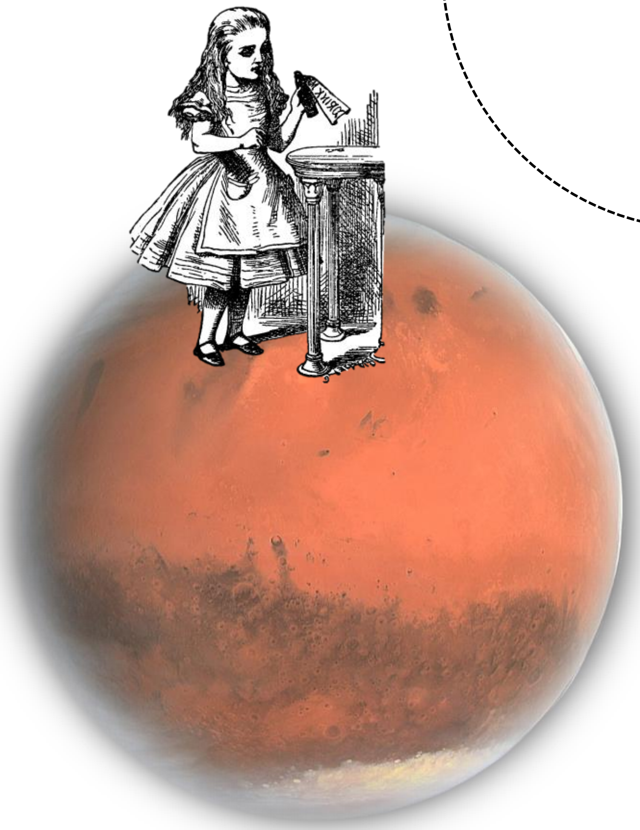


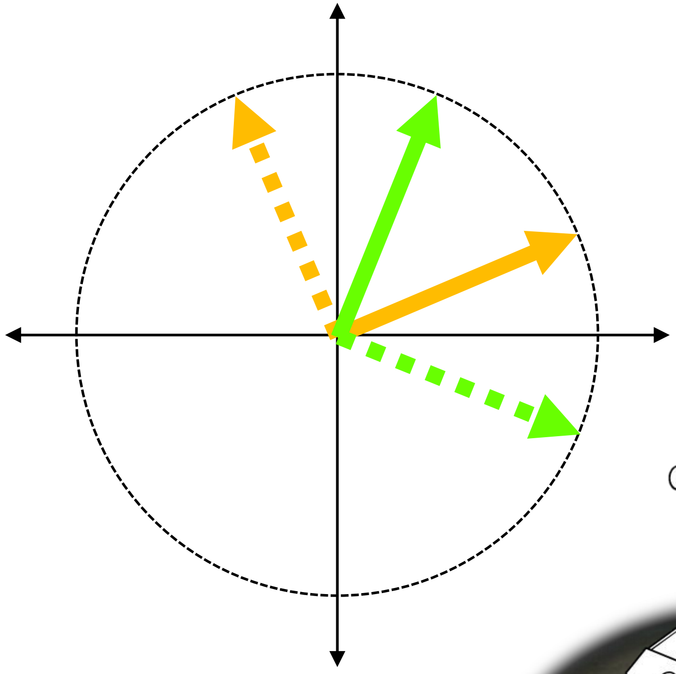
Red is “close” to Orange (22.5°)
Orange is “close” to Yellow (22.5°)
Yellow is “close” to Green (22.5°)
Green is “**far**” from Red (90°−22.5°)

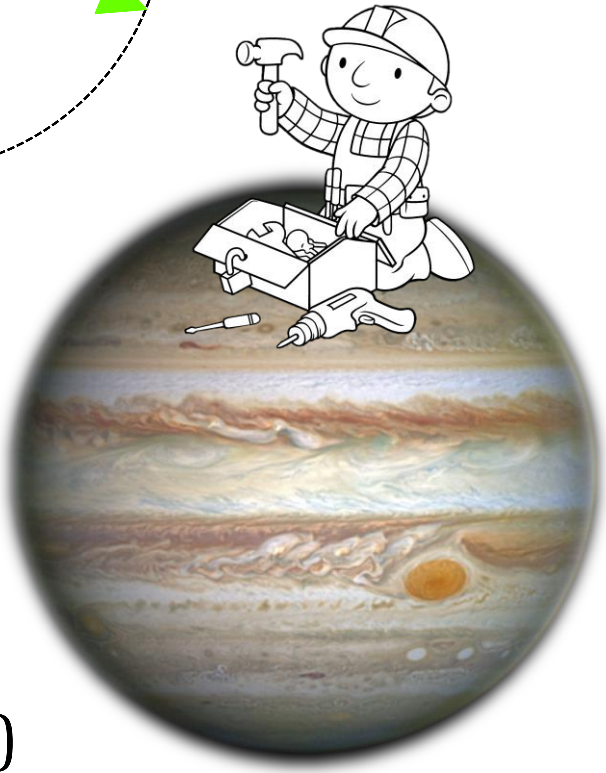




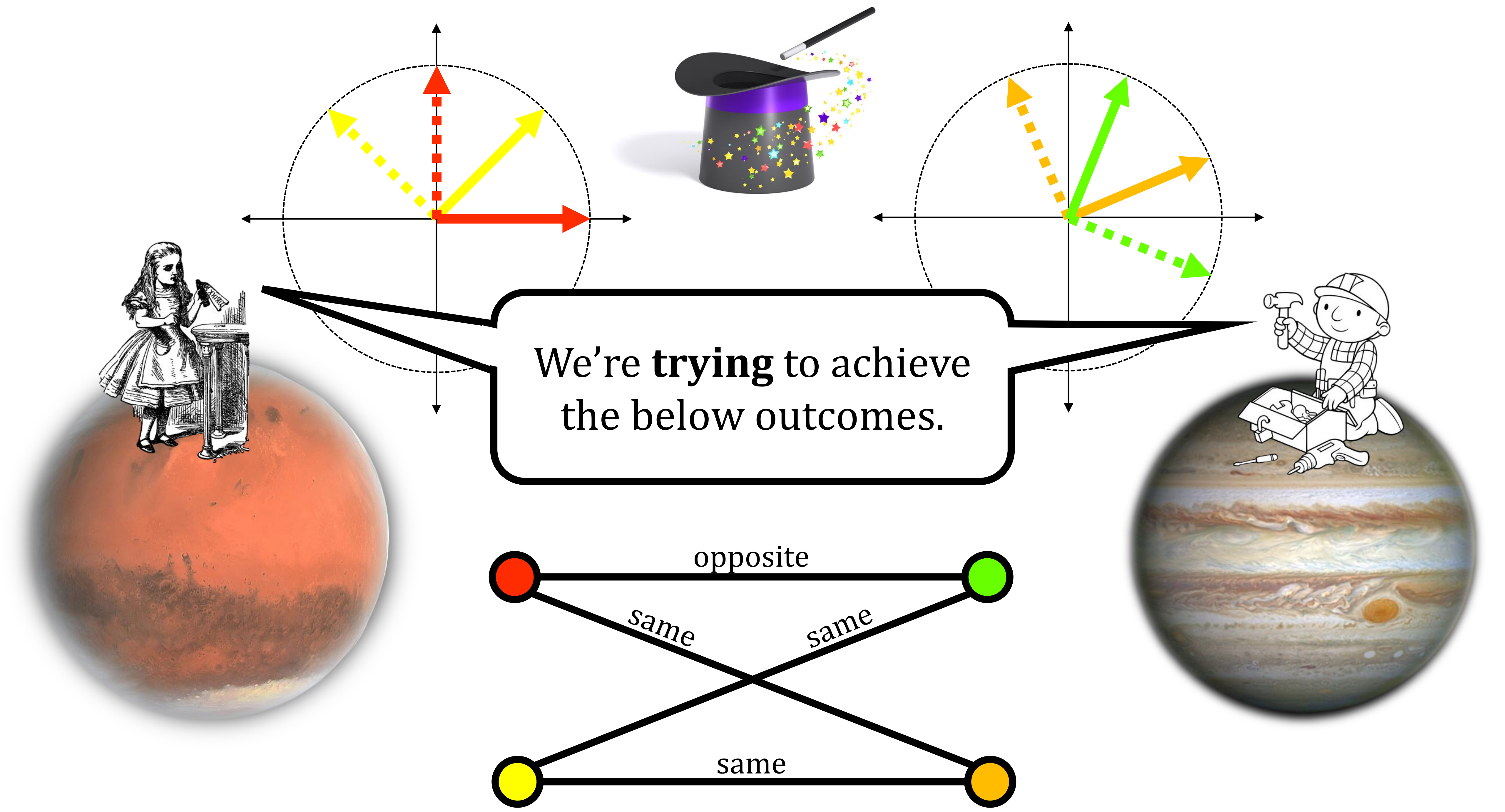


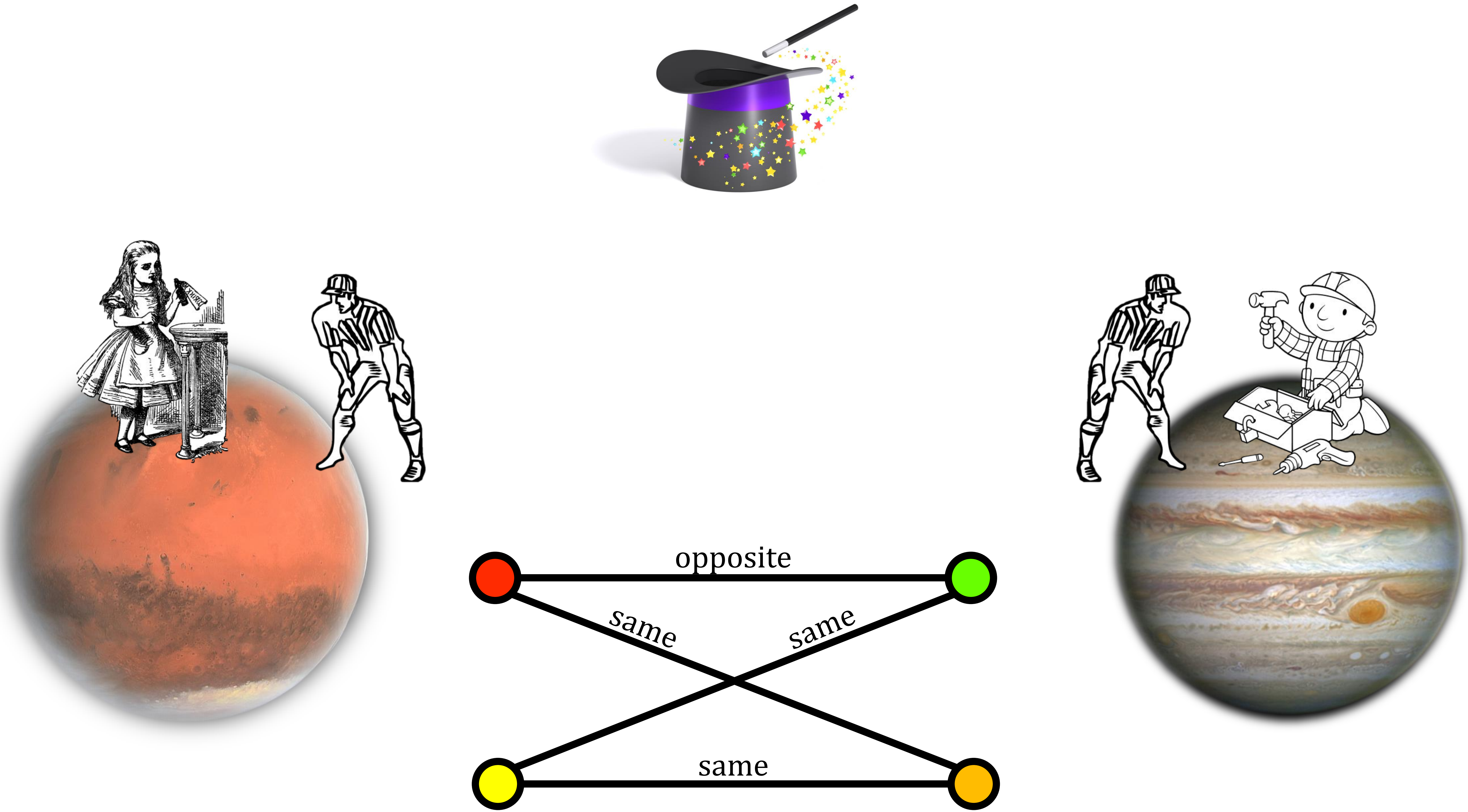




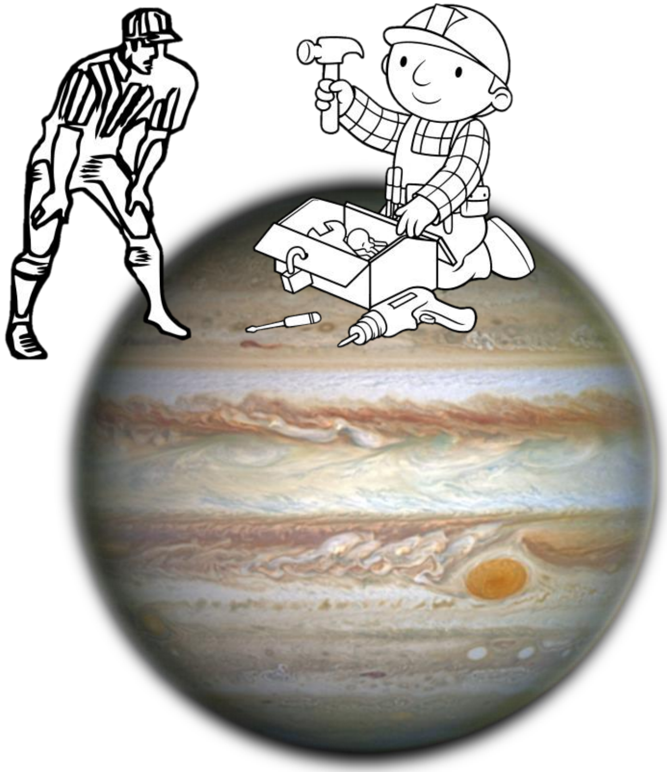


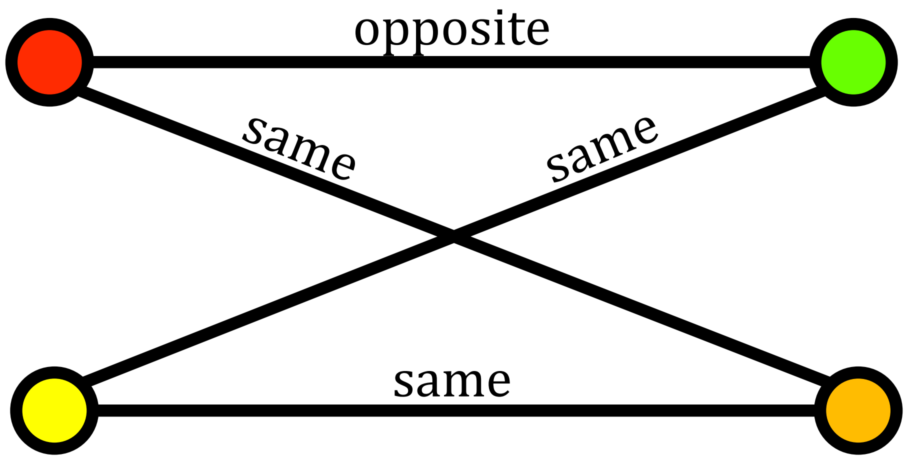
| | | | |
|---|---|---|---|
| Red | is “close” to | Orange | (22.5°) |
| Orange | is “close” to | Yellow | (22.5°) |
| Yellow | is “close” to | Green | (22.5°) |
| Green | is “**far**” from | Red | (90°−22.5°) |

