Homework 4 — CS 819 C — Spring 2002
March 5, 2002 — due March 12

Problem 1

Prove that

$$(\mathsf{tree}\ \tau\ (i)\ *\ \mathbf{true}) \Rightarrow \mathsf{dag}\ \tau\ (i).$$

Problem 2

The procedure $\mathsf{subst1}(\mathsf{a}, \mathsf{j}\ ;\ \mathsf{i})$ defined in the fifth section of the class notes will perform a mutation operation on every field of the tree structure whose root is i. Except when a field contains the atom a, however, the mutation operation will merely overwrite the current value with the same value.

Write an alternative version of this procedure that satisfies the same specification:

$$\{\mathsf{tree}\ \tau\ (\mathsf{i})\ *\ (\mathsf{p} \wedge \mathsf{dag}\ \tau'\ (\mathsf{j}))\}$$
$$\mathsf{subst1}(\mathsf{a}, \mathsf{j}\ ;\ \mathsf{i})$$
$$\{\mathsf{tree}\ (\tau/\mathsf{a} \to \tau')\ (\mathsf{i})\ *\ \mathsf{p}\},$$

but avoids the unnecessary mutation operations. (Hint: Define a subsidiary procedure that accepts, instead of i, a location containing i, and mutates this location only if i changes.)

Give a formal proof that your procedure meets the above specification.