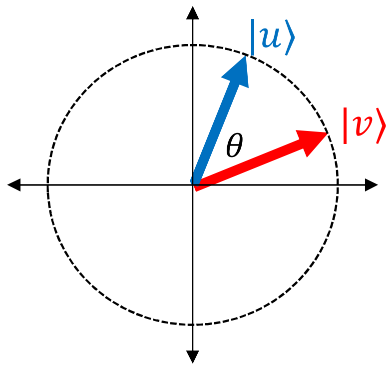
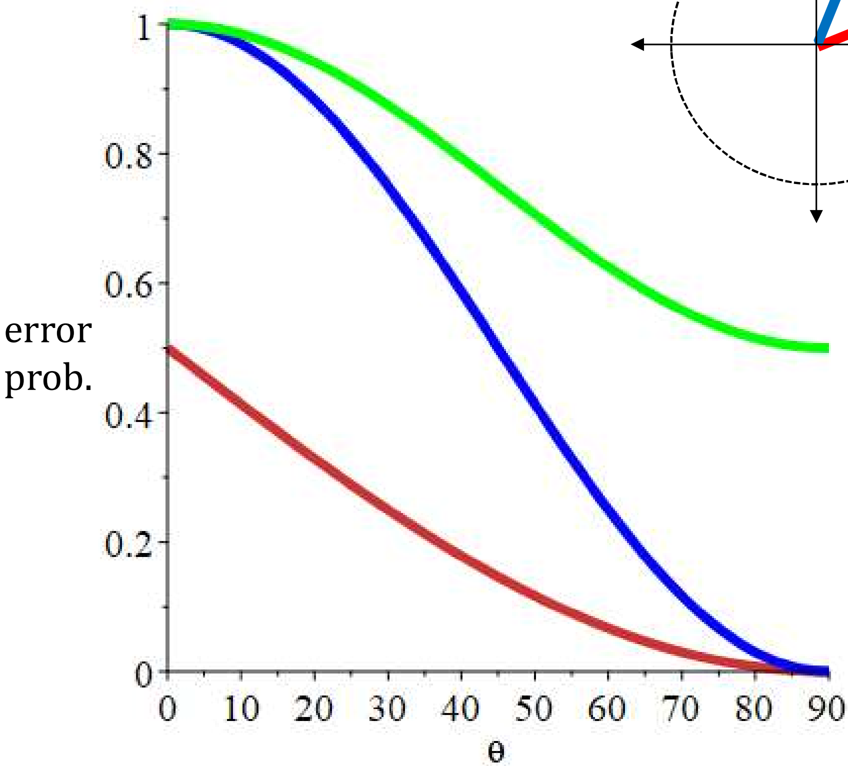
Lecture 5.5:

