Computer Science 15-499C / 15-881, Fall 1997

Introduction to Geometry

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1 Course Outline

This course will cover elementary differential and computational geometry. The purpose of the course is to prepare a student for advanced geometrical work in robotics and computer science. Increasingly, cutting-edge results in these areas require a working knowledge of differential geometry, algebraic geometry, algebraic topology, and computational geometry. Much of this work is inaccessible to a student just entering the field. In this course we will convey the basic tools, definitions, and results of differential geometry and the basic algorithms of computational geometry, so that a student can, either by self-study or through further courses, understand and implement the advanced results in computer science and robotics discovered in the past decade. We will touch upon some of these applications in the course. In particular, we will consider the robot motion planning problem as a core application.

The topics are:

- Motion Planning:

  Configuration Space, Visibility Graph, Non-Holonomic Motion Planning, Forces in Cspace.

- Frame Fields:

  Curves, Frenet Formulas, Covariant Derivatives, Differential Forms, Connection Forms, Structural Equations.

- Calculus on Surfaces:

  Surfaces, Patches, Tangent Vectors, Mappings, Differential Forms, Integration on Surfaces, Manifolds.

- Shape Operators:

  Surface Shape Operator, Normal Curvature, Gaussian Curvature.

- Point and Range Queries:

  One shot, repeated query, slab method, multidimensional binary tree.

- Convex Hull Algorithms:

  Lower bounds, sorting reduction, 2D algorithms (Graham's Scan, Jarvis' March, Quickhull, Mergehull), dynamic convex hull, higher-dimensional algorithms (gift-wrapping, beneath-beyond), 3D convex hull.

- Proximity Algorithms:

  Voronoi diagram, triangulations, Euclidean minimum spanning tree.

- Plane Sweep:

  Polygon union and intersection.

Prerequisites for this course are minor, but important. Calculus and linear algebra should be sufficient from the mathematics side, while fundamental data structures and algorithms should be sufficient from the computer science side. The course will consist of a series of intensive lectures. It is expected that a student will spend a few hours each day digesting the material of the lectures and solving some related homework problems. Grades will be based primarily on the homework, and possibly an exam. The course may be taken at either the undergraduate or the graduate levels. Students taking the class for graduate credit will be asked to prepare a project, either a paper or an implementation of an advanced topic.

2 Bibliography

Here are some good texts for background reading. The course texts are the books by O'Neill and Preparata & Shamos.

1. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, Academic Press, New York, 1975.

2. H. Edelsbrunner, *Algorithms in Combinatorial Geometry*, Springer-Verlag, Berlin, 1987.

3. J.-C. Latombe, *Robot Motion Planning*, Kluwer Academic Publishers, Boston, 1991.

4. B. O'Neill, *Elementary Differential Geometry*, Academic Press, New York, 1966. 2nd Edition: 1997.

5. F. P. Preparata and M. I. Shamos, *Computational Geometry*, Springer-Verlag, New York, 1985. (Corrected and expanded printing: 1988.)

6. M. Spivak, *Differential Geometry*, Publish or Perish, Berkeley, 1979.

3 Schedule

Here is an approximate schedule for the semester. This is subject to change as we see how quickly we cover the material.

August

Lecture	Date	Day	Topic
1	Aug 26	T	Overview
2	Aug 28	R	Calculus on Euclidean Space: space, tangent vectors, curves

September

Lecture	Date	Day	Topic
3	Sep 2	T	Dual Spaces, 1-Forms
4	Sep 4	R	Differential forms
5	Sep 9	T	Mappings
6	Sep 11	R	Curves, speed, unit speed, $[T, N, B]$
7	Sep 16	T	Frenet Formulas
8	Sep 18	R	Covariant Derivatives, Lie Brackets, Frame Fields
9	Sep 23	T	Non-holonomic Motions, Connection Forms, Structural Equations
10	Sep 25	R	Surfaces: definitions, examples, patches, manifolds
11	Sep 30	T	Manifolds: configuration space, embeddings, differential forms

October

Lecture	Date	Day	Topic
12	Oct 2	R	Mappings, pullbacks, line integrals, 2D integration, Stokes' Theorem
13	Oct 7	T	Shape Operator, examples, normal acceleration
14	Oct 9	R	Curvature: normal, Gaussian, mean, methods of computation
15	Oct 14	T	Gauss Theorem, surface area
16	Oct 16	R	open
17	Oct 21	T	Datastructure preliminaries (e.g., segment tree, DCEL)
18	Oct 23	R	Geometric preliminaries (e.g., homog. coords., duality, edge-edge inter.)
19	Oct 28	T	Range searching, point location
20	Oct 30	R	Range query and point location (cont)

November

Lecture	Date	Day	Topic
21	Nov 4	T	Convex Hull (definitions, reductions, extreme points, etc.)
22	Nov 6	R	Convex Hull in 2D (definitions, theorems, Graham's scan)
23	Nov 11	T	Convex Hull in 2D (Jarvis' March, Quickhull, Mergehull, Dynamic)
24	Nov 13	R	Higher Dimensional Convex Hull
25	Nov 18	T	Proximity problems in the plane, Voronoi Diagram definition
26	Nov 20	R	Voronoi Diagram algorithm, Euclidean Minimum Spanning Tree, Triang.
27	Nov 25	T	open
	Nov 27	R	Thanksgiving

December

Lecture	Date	Day	Topic
28	Dec 2	T	Polygon union and intersection, plane sweep
29	Dec 4	R	open