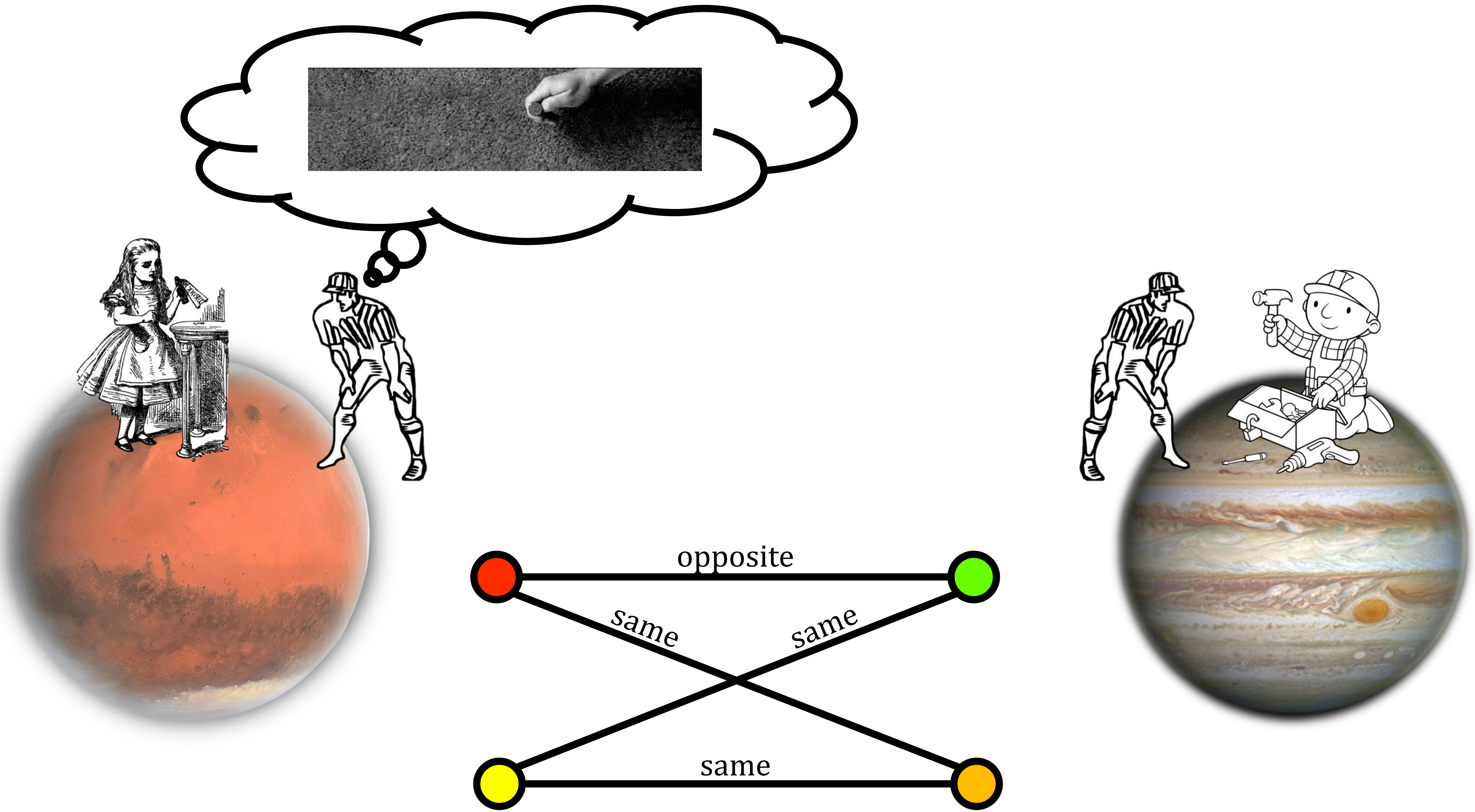

We're **trying** to achieve the below outcomes.
opposite
same
same
same

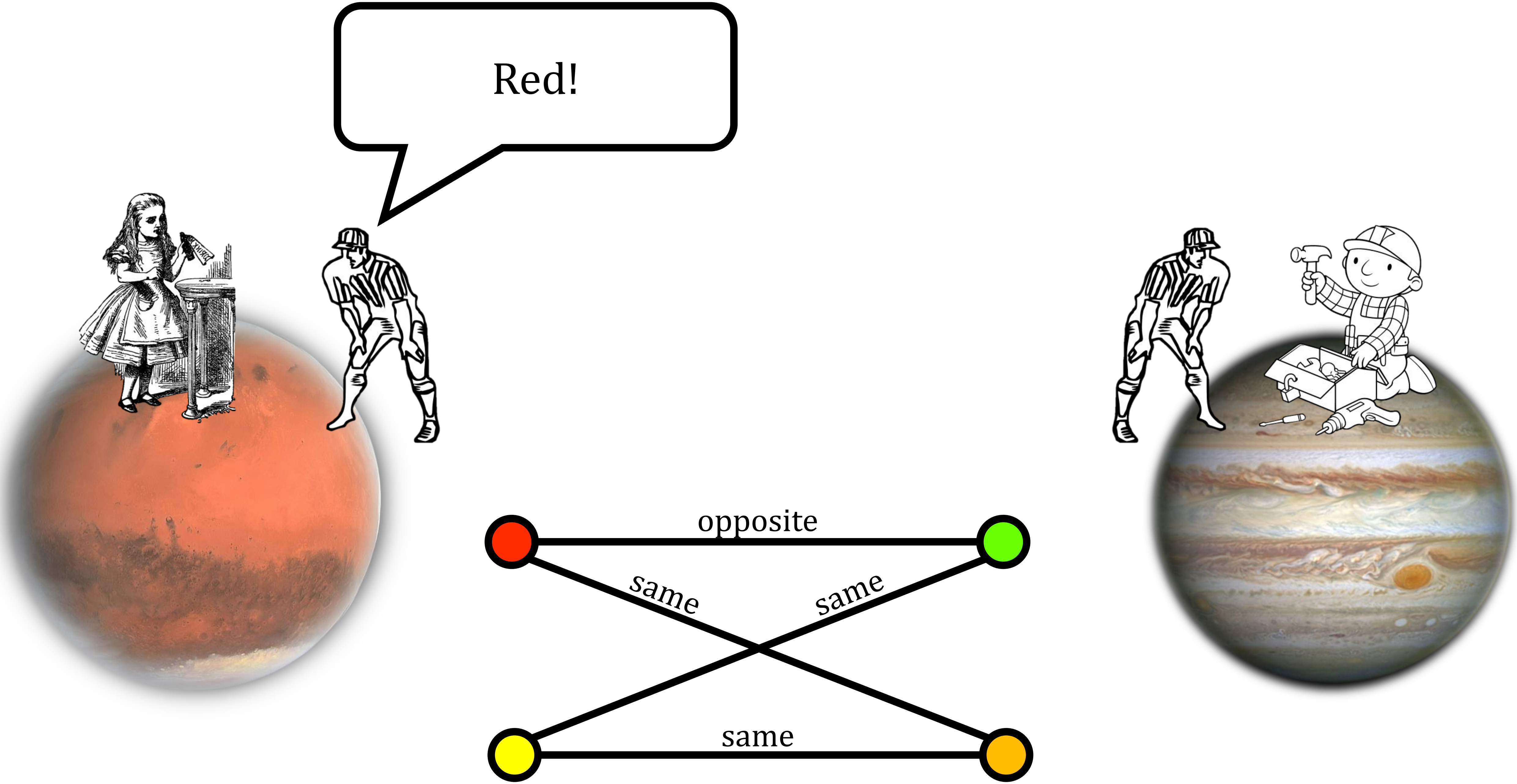

opposite
same
same
same

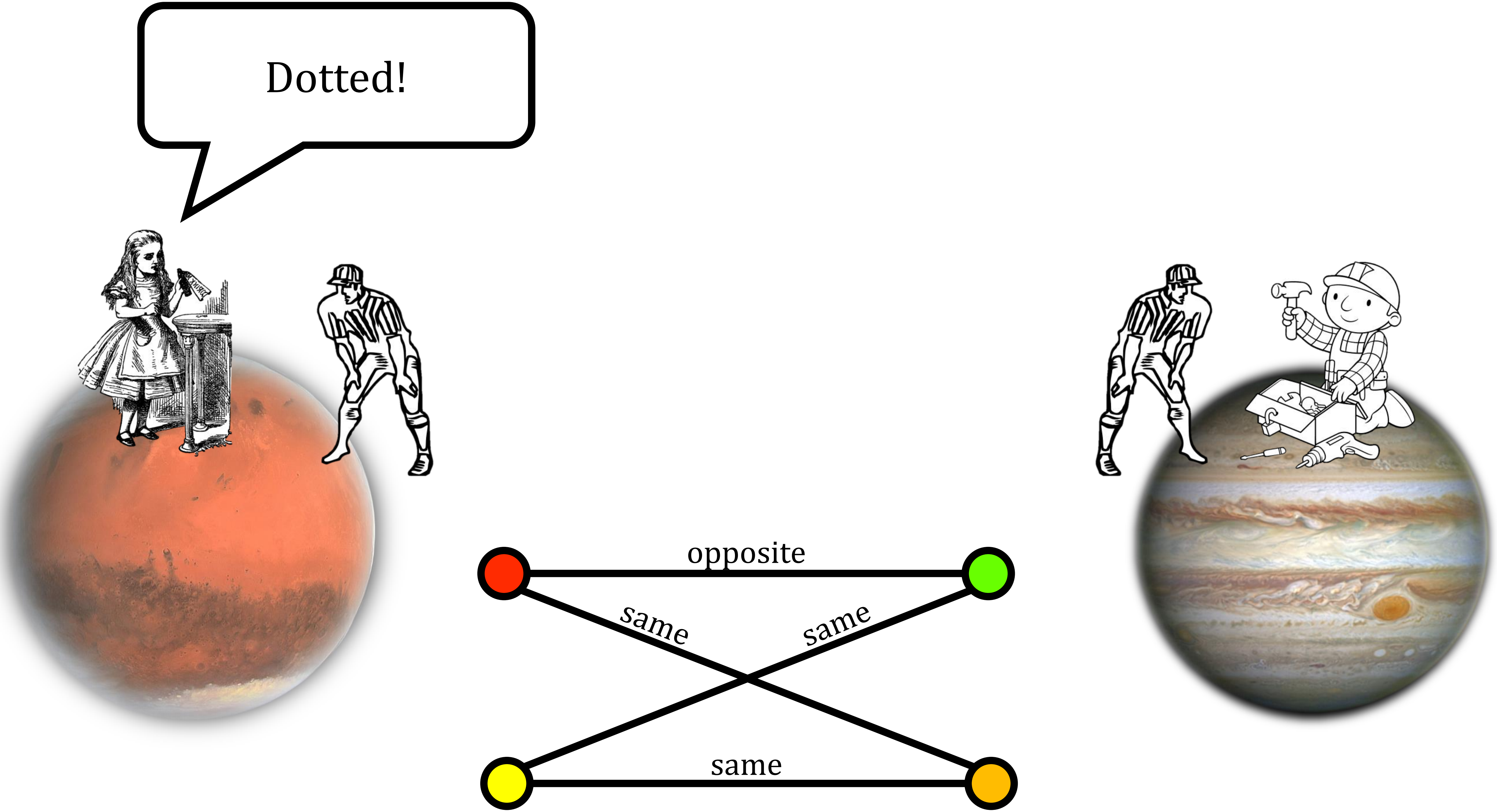
Flips a fair coin to choose
either “Red” or “Yellow”,
as a “challenge”.