Multiplying By A Global Phase Doesn't Matter

Discriminating Two Quantum States at Angle θ



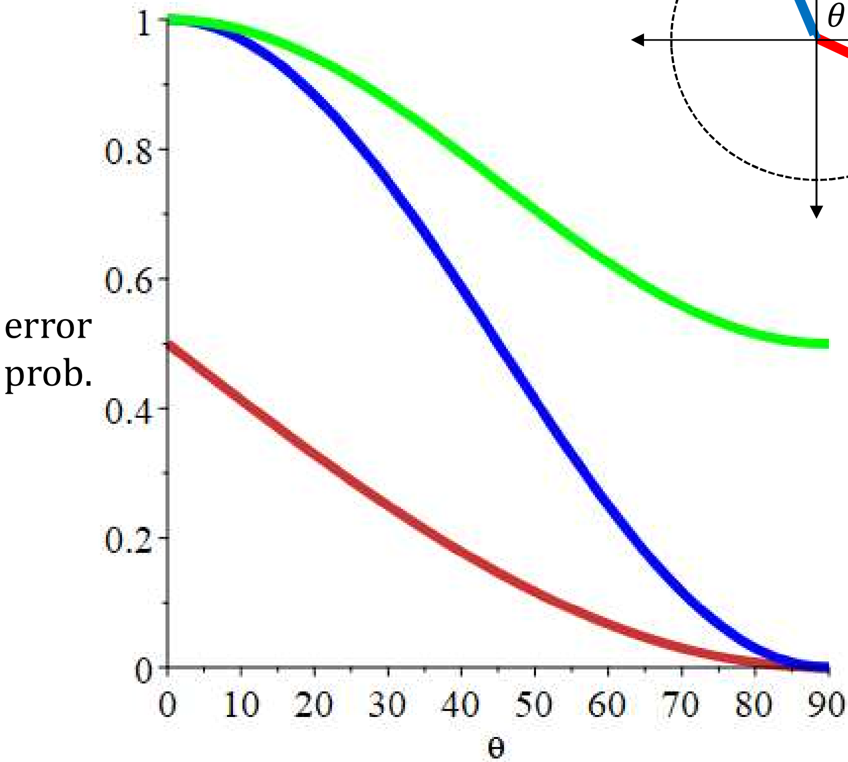


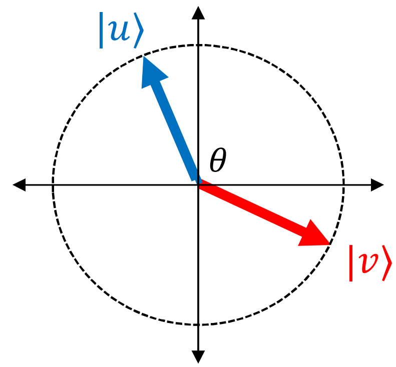
Two-sided error: $\frac{1}{2} - \frac{1}{2}\sin\theta$

One-sided error: $1 - (\sin\theta)^2$

Zero-sided error: $1 - \frac{(\sin\ \ )^2}{2}$

Discriminating Two Quantum States at Angle θ





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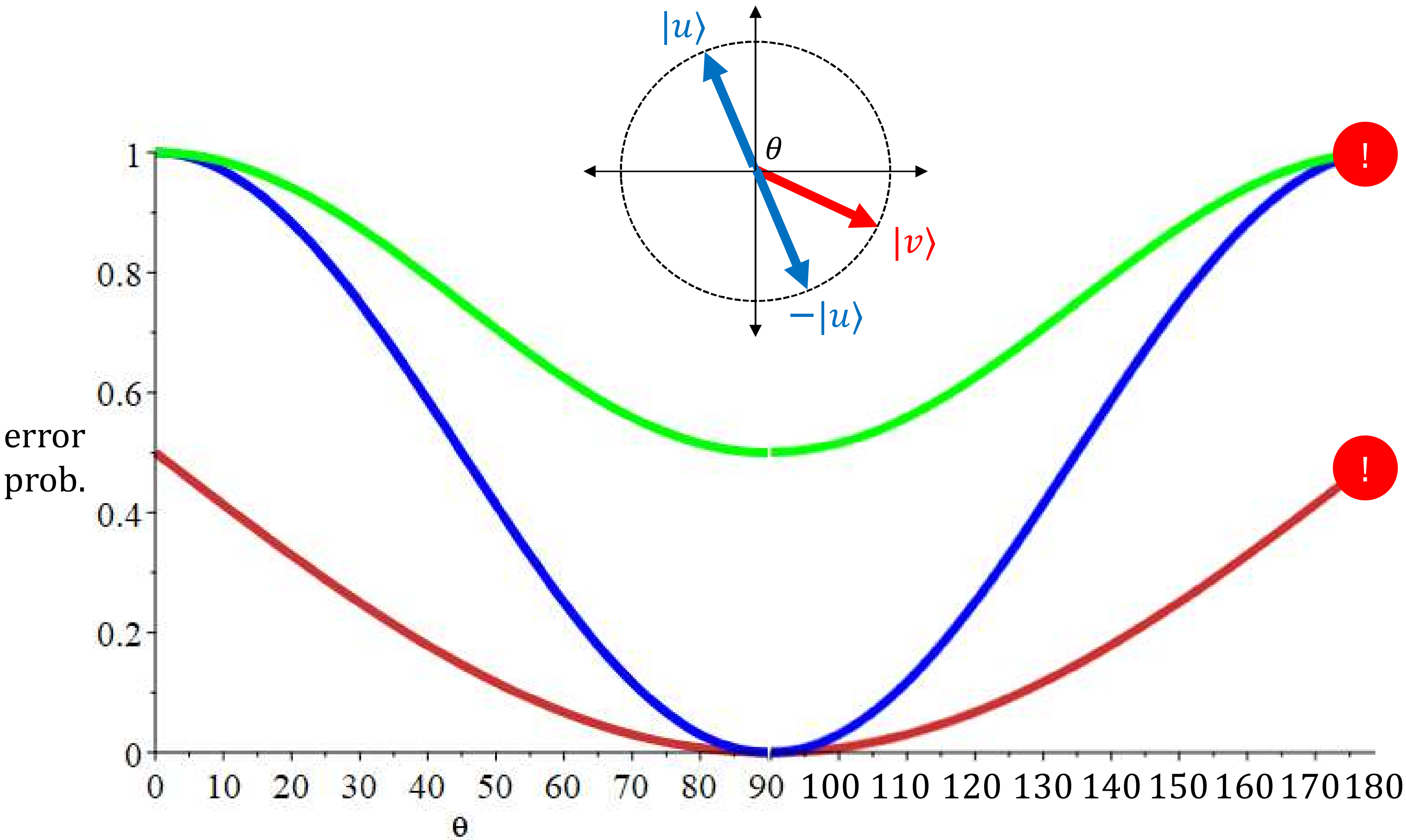
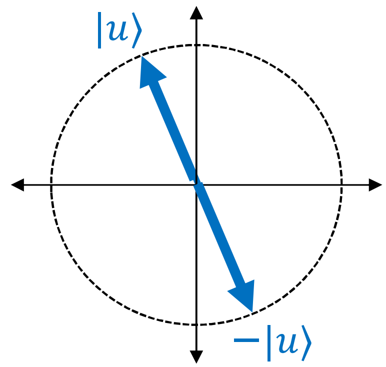
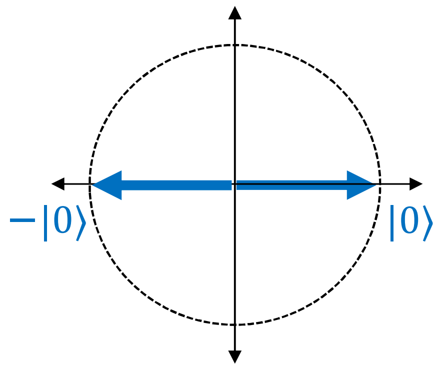




