CS294-3: Algorithms in the real world Assignment 3 (Linear and Integer Programming)
Due: Wednesday Oct 15, 1997

Please feel free to send me email if there are any ambigouous questions.

Problem 1: Strang Chapter 8 problem 8.1.9

Problem 2: Strang Chapter 8 problem 8.2.1.

Problem 3: Strang Chapter 8 problem 8.2.13.

Problem 4:
Consider the following problem

$$\begin{array}{ll} \text{maximize} & x_1 + x_2 \\ \text{subject to} & 2x_1 + x_2 \leq 4 \\ & x1, x2 \geq 0 \end{array}$$

(a) Starting with the initial solution $(x_1, x_2) = .1, .1$ solve this problem using the affine scaling interior-point method.

(b) Illustrate the progress of the algorithm on the the graph in $x_1 - x_2$ space.

Problem 5:
Let $S = \{x \in R^n : Ax \leq b\}$ and $T = \{x \in R^n : Bx \leq d\}$. Assuming that S and T are nonempty, describe a polynomial time algorithm (in n) for checking whether $S \subset T$.

Problem 6:
Formulate the following problem as a mixed integer programming problem.

$$\begin{array}{lll} \text{maximize} & f_1(x) + f_2(x) & \\ & & x_1 + 3x_2 \leq 12 \\ \text{subject to} & \text{at least two of the following} & 2x_1 + x_2 \leq 16 \\ & & x_1 + x_2 \leq 9 \\ & x_1, x_2 \geq 0 & \end{array}$$

where

$$f_1(x_1) = \begin{cases} 10 + 2x_1, & 0 \leq x_1 \leq 5 \\ 15 + x_1, & \text{otherwise} \end{cases}$$

$$f_2(x_2) = \begin{cases} 8 + x_2, & 0 \leq x_2 \leq 2 \\ 4 + 3x_2 & \text{otherwise} \end{cases}$$

Problem 7:
Solve the following problem using the branch-and-bound integer programming procedure describe in class. Feel free to do it graphically.

$$\begin{array}{ll} \text{maximize} & 2x_1 + x_2 \\ \text{subject to} & 2x_1 - 2x_2 \leq 3 \\ & -2x_1 + x_2 \leq 2 \\ & 2x_1 + 2x_2 \leq 13 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \text{ integers} \end{array}$$

Problem 8:
Solve the following problem using implicit enumeration (the 0-1 programming technique) describe in class.

$$\begin{array}{ll} \text{minimize} & 3x_1 + 2x_2 + 5x_3 + x_4 \\ \text{subject to} & -2x_1 + x_2 - x_3 - 2x_4 \leq -2 \\ & -x_1 - 5x_2 - 2x_3 + 3x_4 \leq -3 \\ & 2x_1 + 2x_2 \leq 13 \\ & x_1, x_2, x_3, x_4 \text{ binary} \end{array}$$