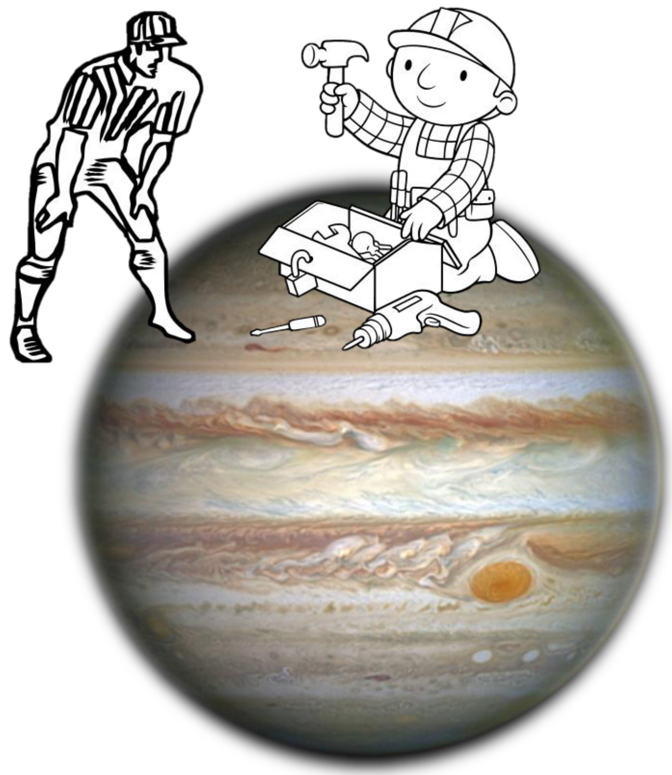


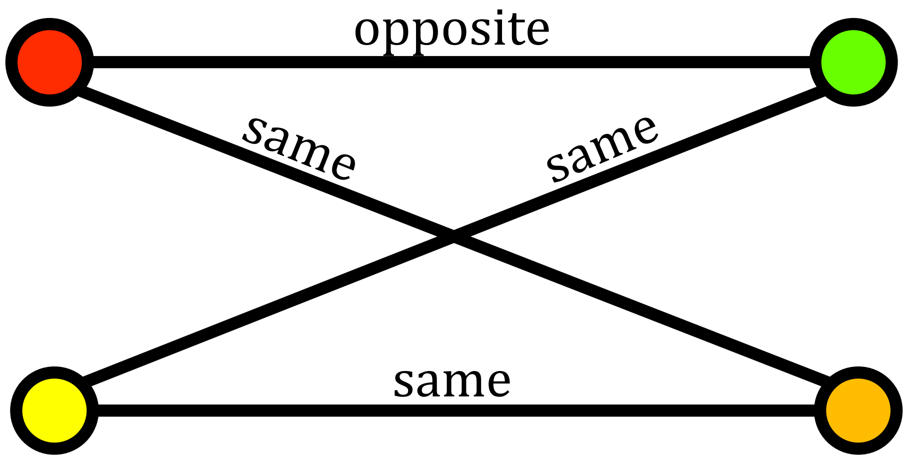

opposite
same
same
same

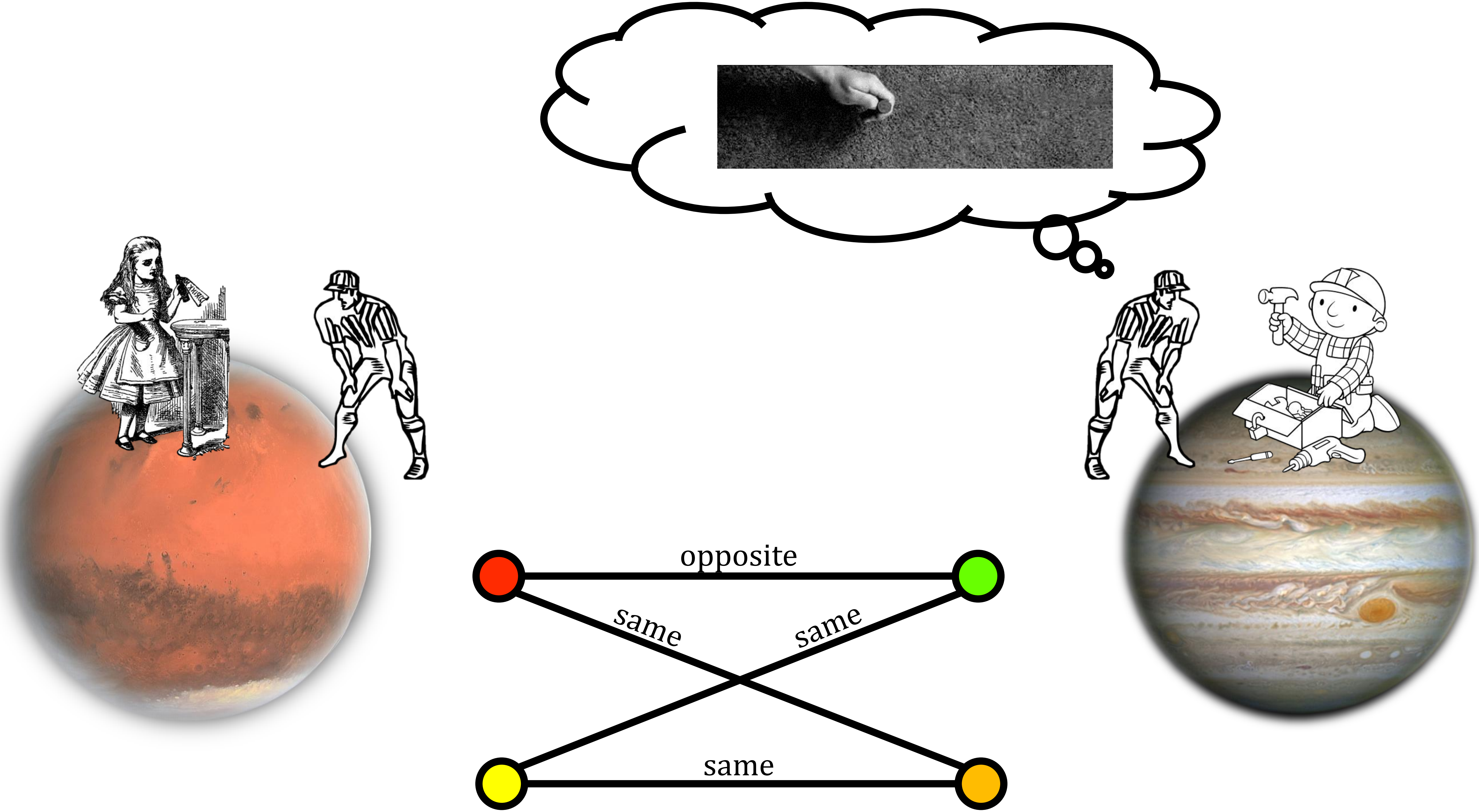

Red!
opposite
same
same
same

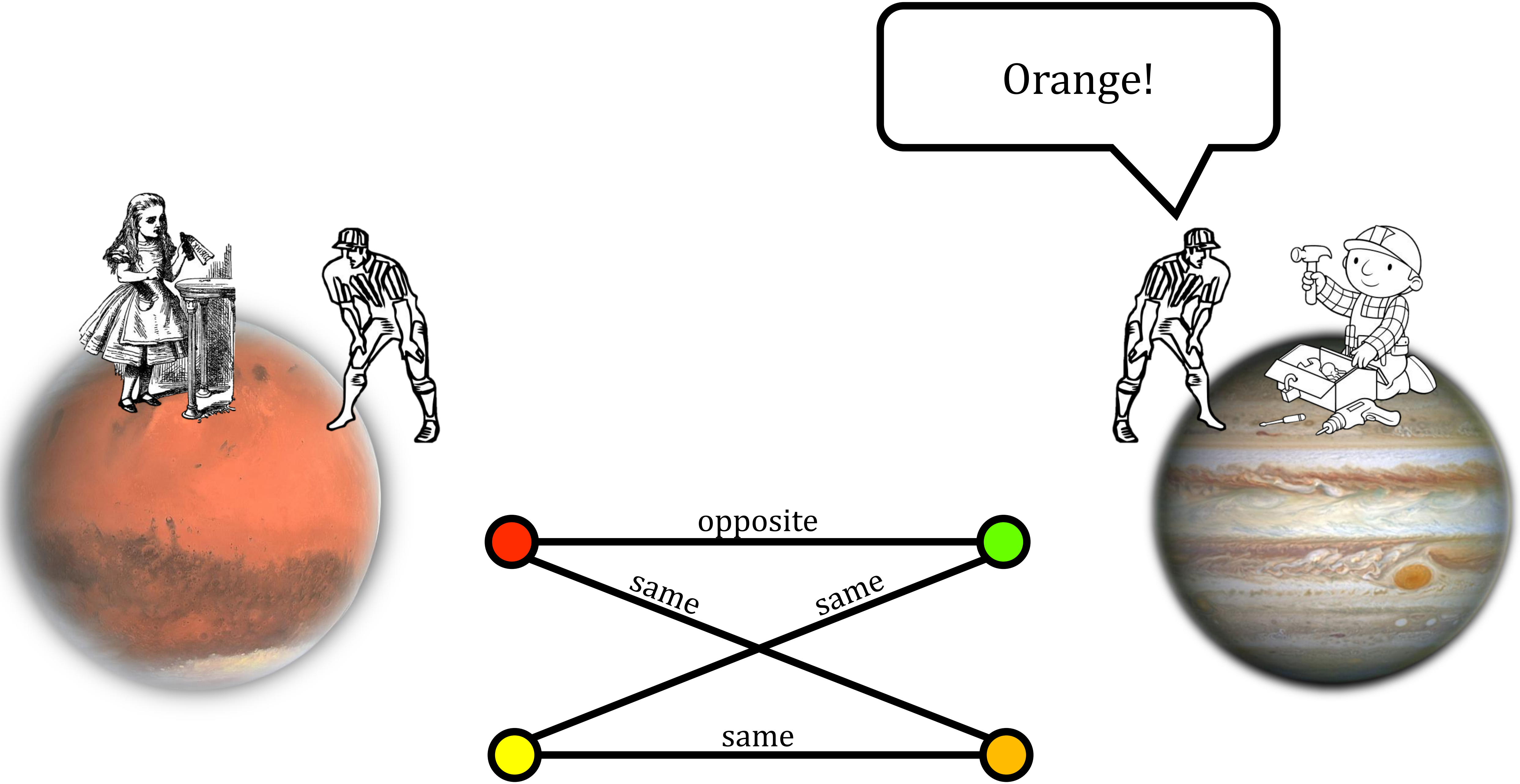

Dotted!
opposite
same
same
same

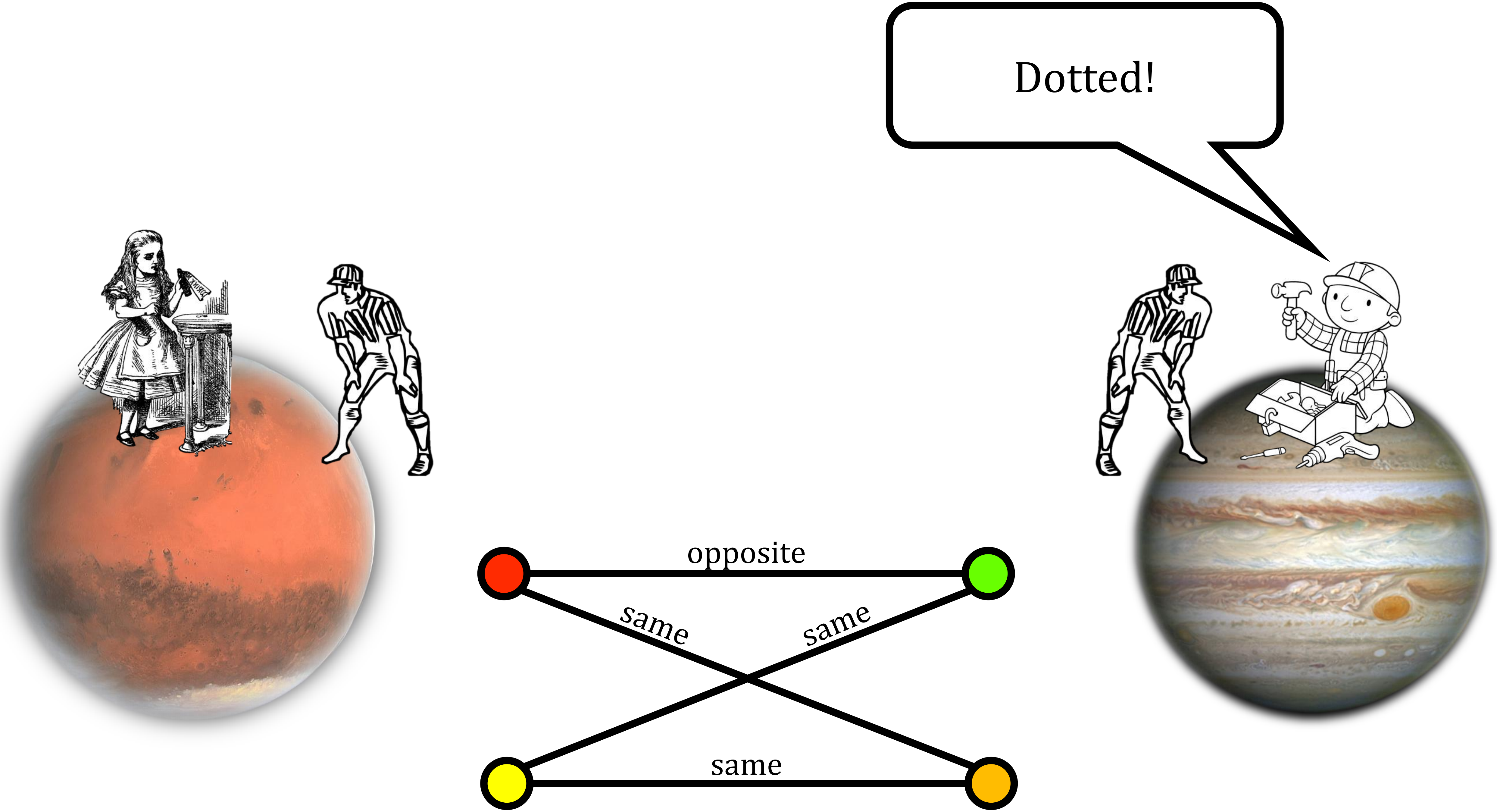
Flips a fair coin to choose
either “Green” or “Orange”,
as a “challenge”.






opposite
same
same
same

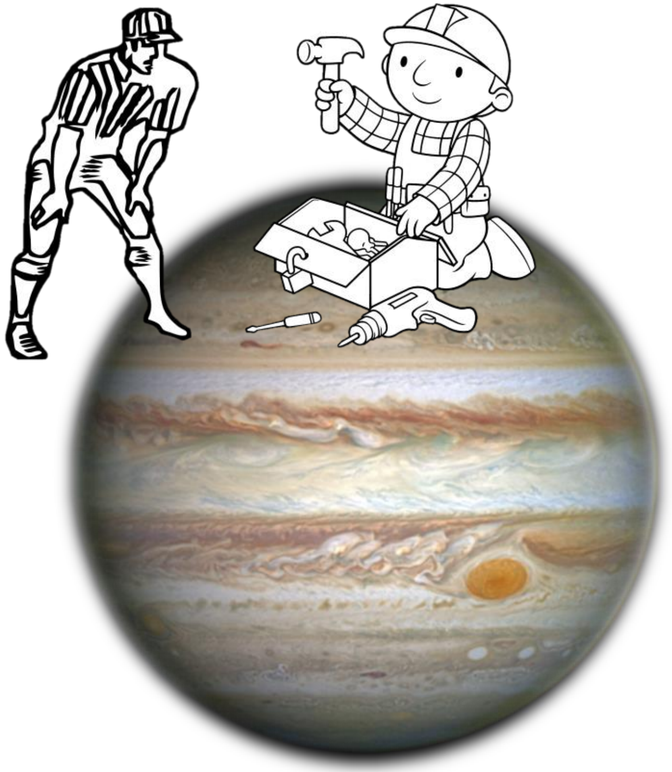

Orange!
opposite
same
same
same

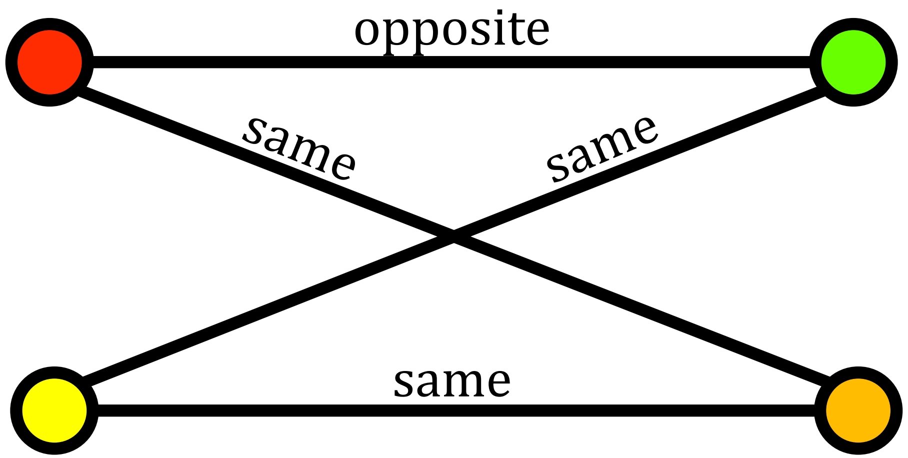

Dotted!
opposite
same
same
same

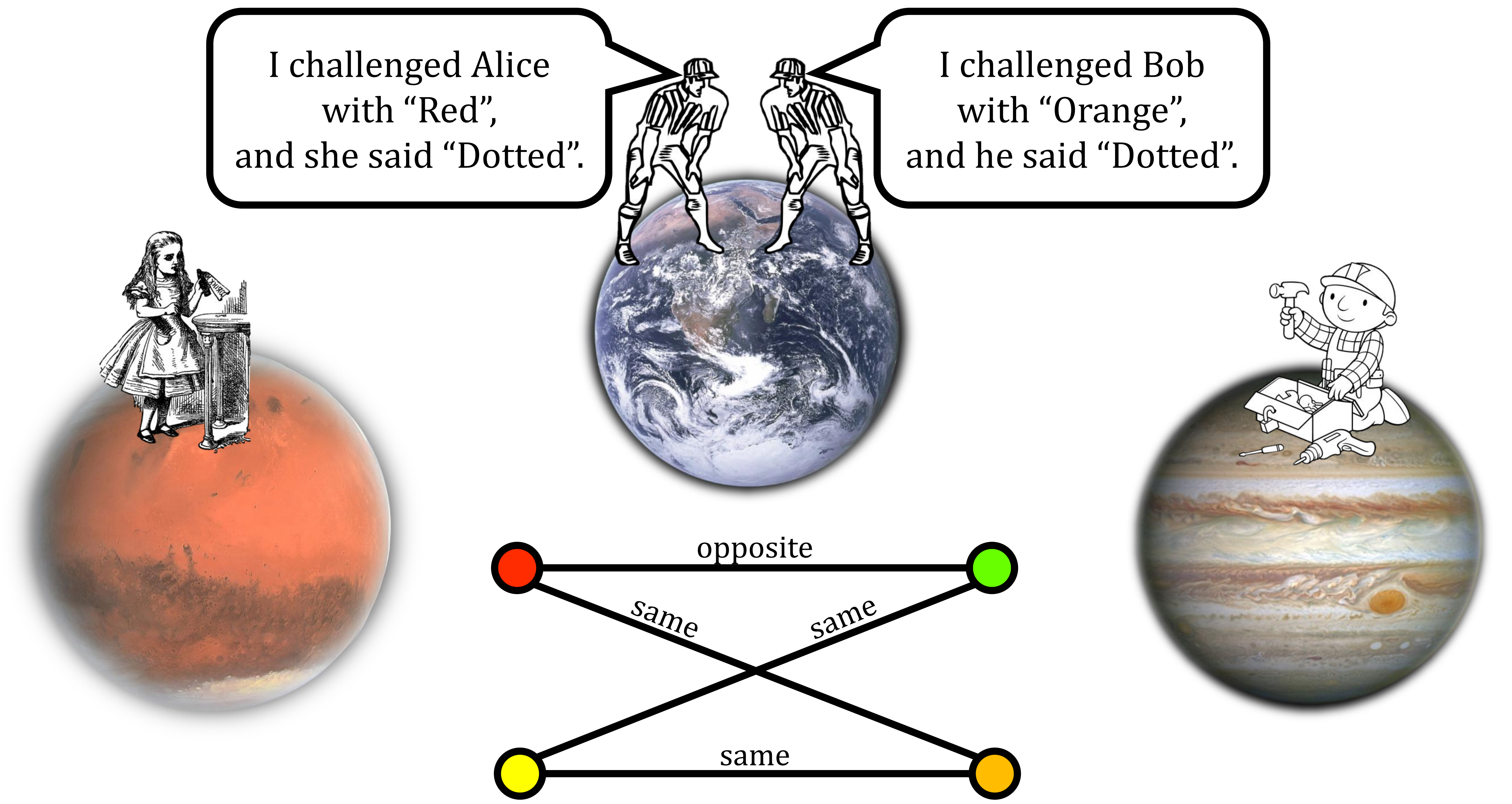
Say the referees:

- synchronize their watches
- challenge at the stroke of midnight, Pittsburgh time
- give Alice and Bob 10 seconds to respond

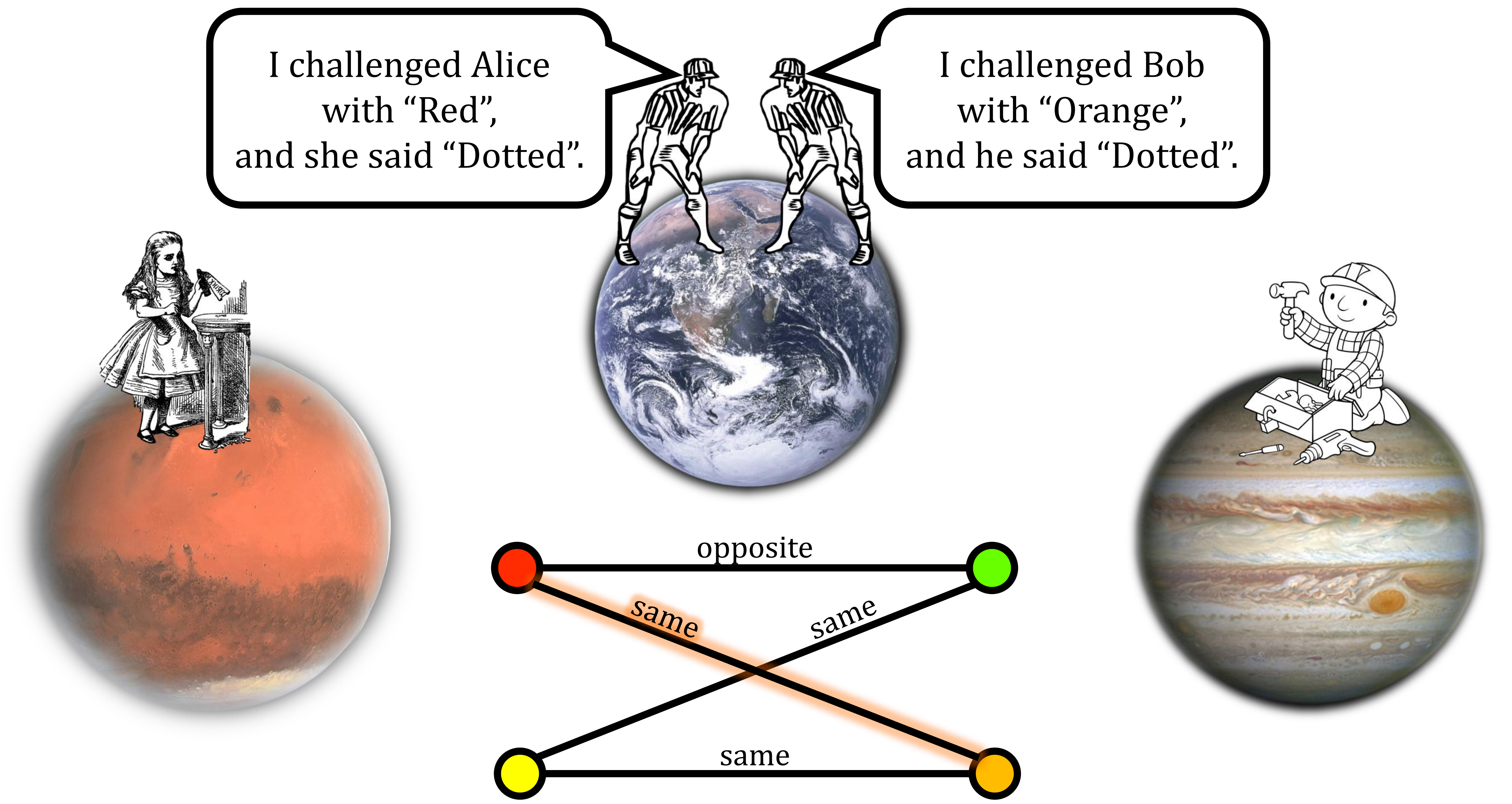
(Mars is at least 30 light-minutes from Jupiter; no time for Alice to secretly communicate with Bob.)