Could **rotate** $|\psi\rangle$.

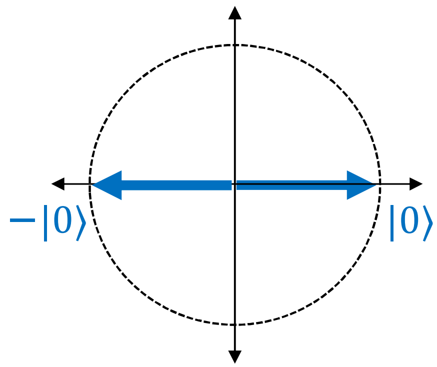
The state $|u\rangle$ and the state $-|u\rangle$ are **indistinguishable**.

Given an unknown qubit $|\psi\rangle$, promised to be either $|u\rangle$ or $-|u\rangle$, there is **no physical experiment** you can do to distinguish them.



Could **rotate** $|\psi\rangle$.

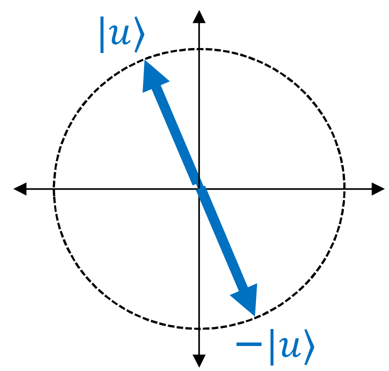
The state $|0\rangle$ and the state $-|0\rangle$ are **indistinguishable**.



Could **measure** $|\psi\rangle$.

You'll read out "$|0\rangle$" with probability **100%,**

and nothing will change.

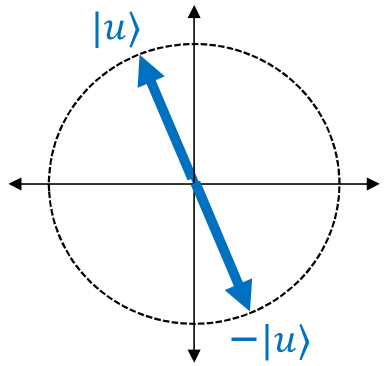


The state $|u\rangle$ and the state $-|u\rangle$ are **indistinguishable**.

Also: The state $|u\rangle$ and the state $i\,|u\rangle$ are **indistinguishable**.

Also: The state $|u\rangle$ and the state $c\,|u\rangle$ are **indistinguishable** whenever c is a complex number of magnitude 1.

Such a c is called a “global phase”.



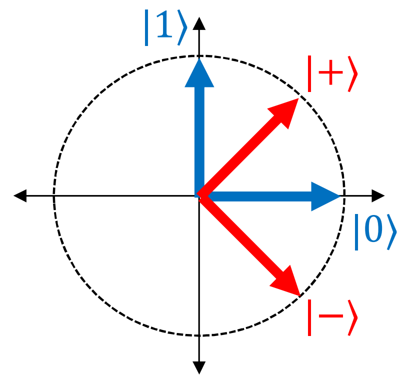
The state $|u\rangle$ and the state $-|u\rangle$ are **indistinguishable**.

Means our notation for quantum states is slightly clunky.

Would be better if indistinguishable states had identical notations.

In fact, when we later study "**mixed quantum states**", this clunkiness will be fixed!

The following "mixed quantum states" are *also* **indistinguishable**…!



Scenario ρ_1

A fair coin is flipped.

If Heads: $|\psi\rangle$ set to $|0\rangle$

If Tails: $|\psi\rangle$ set to $|1\rangle$

Scenario ρ_2

A fair coin is flipped.

If Heads: $|\psi\rangle$ set to $|+\rangle$

If Tails: $|\psi\rangle$ set to $|-\rangle$