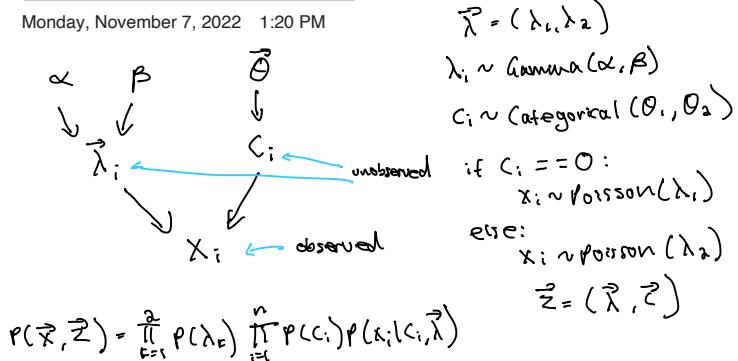
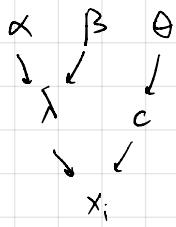


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$$\begin{aligned} \text{ELBO}(q) &= \mathbb{E}_q[\log p(\vec{x}, \vec{z})] - \mathbb{E}_q[\log q(\vec{z})] \\ q(\vec{z})q(\vec{x}, \vec{z}) &= \left[\prod_{k=1}^K q(\lambda_k | a_k, b_k) \right] \left[\prod_{i=1}^n q(c_i | p_i) \right] \\ \text{ELBO}(\vec{a}, \vec{b}, \vec{p}) &= \sum_{k=1}^K \mathbb{E}[\log p(\lambda_k | a_k, b_k)] + \sum_{i=1}^n \left[\right. \\ &\quad \left. - \sum_{k=1}^2 \mathbb{E}[\log q(\lambda_k | a_k, b_k)] \right] \end{aligned}$$

$$\begin{aligned}
q(c_i | p_{ii}) &\propto \exp(\mathbb{E}_{q-c_i} [\log p(c_i | \lambda_1, \lambda_2, \vec{c}_{-i}, \vec{x})]) \\
&\propto \exp(\log p(c_i) + \mathbb{E}_{\lambda_1, \lambda_2} [\underbrace{\log p(x_i | c_i, \lambda_1, \lambda_2)}_{\substack{\downarrow \\ \text{constant}}} | \vec{a}, \vec{b}]) \\
&\propto \exp(\sum_{k=1}^2 c_{ik} \mathbb{E}_{\vec{x}} [\log p(x_i | \lambda_k) | a_k, b_k]) \\
&\propto \exp(\sum_{k=1}^2 c_{ik} \mathbb{E}[x_i \log \lambda_k - \lambda_k | a_k, b_k]) \\
&= \exp(\sum_{k=1}^2 c_{ik} \mathbb{E}[\log \lambda_k | a_k, b_k] x_i - \mathbb{E}[\lambda_k | a_k, b_k]) \\
p_{ii} &\propto \exp(\mathbb{E}[\log \lambda_1 | a_1, b_1] x_i - \mathbb{E}[\lambda_1 | a_1, b_1])
\end{aligned}$$



$$\begin{aligned}\bar{\lambda} &= (\lambda_1, \lambda_2) \\ \lambda_k &\sim \text{Gamma}(\alpha, \beta) \\ \text{For } i = 1 \text{ to } n' \\ C_i &\sim \text{Categorical}(\theta_1, \theta_2) \\ \text{if } C_i = 0: \\ x_i &\sim \text{Poisson}(\lambda_1) \\ \text{else:} \\ x_i &\sim \text{Poisson}(\lambda_2) \\ Z &= \{\lambda, c\}\end{aligned}$$

$$\text{Gamma PDF} \quad \frac{\beta^\alpha x^{\alpha-1}}{\Gamma(\alpha)} e^{-\beta x}$$

$$\text{Poisson PMF} \quad \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\begin{aligned} q(z) \\ \hat{\Lambda}_k &\sim \text{Gamma}(\alpha_k, b_k) \\ c_i &\sim \text{Categorical}(p_{i1}, p_{i2}) \end{aligned}$$

$$q_i(z_i) \propto \exp\left(\mathbb{E}_{q_{-i}}[\log p(z_i | z_{-i}, x)]\right)$$

$$q(\lambda_k) \propto \exp\left(\log p(\lambda_k) + \sum_{i=1}^n \mathbb{E}_{c_i}\left[\log p(x_i | c_i, \lambda) ; p_i, a_{ik}, b_{ik}\right]\right)$$

$$\begin{aligned}
 \log(g(\lambda_k)) &= \text{const.} + \log p(\lambda_k) + \sum_{i=1}^n \mathbb{E}_{C_i} [\log p(x_i | C_i, \lambda)] ; p_i, a_{ik}, b_{ik} \\
 &\quad \lambda_k \sim \text{Gamma}(\alpha, \beta) \quad \log \left\{ \prod_{k=1}^n p(x_i | \lambda_k)^{c_{ik}} \right\} = \sum_{k=1}^n c_{ik} \log p(x_i | \lambda_k) \\
 &= \log \left\{ \lambda_k^{\alpha-1} e^{-\beta \lambda_k} \right\} + \sum_{i=1}^n \mathbb{E}_{C_i} \left[c_{ik} \log p(x_i | \lambda_k) ; p_i, a_{ik}, b_{ik} \right] + \text{const.} \\
 &\quad \log(x_i | \lambda_k) \cdot \mathbb{E}_{C_i} [c_{ik} ; p_i, a_{ik}, b_{ik}] \\
 &\quad = p_{ik} \\
 &= \log \left\{ \lambda_k^{\alpha-1} e^{-\beta \lambda_k} \right\} + \sum_{i=1}^n (\log(p(x_i | \lambda_k))) p_{ik} + \text{const.} \\
 &\quad \lambda_k^{a_{ik}-1} e^{-\lambda_k b_{ik}} \\
 &= \log \left\{ \lambda_k^{\alpha-1} e^{-\beta \lambda_k} \right\} + \sum_{i=1}^n \log \left(\lambda_k^{x_i p_{ik}} e^{-\lambda_k p_{ik}} \right) \\
 &\quad x_i \sim \text{Poisson}(\lambda_k) \quad \log \left(\frac{\lambda_k^{x_i} e^{-\lambda_k}}{x_i!} \right) \\
 &= \log \left\{ \lambda_k^{\alpha-1} e^{-\beta \lambda_k} \right\} + \log \left(\lambda_k^{\sum x_i p_{ik}} e^{-\lambda_k \sum p_{ik}} \right) + \text{const.} \\
 &= \log \left\{ \lambda_k^{(\alpha + \sum x_i p_{ik})-1} \cdot e^{-\lambda_k (\beta + \sum_{i=1}^n p_{ik})} \right\} + \text{const.}
 \end{aligned}$$

$$q(\lambda_k) \propto \lambda_k^{(\alpha + \sum_i x_i p_{ik}) - 1} \cdot e^{-\lambda_k (\beta + \sum_{i=1}^n p_{ik})}$$

$$a_k := \alpha + \sum_{i=1}^n x_i p_{ik}$$

$$b_k := \beta + \sum_{l=1}^n p_{lk}$$