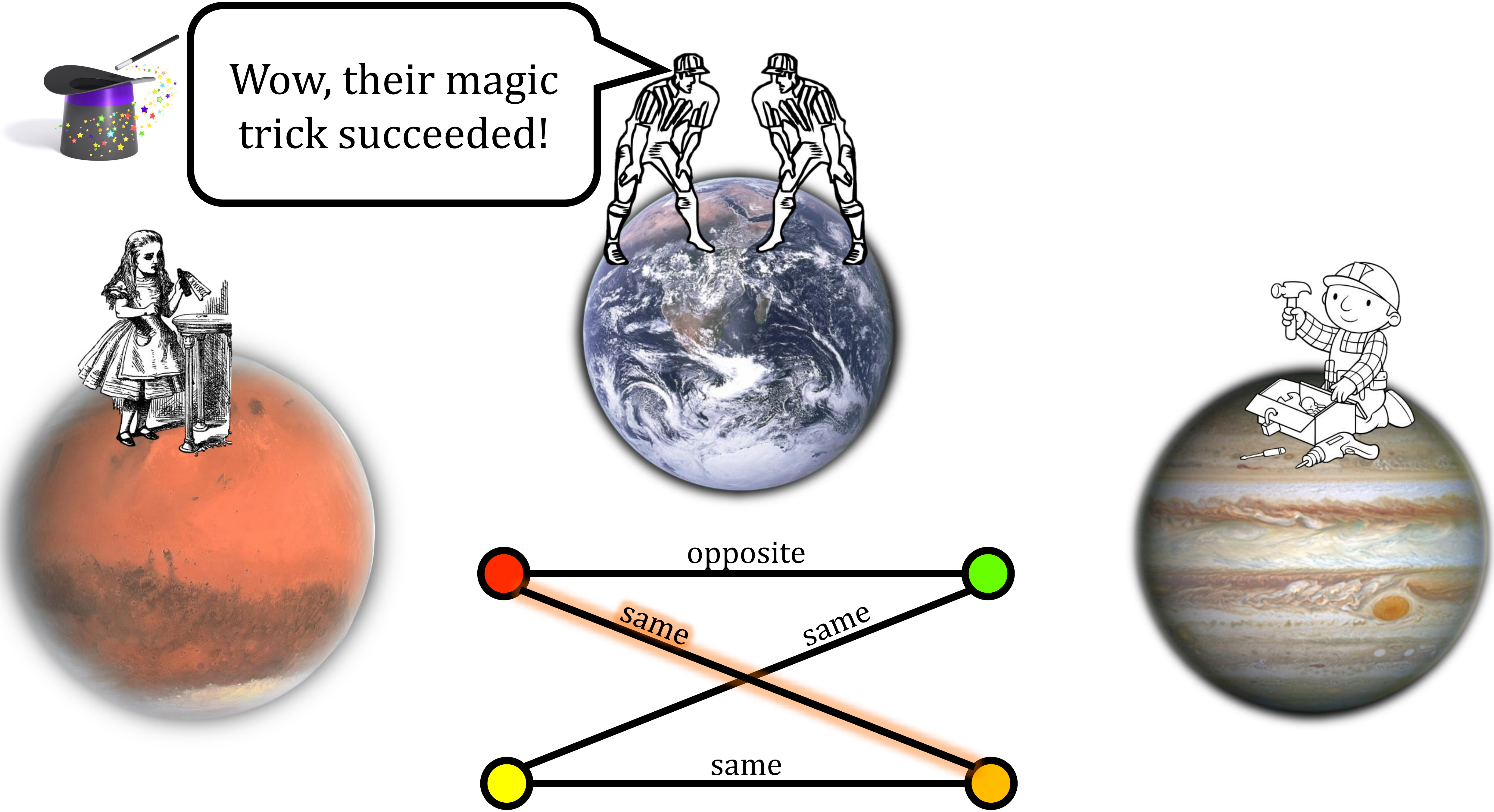


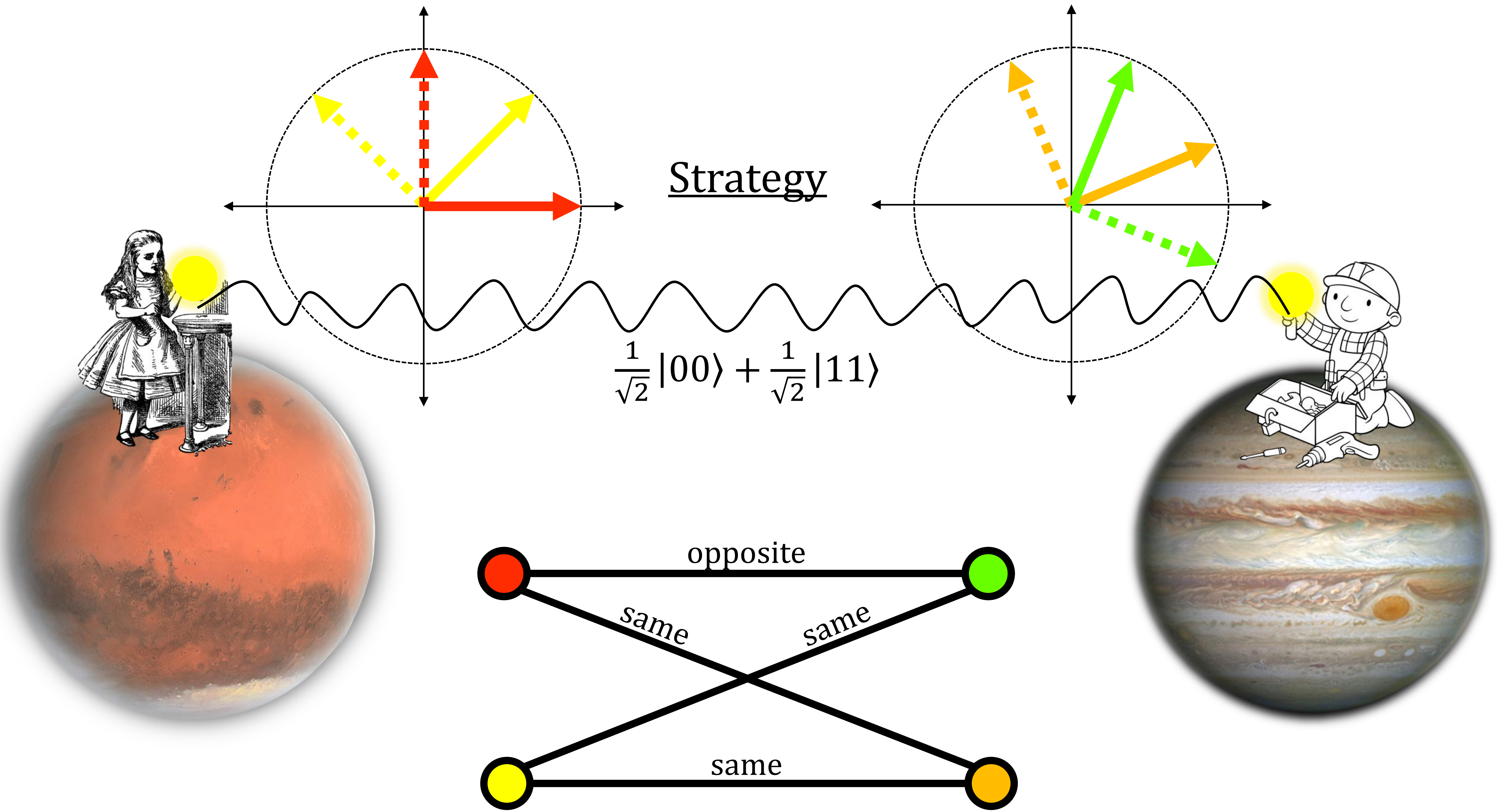


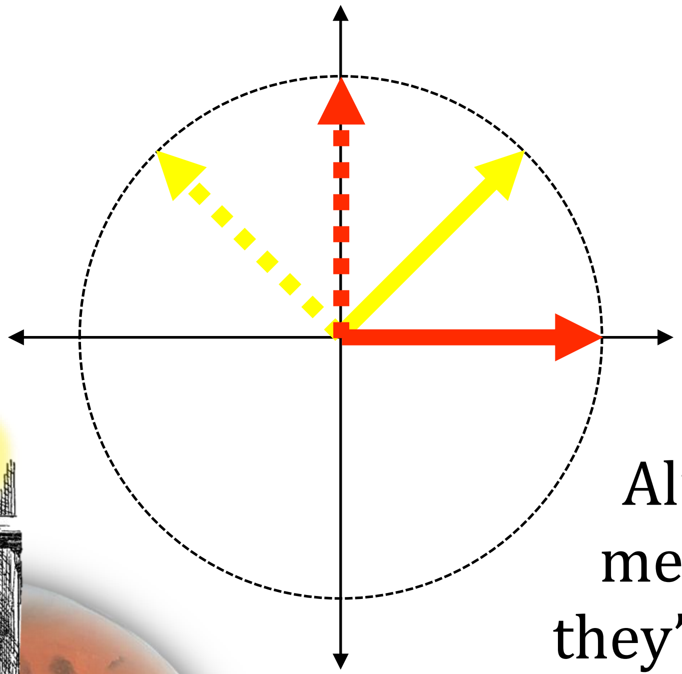


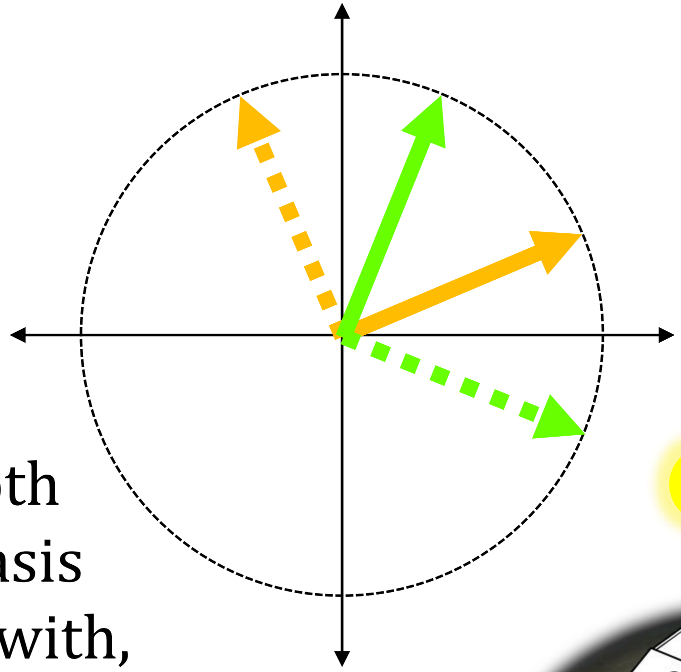
I challenged Alice with “Red”, and she said “Dotted”.
I challenged Bob with “Orange”, and he said “Dotted”.
opposite
same
same
same


I challenged Alice with “Red”, and she said “Dotted”.
I challenged Bob with “Orange”, and he said “Dotted”.
opposite
same
same
same


Wow, their magic trick succeeded!
opposite
same
same
same

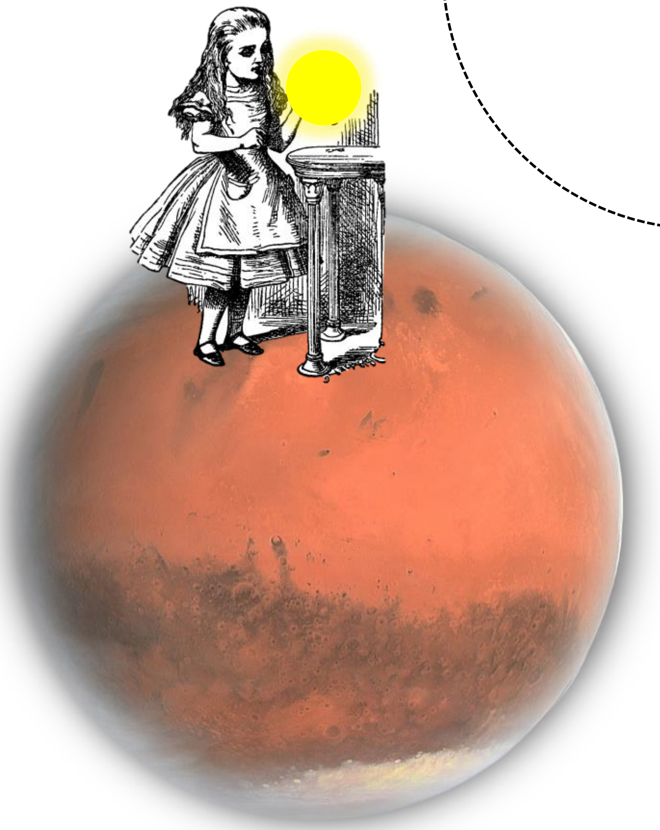

Strategy
$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
opposite
same
same
same

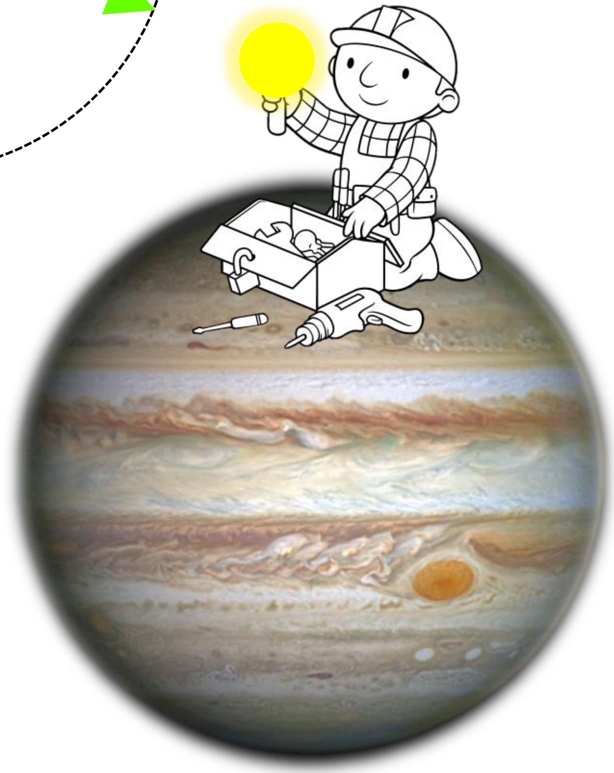


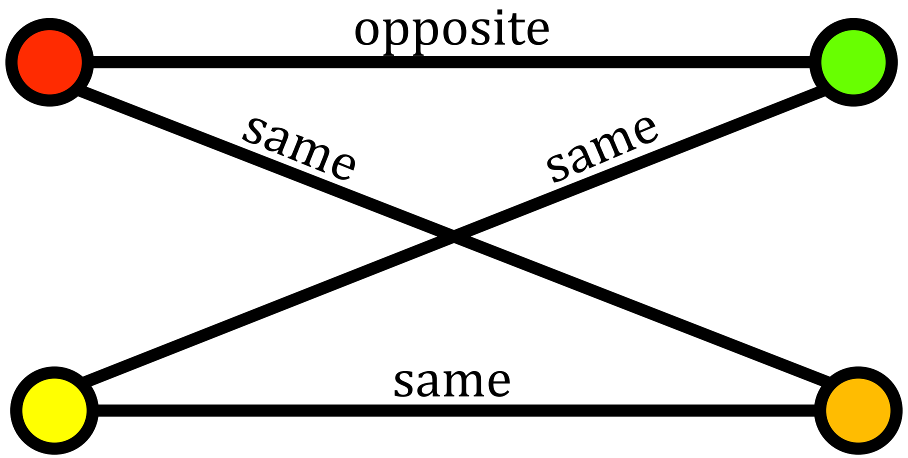


## Strategy

Alice and Bob both measure in the basis they’re challenged with, and respond with the outcome.

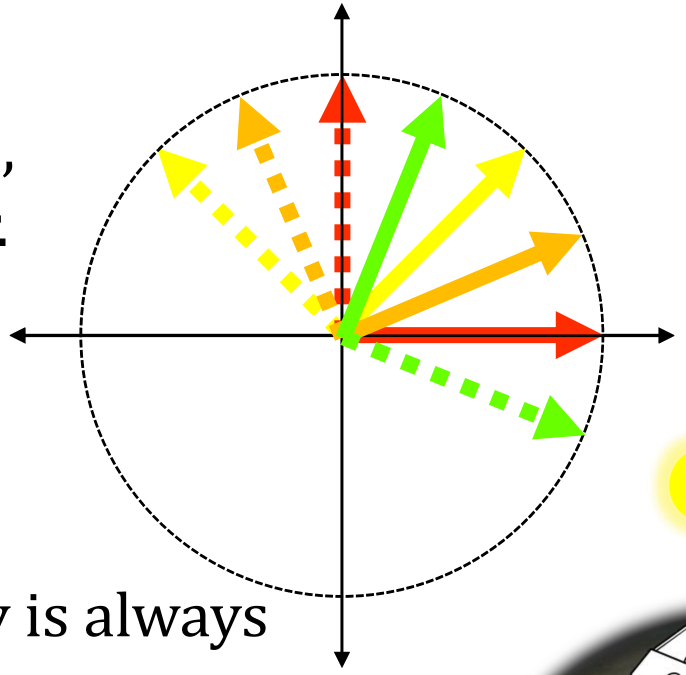




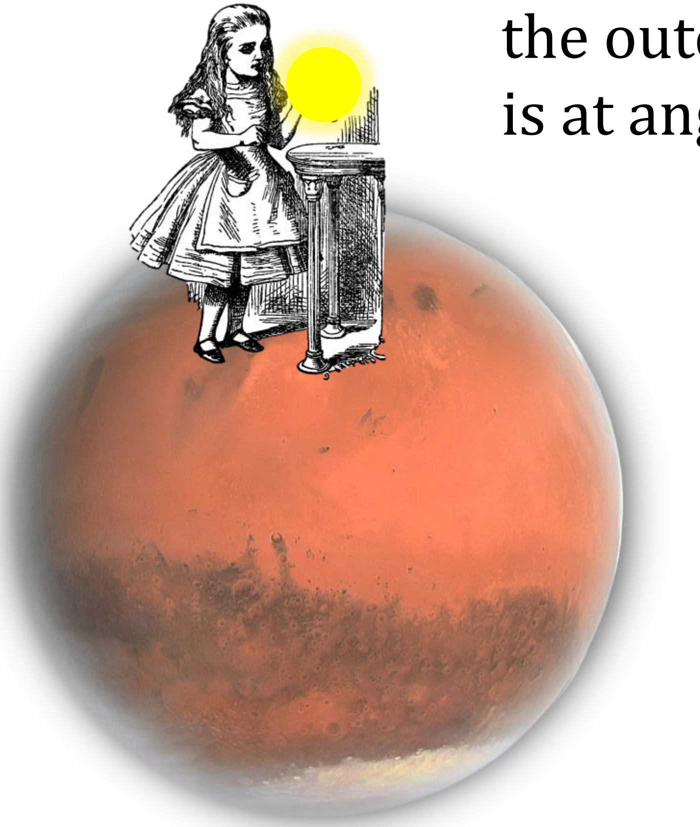


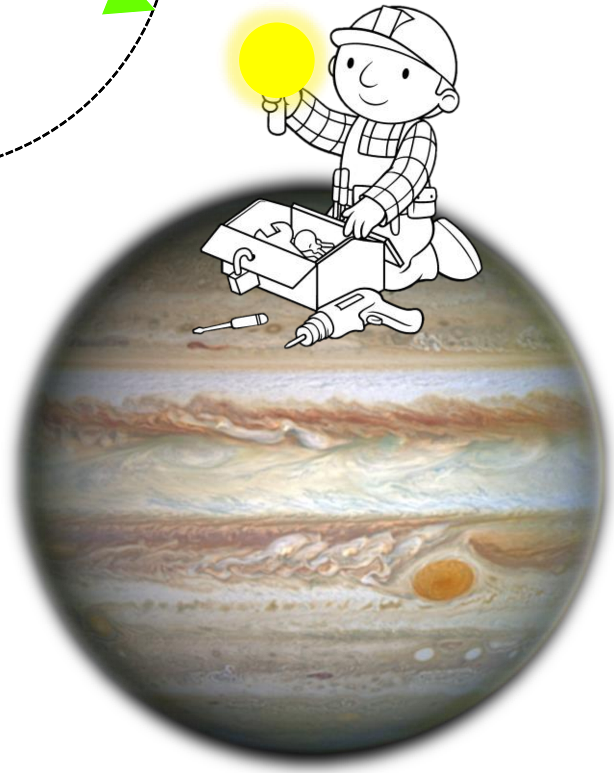
<u>Analysis</u>: May assume Alice measures first.

