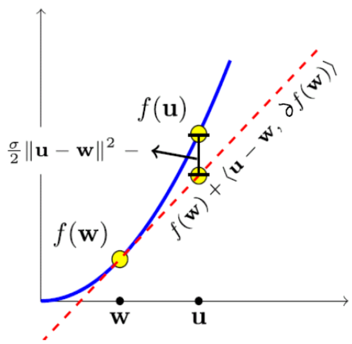
10-725 Recitation 1 Topics

Vector space over scalar field (for us, the real numbers) is a set X with vector addition and scalar multiplication.
Various combinations of vector elements: linear, affine, convex, conic.
Abstract specification of linear function t on X: $t(x+y) = t(x) + t(y)$ and $T(c\,x) = c\,t(x)$
(Real) inner product $\langle \cdot, \cdot \rangle : X \times X \to \mathbb{R}$ is symmetric, linear in first argument, nonnegative, 0 only when 0 is an argument.
Example: standard inner product on vectors (and matrices) $\langle x, y \rangle = \sum_i x_i\, y_i$.
Inner product useful because it provides a concrete implementation of linear functions: $t(x) = \langle m, x \rangle$.
Affine functions are translated linear functions, i.e. $u(x) = \langle m, x \rangle + b$.a

Favorite affine functions? Those tangent to some (convex) function f at a point x: slope is $\nabla f(x)$, intercept is $f(x)$.
First-order definition of convexity. Proof that $f(x) = x^2$ is convex.
Speaking of convexity, here is a strongly convex function:



Speaking of gradients: on the homework, you'll need to take a derivative with respect to a matrix.
Affine functions in n dimensions are linear functions in $n+1$ dimensions. (This is an example of "lifts".) Will be loose about affine/linear.

Various hulls: linear, affine, convex, conic.
"Subspace" or "span" = linear hull. Example: hyperplanes.
Linearly independent vectors: no vector contained in span of others
Basis of vector space: linearly independent and they span the vector space.
Dimension of vector space: size of basis.

Linear vector inequaliy (aka halfspace): $\{x : a_0 + \sum_i \langle a_i, x \rangle \geq 0\}$
(I also mentioned linear matrix inequalities, which each specify infinitely many linear vector inequalities. We will perhaps revisit these.)
Polyhedron: intersection of a finite number of halfspaces
Polytope: convex hull of a finite set of points
Minkowski-Weyl: every bounded polyhedron is a polytope. every polytope is a polyhedron.

For a convex set K, a point in K is
- exposed if it is $K \cap H$ for some halfspace H
- extreme if it is not a convex combination of any two other points in K

Example of extreme but not exposed? The "hockey rink".
Example of exposed but not extreme? Use a higher-dimensional H.
Farkas' lemma: http://demonstrations.wolfram.com/FarkassLemmaInTwoDimensions/