Whichever outcome vector she receives,
Bob's qubit snaps to the same vector.



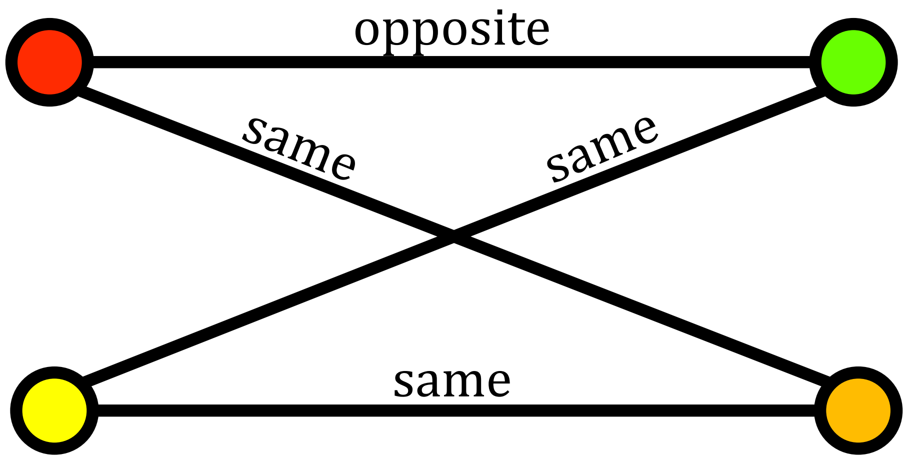
In all cases, when Bob measures,
the outcome vector Bob "wants"
is at angle 22.5° from his state.





Hence success probability is always

$$\cos(22.5°)^2 = \frac{1}{2} + \frac{1}{2\sqrt{2}} \approx \mathbf{85\%}$$

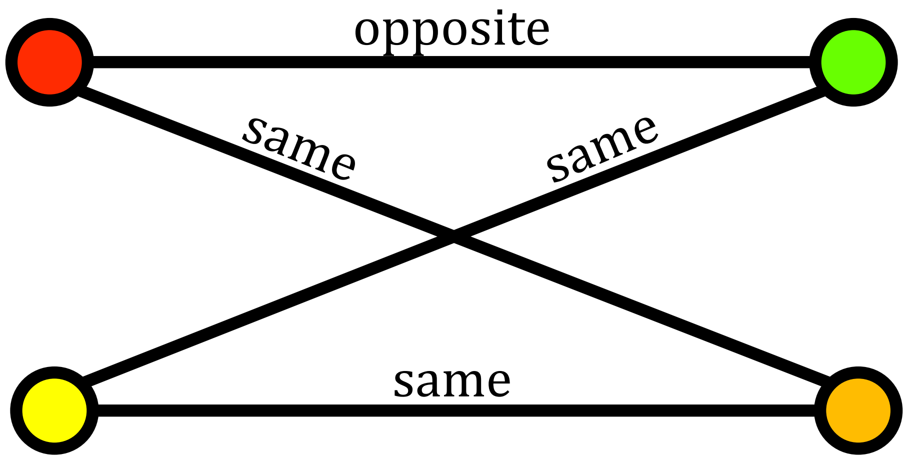


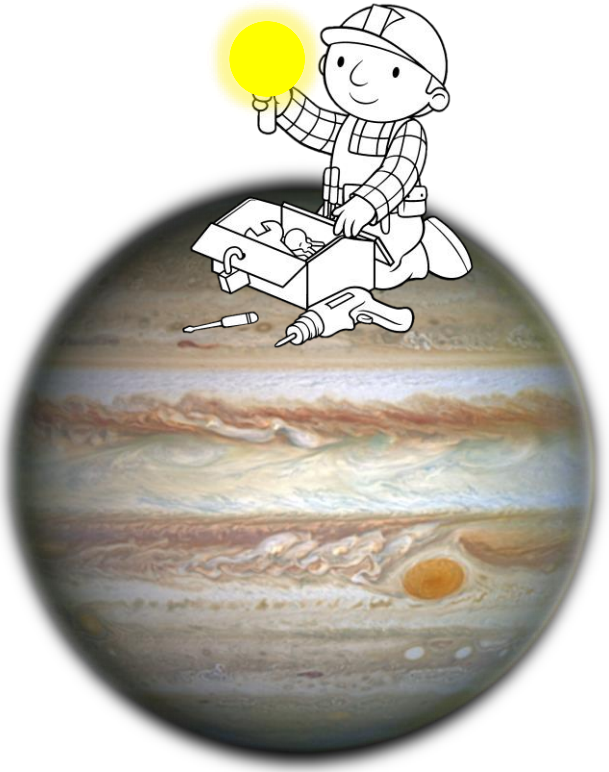
With “shared entanglement” (1 EPR pair) and no communication, Alice and Bob can succeed with probability **85%**.

Is that impressive?

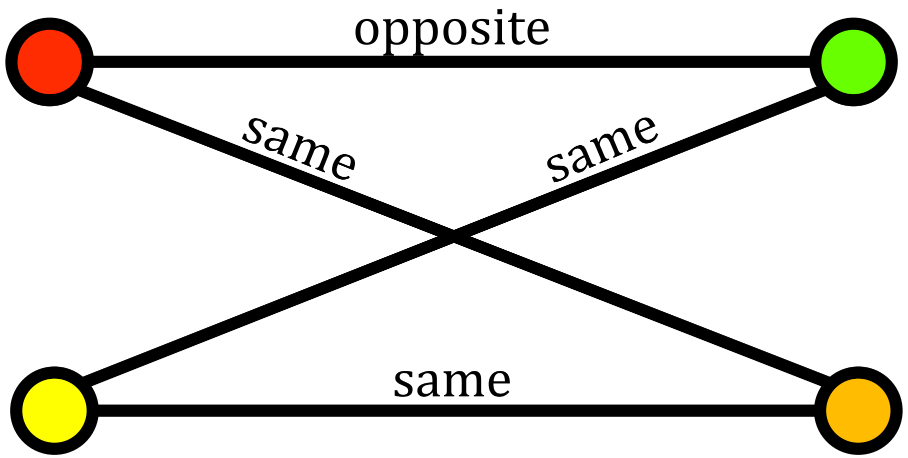
Hence success probability is always

$$\cos(22.5°)^2 = \frac{1}{2} + \frac{1}{2\sqrt{2}} \approx \mathbf{85\%}$$



With **communication allowed**,
Alice and Bob can succeed with probability **100%**!



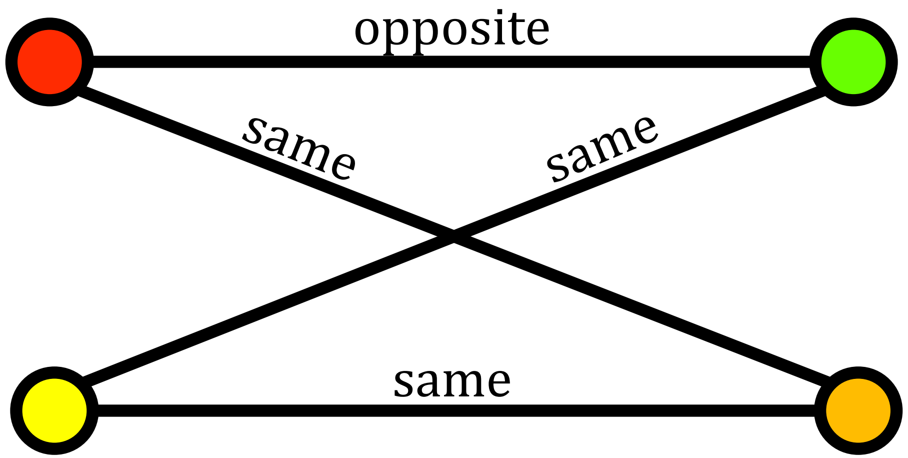

What if Alice and Bob are **deterministic**?

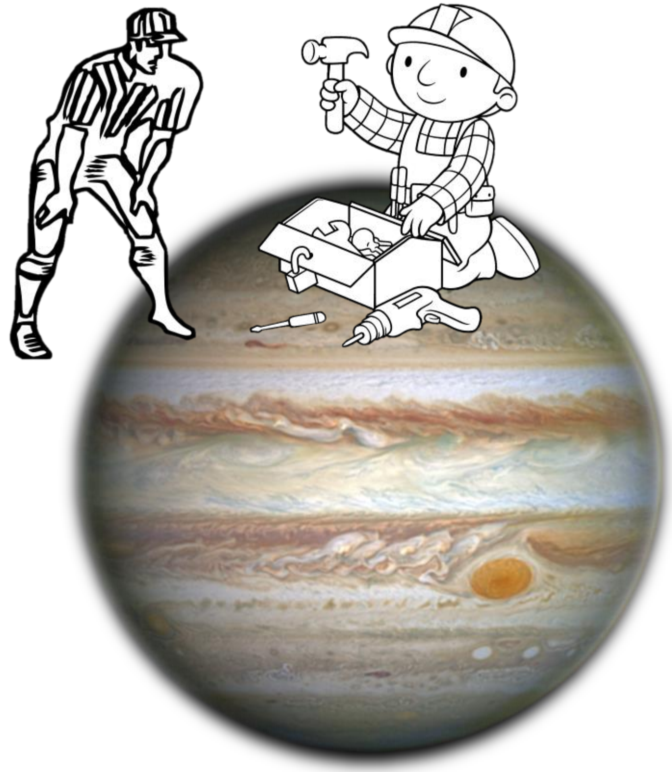
Alice( ) = Solid/Dotted

Alice( ) = Solid/Dotted

Bob( ) = Solid/Dotted

Bob( ) = Solid/Dotted



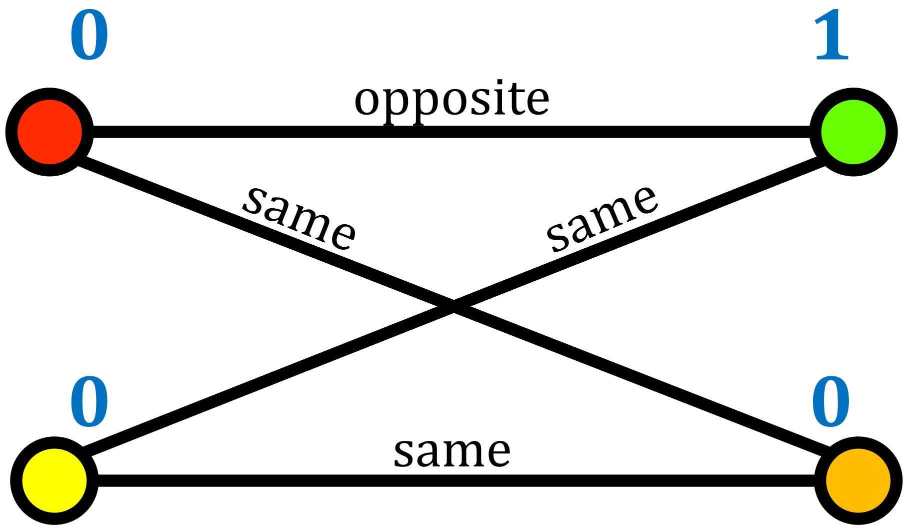
What if Alice and Bob are **deterministic**?

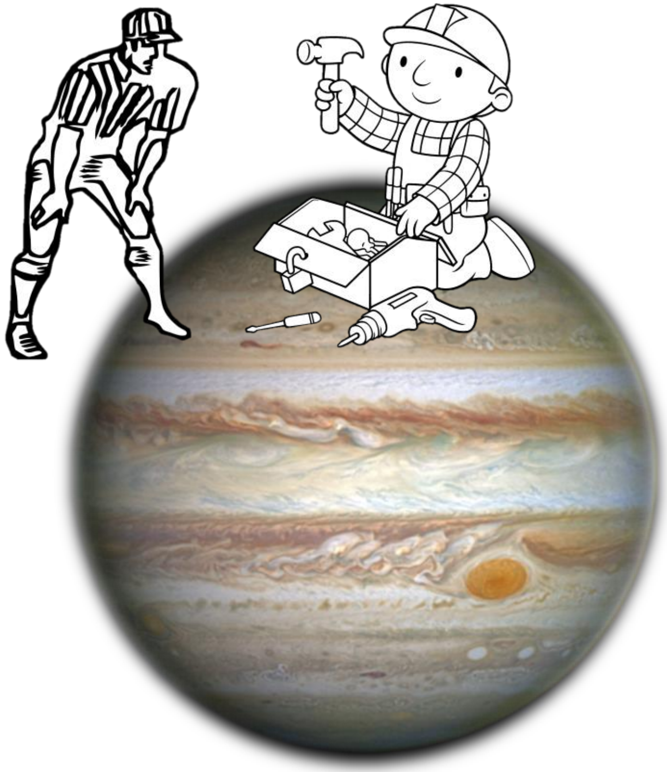
Alice(🔴) = **0**/**1**

Alice(🟡) = **0**/**1**

Bob(🟢) = **0**/**1**

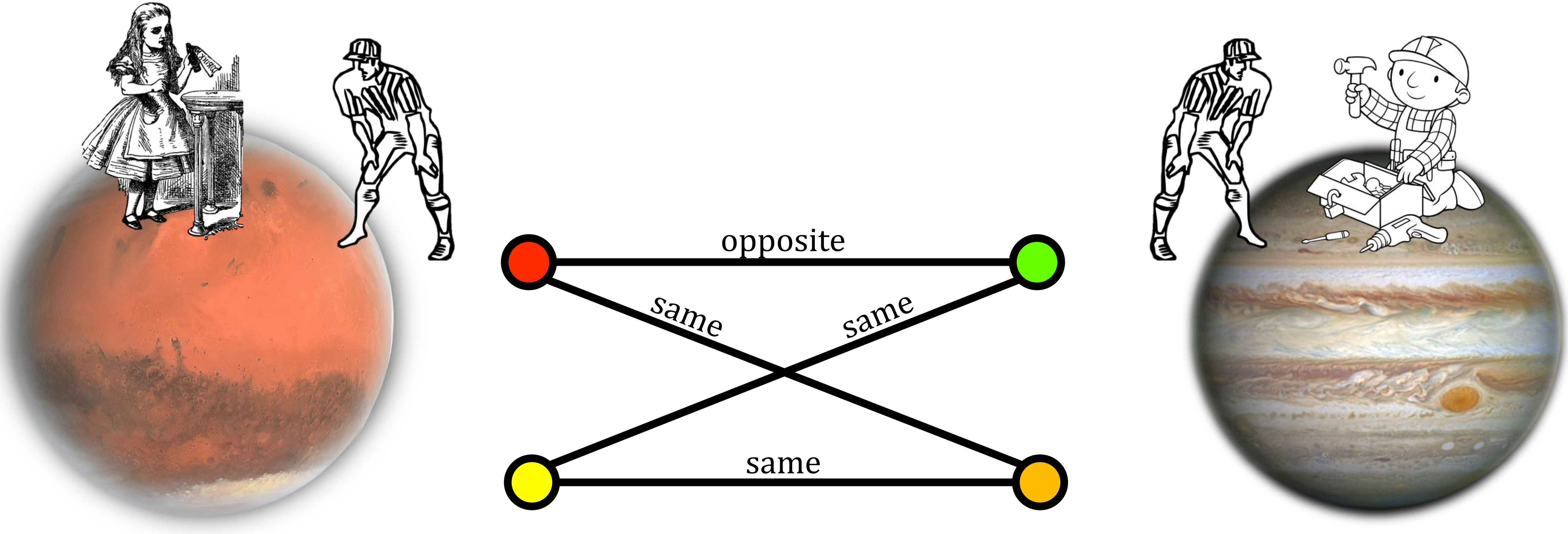
Bob(🟠) = **0**/**1**



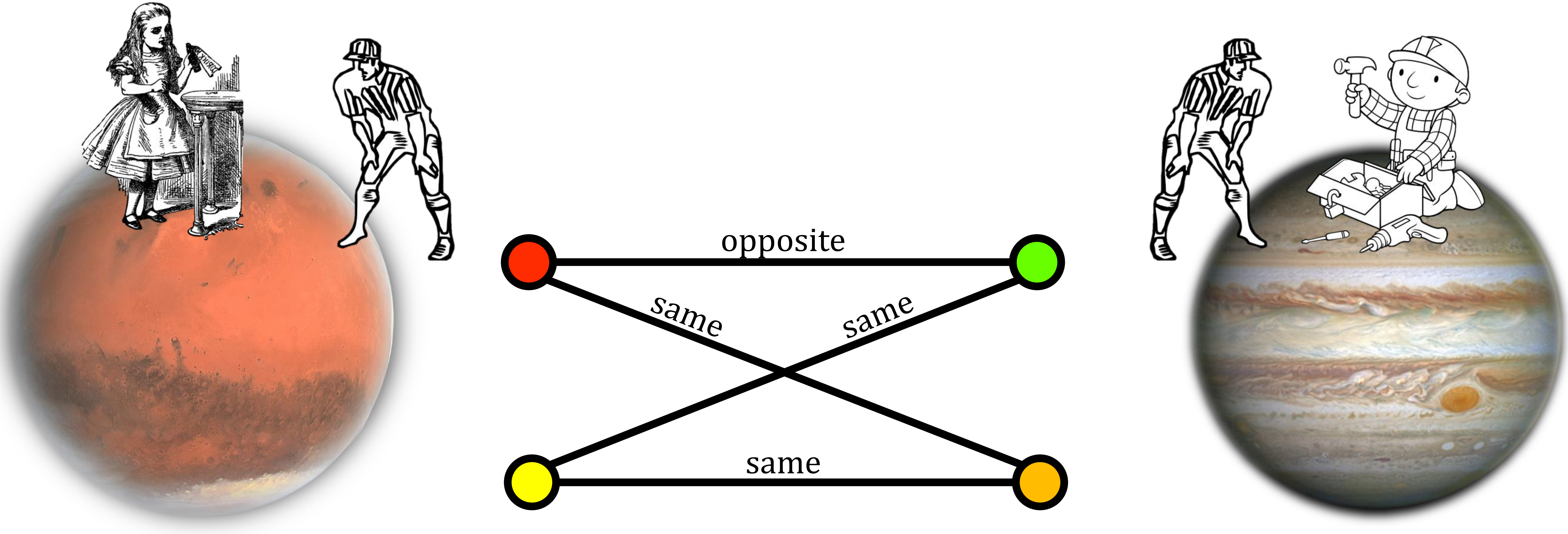



What if Alice and Bob are **deterministic**?

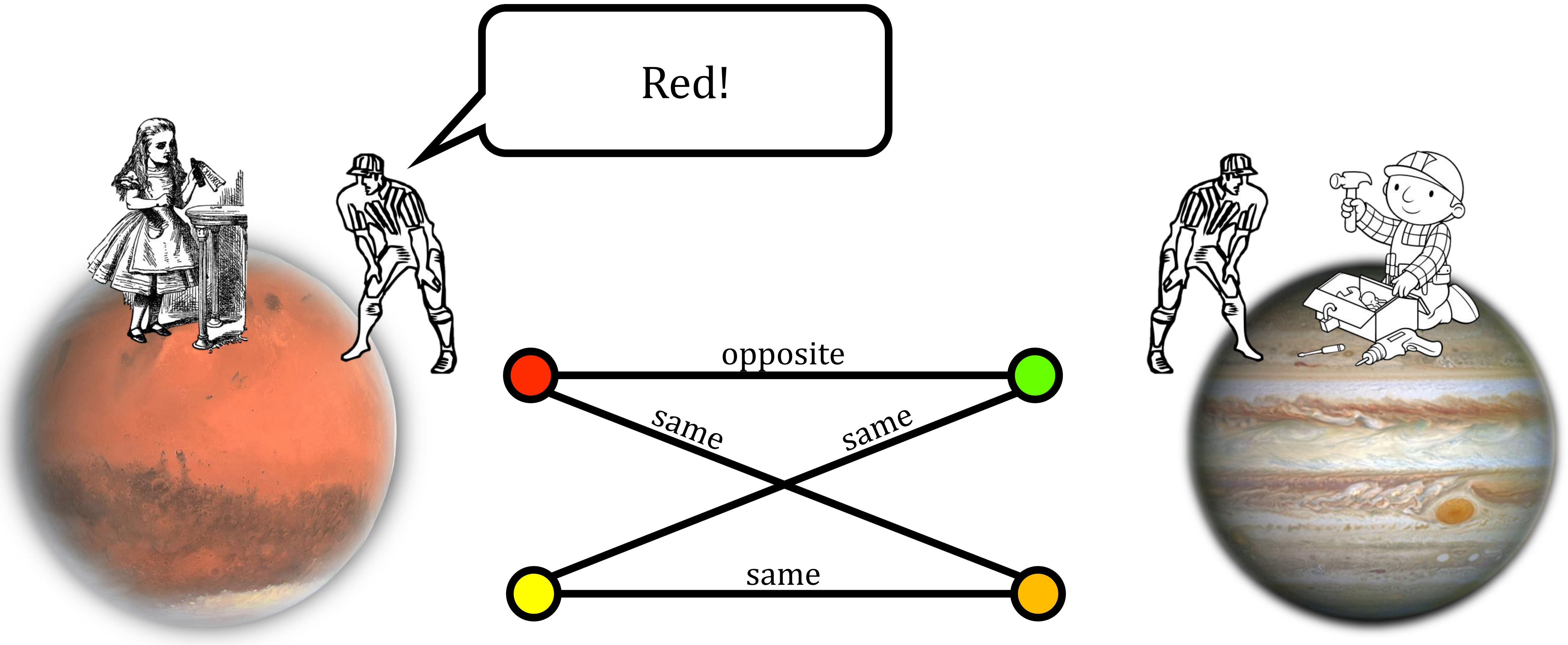
Success probability $\leq \frac{3}{4} = \mathbf{75\%}$



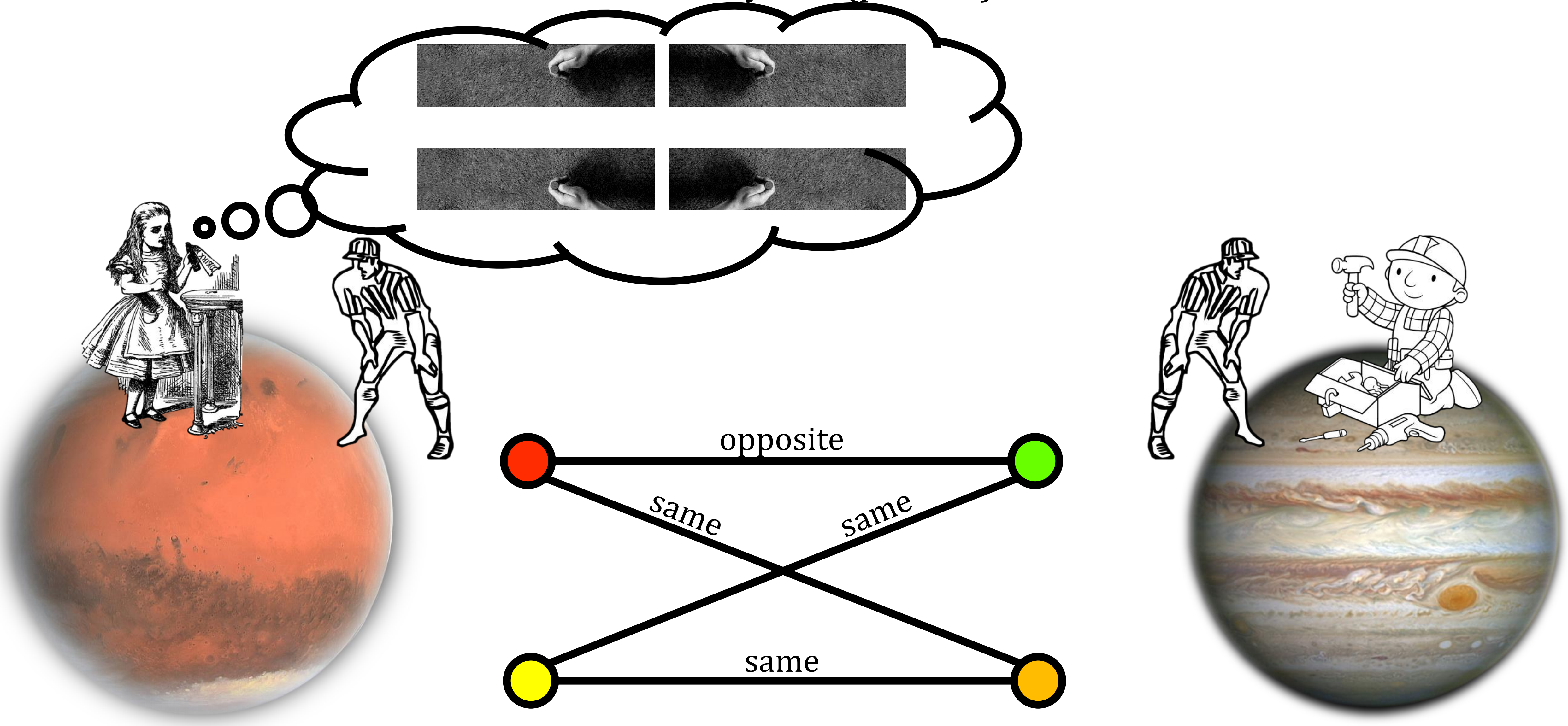
What if Alice and Bob may use (private) **randomness**?



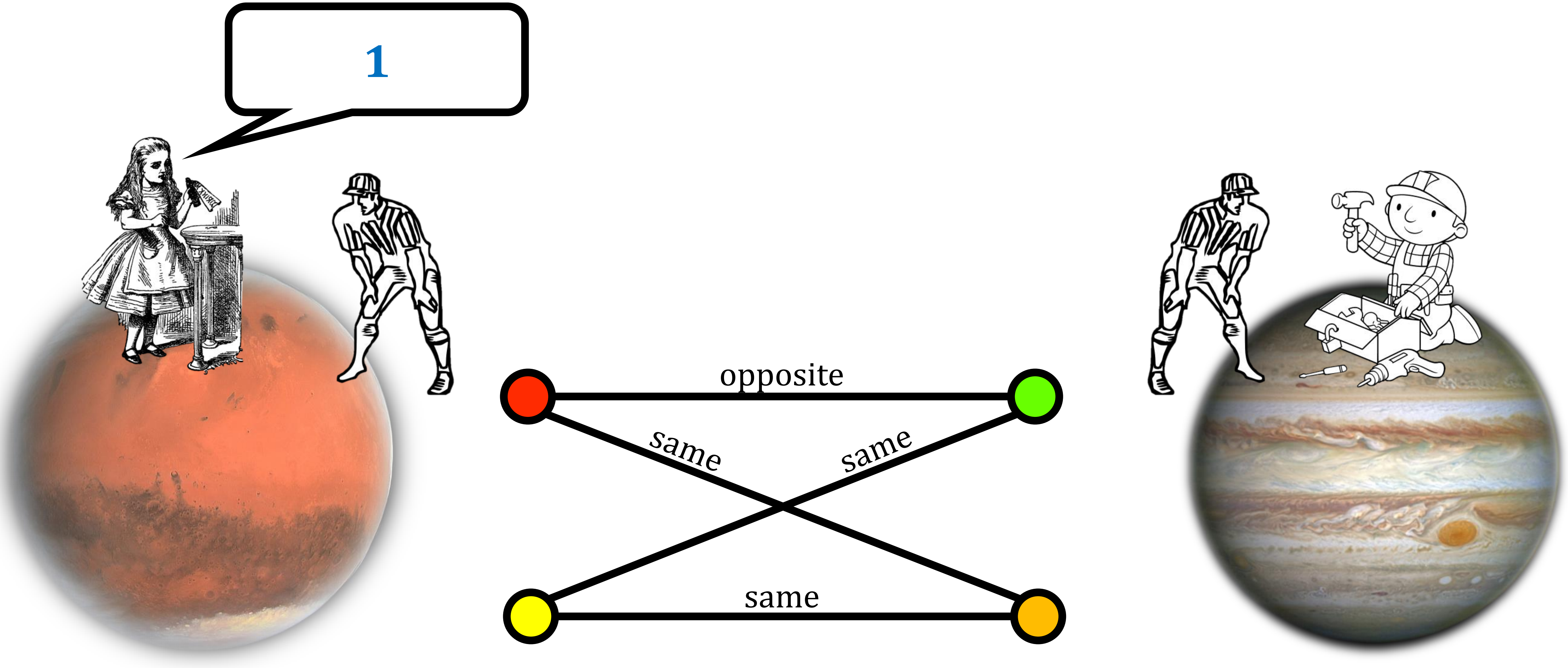
What if Alice and Bob may use (private) **randomness**?



What if Alice and Bob may use (private) **randomness**?



# What if Alice and Bob may use (private) **randomness**?



## What if Alice and Bob may use (private) **randomness**?

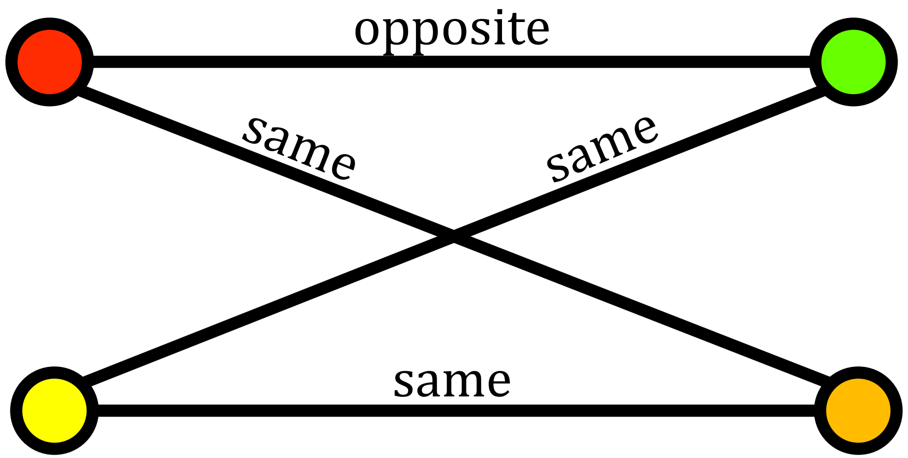
Alice( , her coin flips) = **0/1**

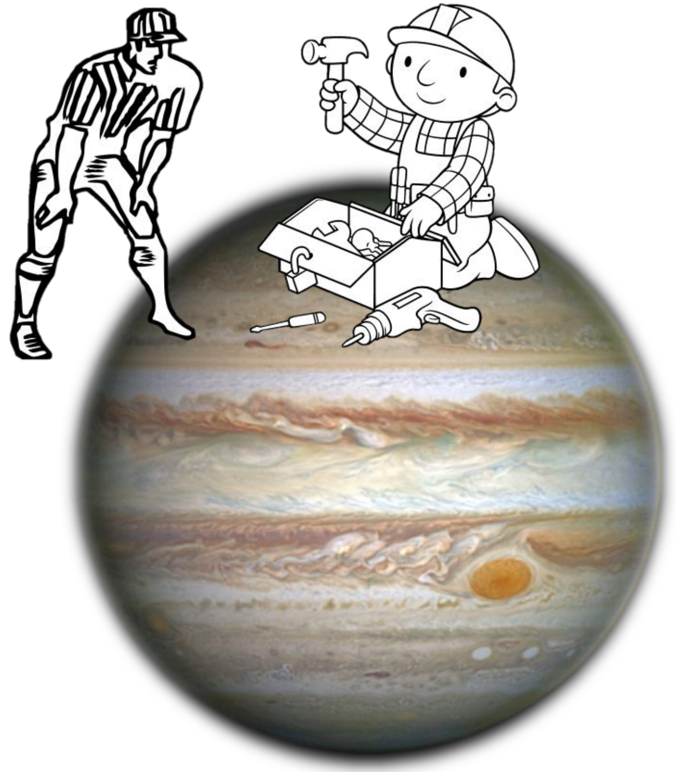
Alice( , her coin flips) = **0/1**

Bob( , his coin flips) = **0/1**

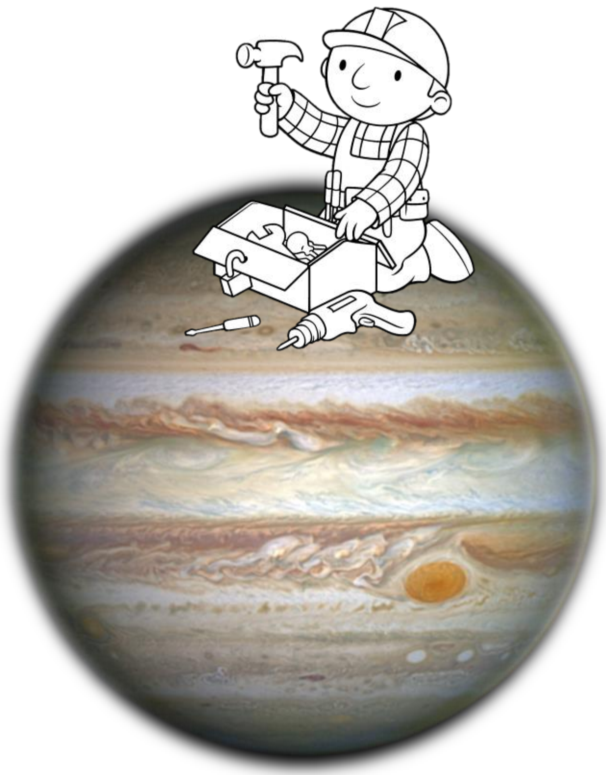
Bob( , his coin flips) = **0/1**

Success probability $\leq$ **75%**

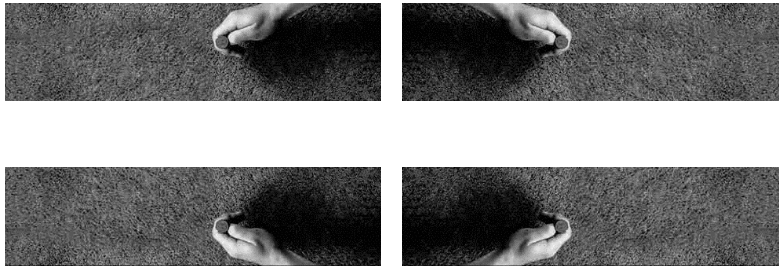


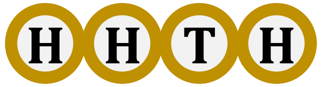
# What if Alice and Bob may use **shared randomness**?



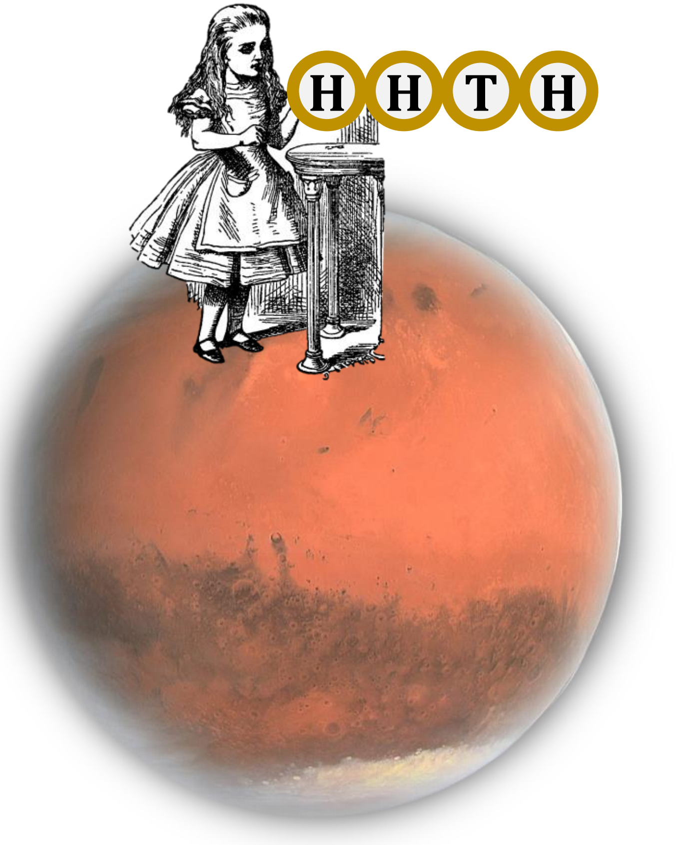
# What if Alice and Bob may use shared **randomness**?

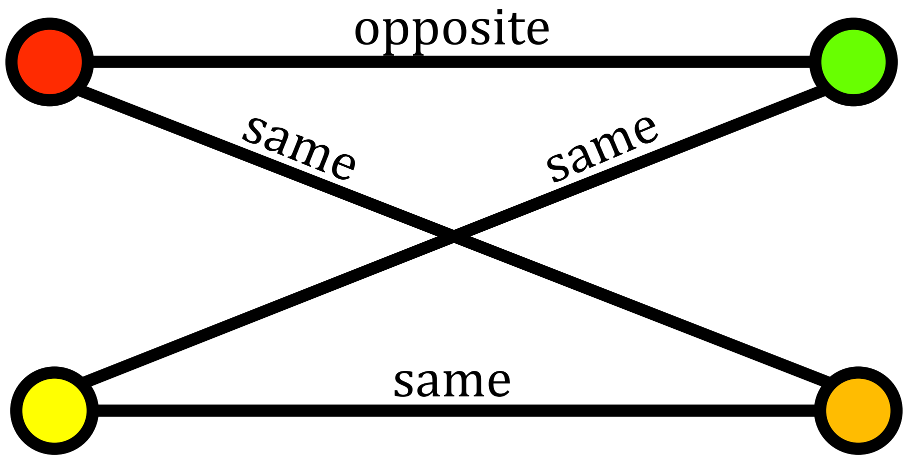


What if Alice and Bob may use **shared randomness**?




What if Alice and Bob may use **shared randomness**?




What if Alice and Bob may use **shared randomness**?

Alice( , their coin flips) = **0/1**

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Bob( , their coin flips) = **0/1**

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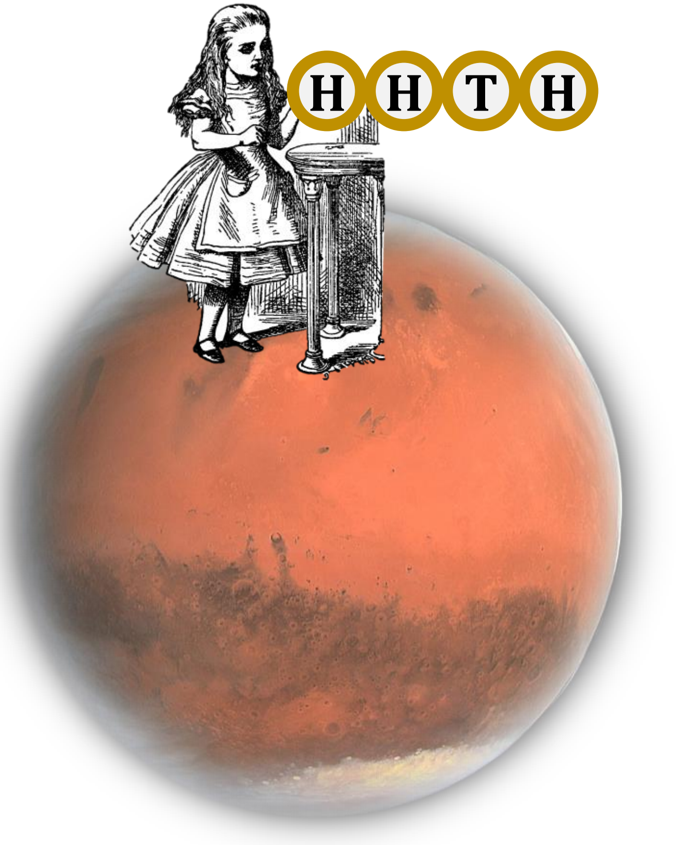


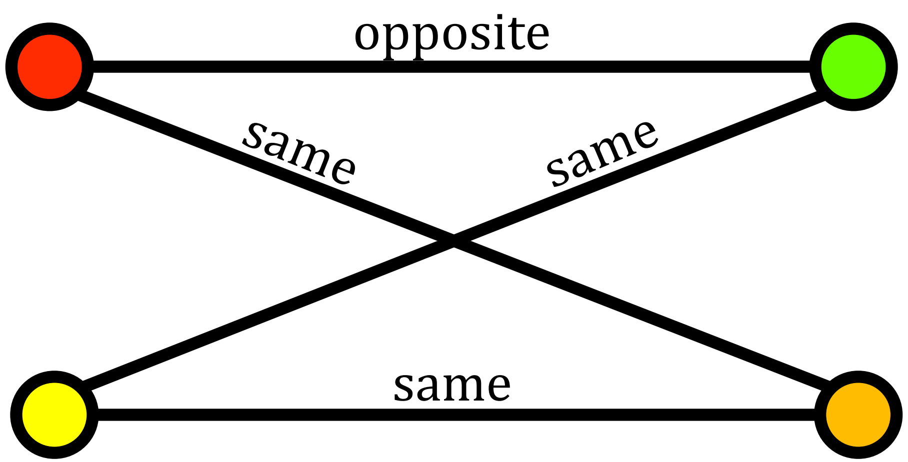


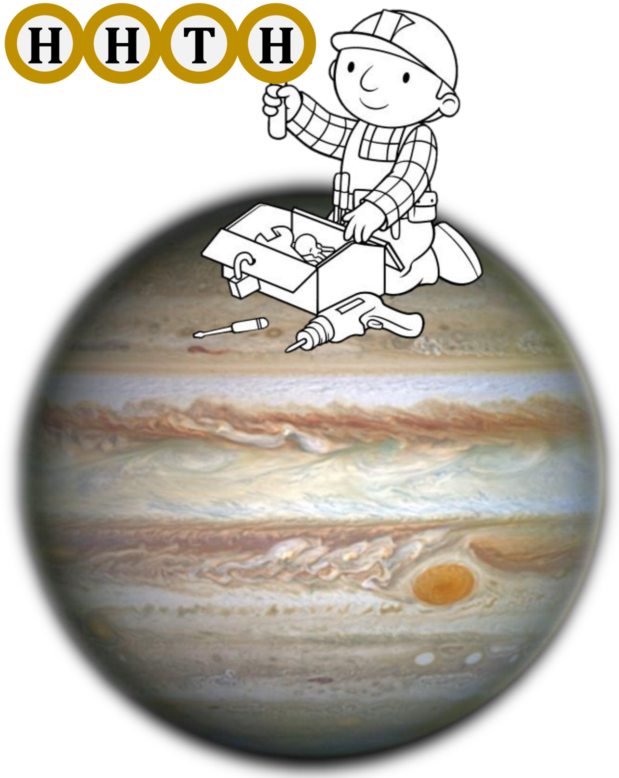

What if Alice and Bob may use **shared randomness**?

Alice(●, their coin flips) = **0/1**  Bob(●, their coin flips) = **0/1**

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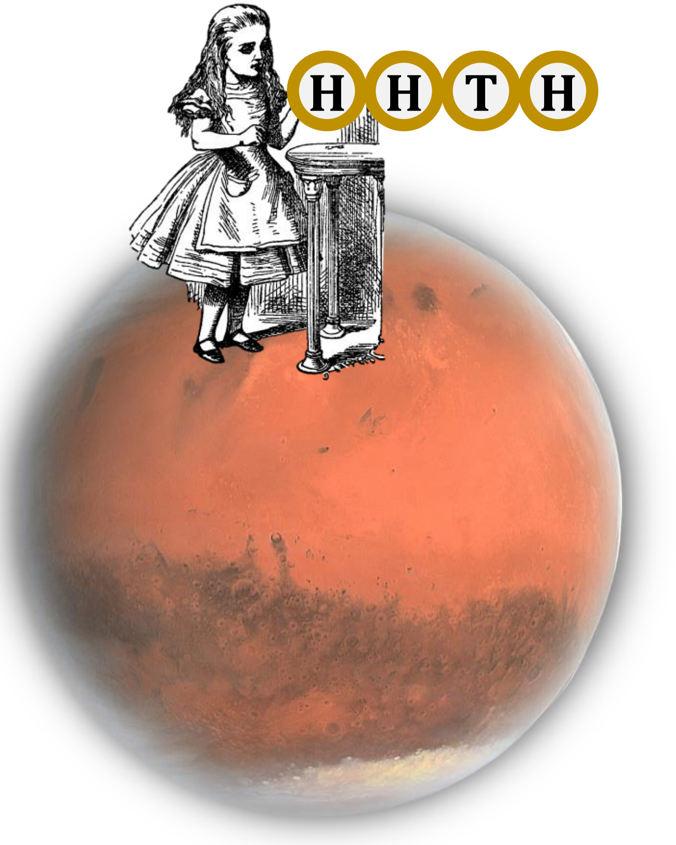
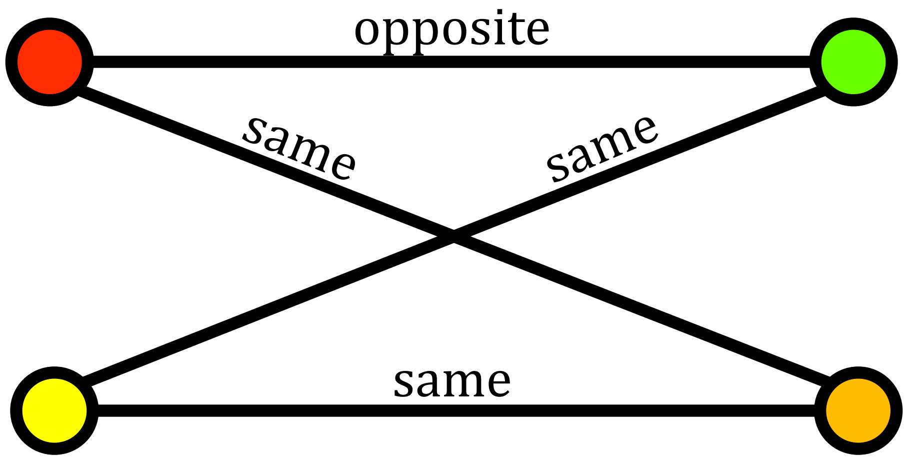



**Claim:**

Success probability still ≤ **75%**

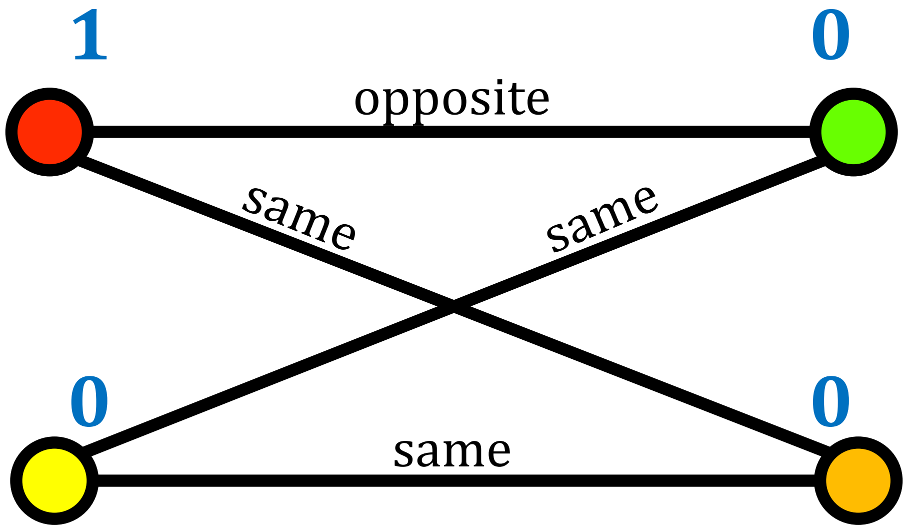



What if Alice and Bob may use **shared randomness**?

Alice(🔴, HHTH) = **0/1**  Bob(🟢, HHTH) = **0/1**

Alice(🟡, HHTH) = **0/1**  Bob(🟠, HHTH) = **0/1**


Conditional success probability $\leq$ **75%**




What if Alice and Bob may use **shared randomness**?

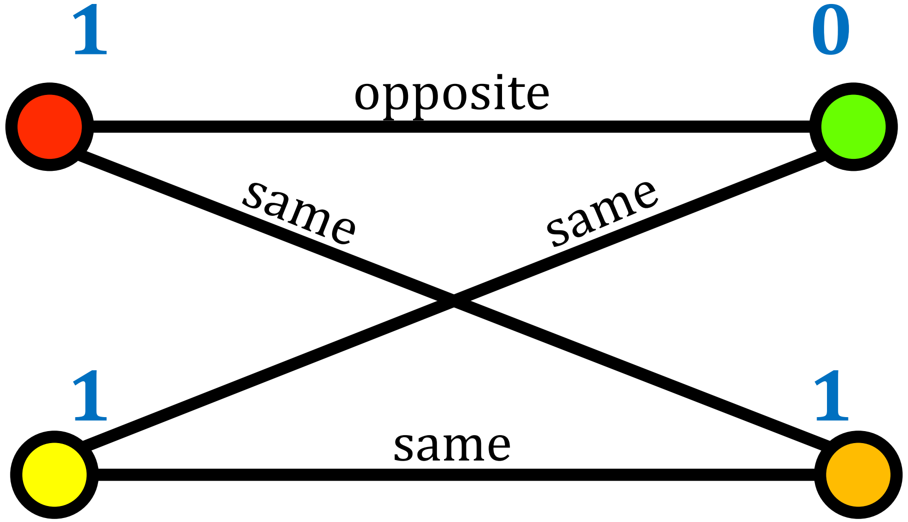
Alice( , T T T T ) = **0/1**

Alice( , T T T T ) = **0/1**

Bob( , T T T T ) = **0/1**

Bob( , T T T T ) = **0/1**


Conditional success probability ≤ **75%**




What if Alice and Bob may use **shared randomness**?

Alice( , their coin flips) = **0/1**

Bob( , their coin flips) = **0/1**

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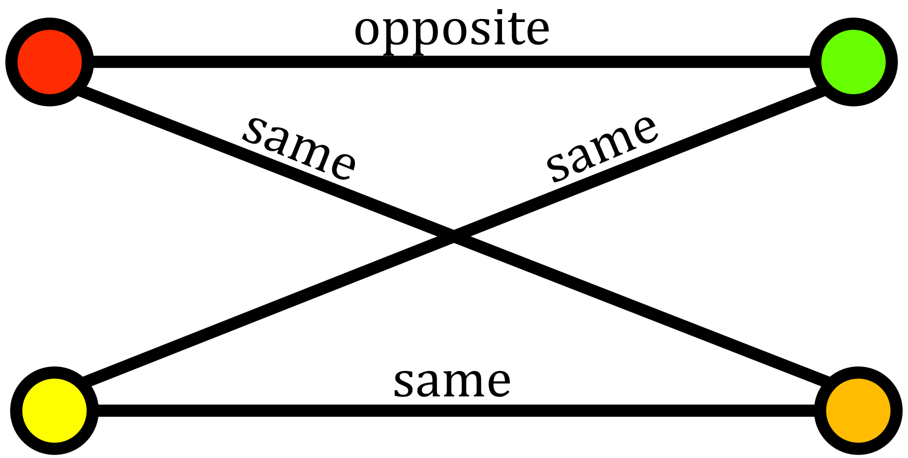
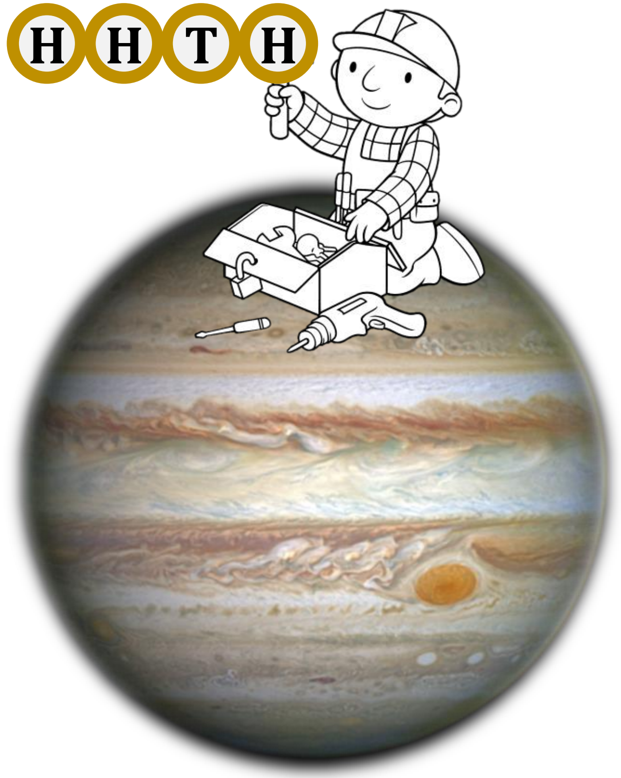
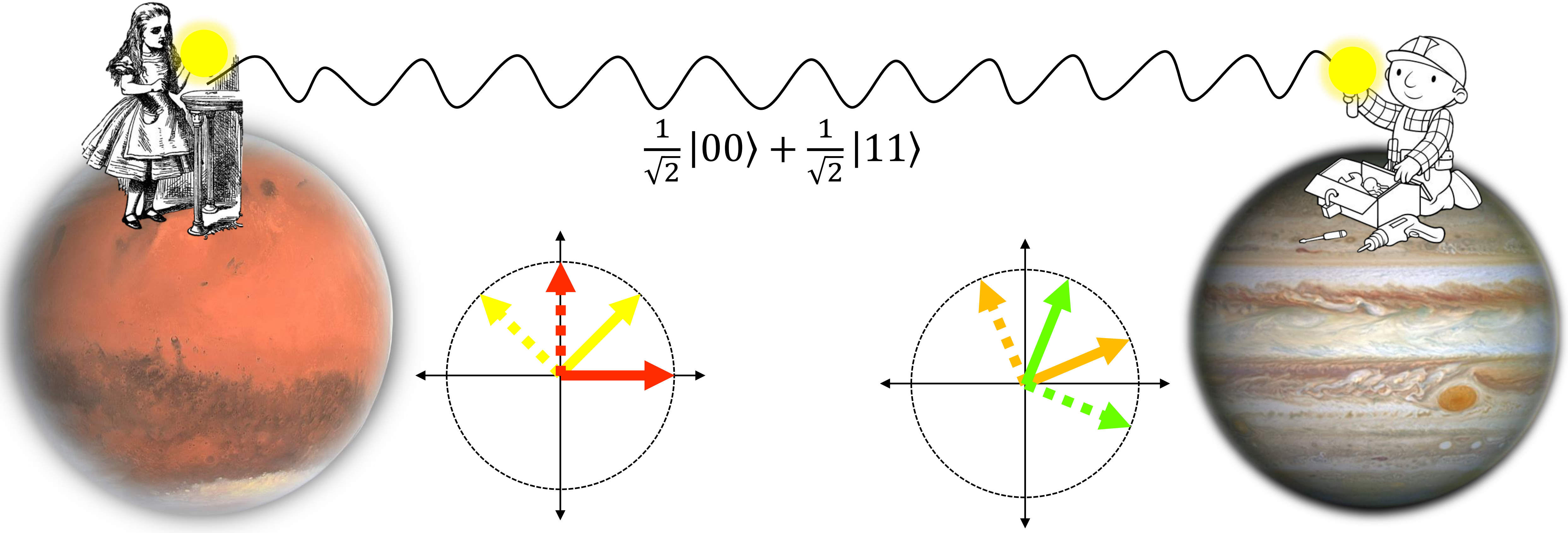
Overall success probability ≤ **75%**



With **shared quantum entanglement**:

Success probability $\geq$ **85%**

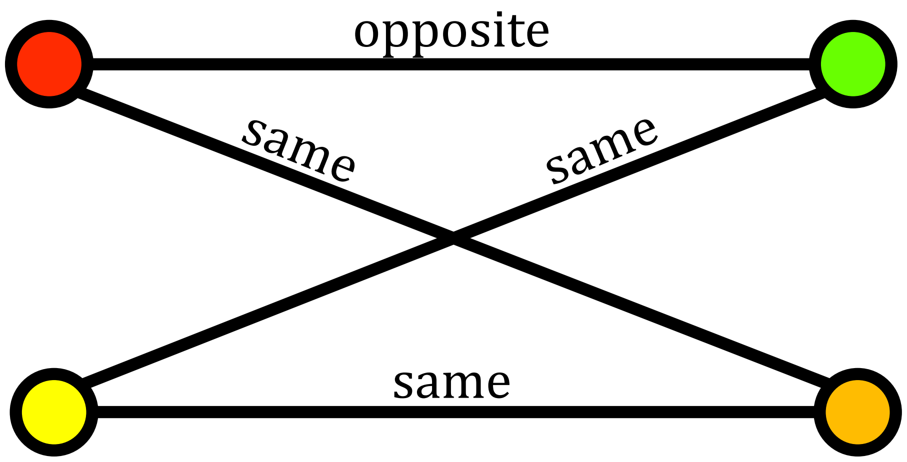


$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

# Summary

Best success probability Alice and Bob
can achieve in ~~this Magic Trick...~~ the "CHSH experiment"...

| | |
|---|---|
| Deterministic: | 75% |
| Private randomness: | 75% |
| Shared randomness: | 75% |
| *Shared quantum entanglement:* | **85%** |




(Tsirelson 1980: The 85% strategy
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# Summary

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(Tsirelson 1980: The 85% strategy we saw is optimal.)

**J. Bell** (1964)

**C**lauser – **H**orne – **S**himony – **H**olt (1969)

# Summary

Best success probability Alice and Bob can achieve in the "CHSH experiment"…

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**J. Bell** (1964)

Bell's Inequality

"Bell Inequality violation"

# Summary

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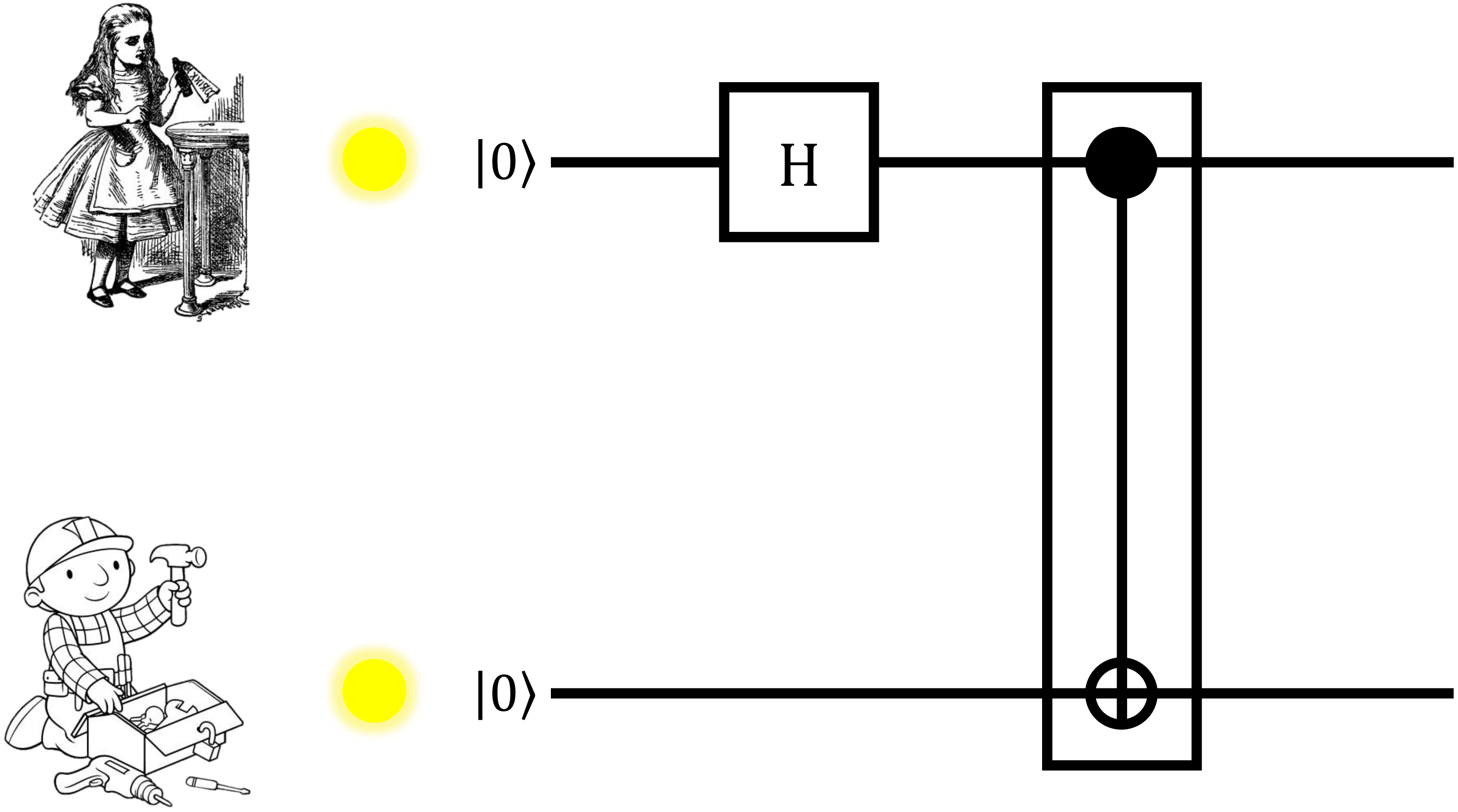
"Local Hidden Variables"


|0⟩
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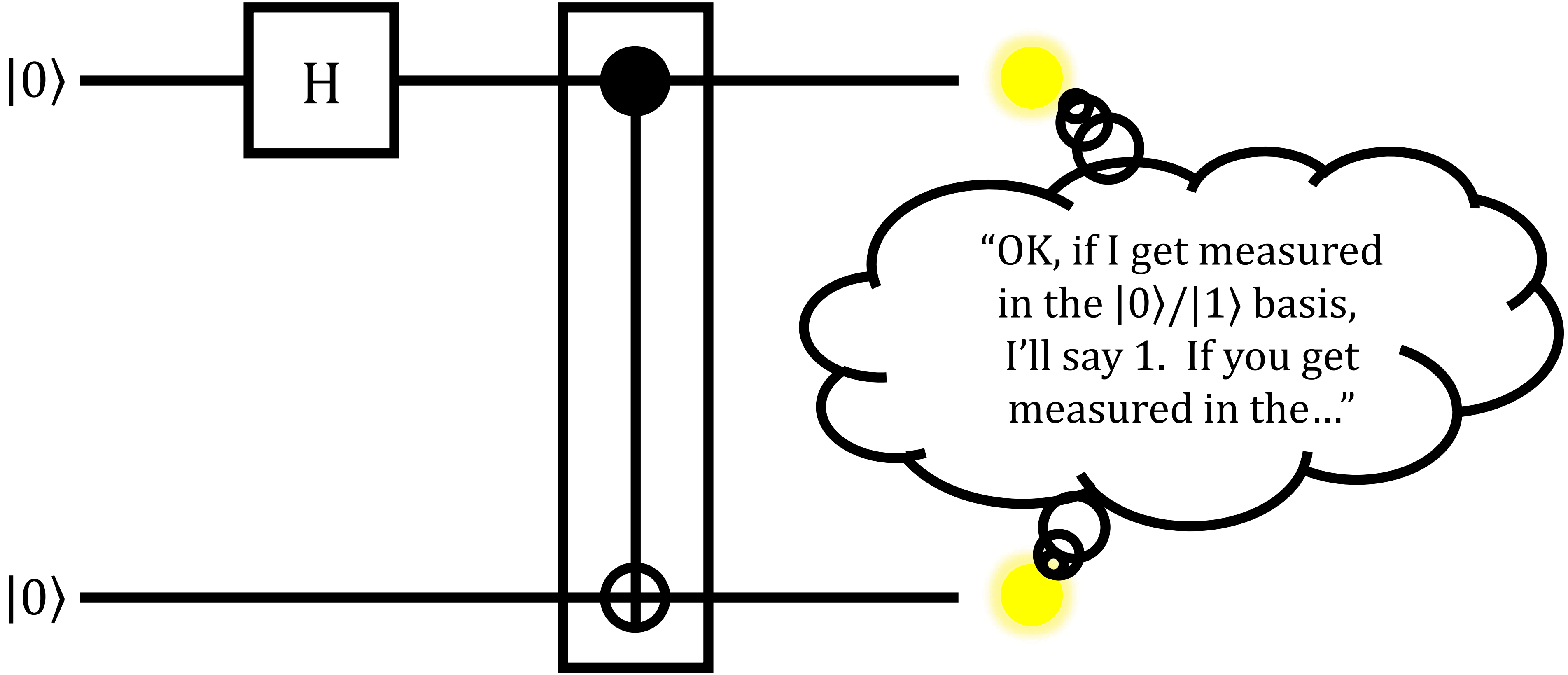
|0⟩


|0⟩
H
|0⟩
"OK, if I get measured in the |0⟩/|1⟩ basis, I'll say 1. If you get measured in the…"

## Summary

Best success probability Alice and Bob can achieve in the "CHSH experiment"...

| | |
|---|---|
| Deterministic: | 75% |
| Private randomness: | 75% |
| Shared randomness: | 75% |
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"Local Hidden Variables"

***Can the CHSH experiment be done in practice?***

# *Can the CHSH experiment be done in practice?*

**A. Aspect** et al., early ’80s

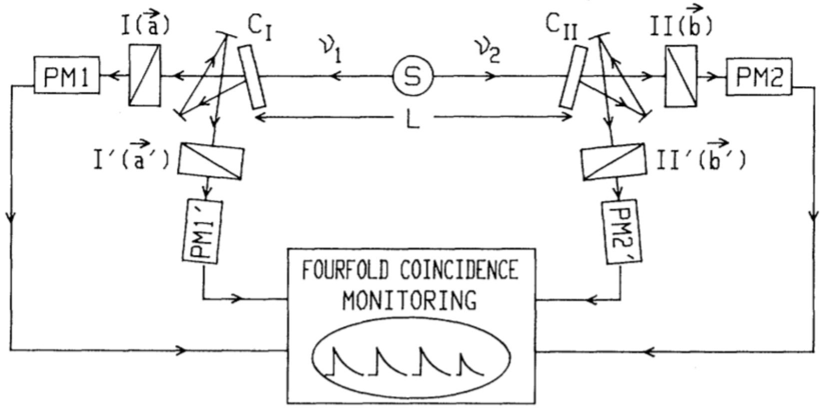
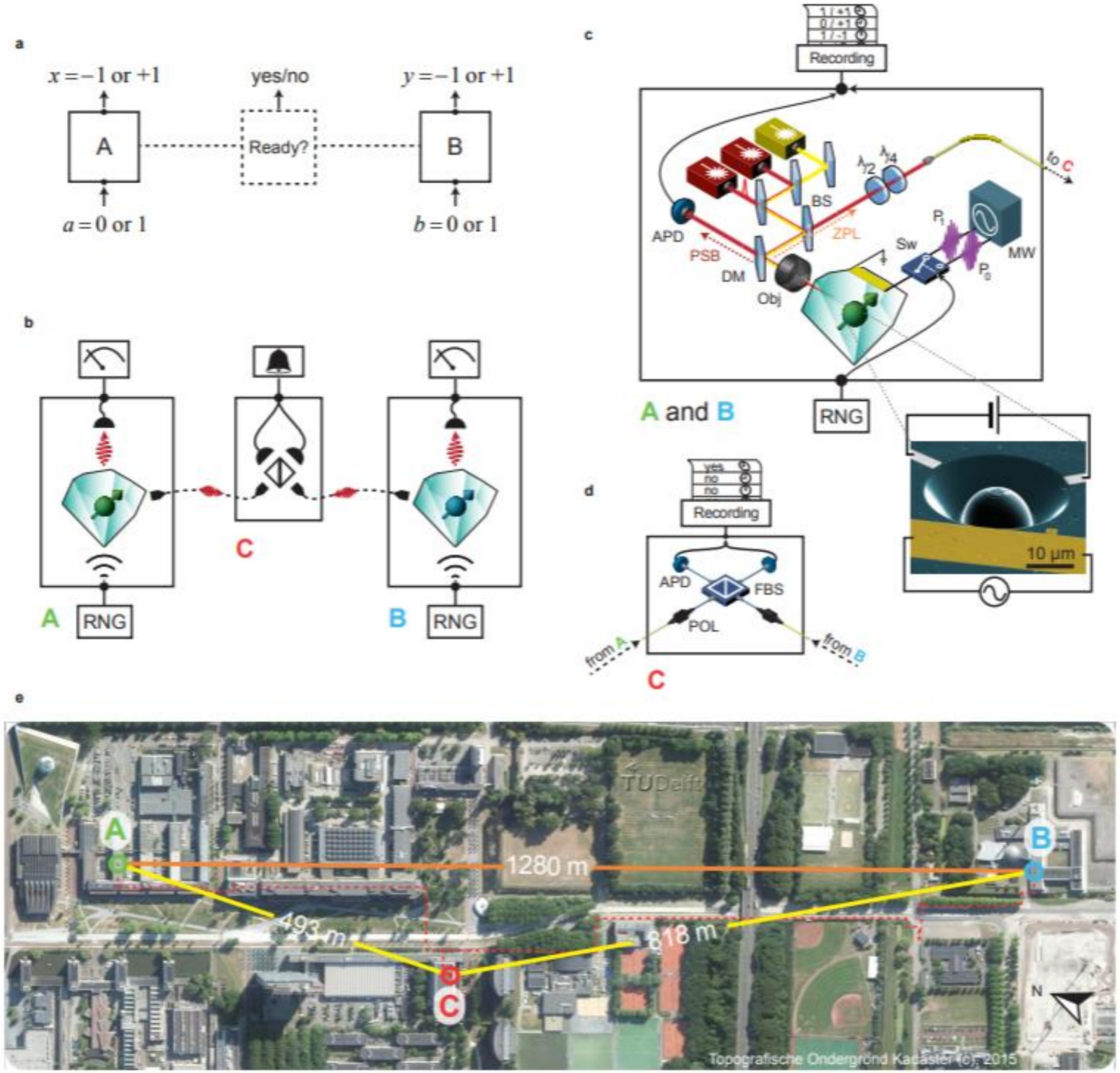



FIG. 2. Timing experiment with optical switches. Each switching device ($C_I$, $C_{II}$) is followed by two polarizers in two different orientations. Each combination is equivalent to a polarizer switched fast between two orientations.

**84%** success rate!




R. Hanson lab, 2014
Delft University of Technology

*Experimental loophole-free violation of a Bell inequality using entangled electron spins separated by 1.3 km*