Assignment #2: Animating Thin Shells

15-864 Advanced Computer Graphics, Carnegie Mellon University
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Due: Monday, March 20, 2006 (midnight)

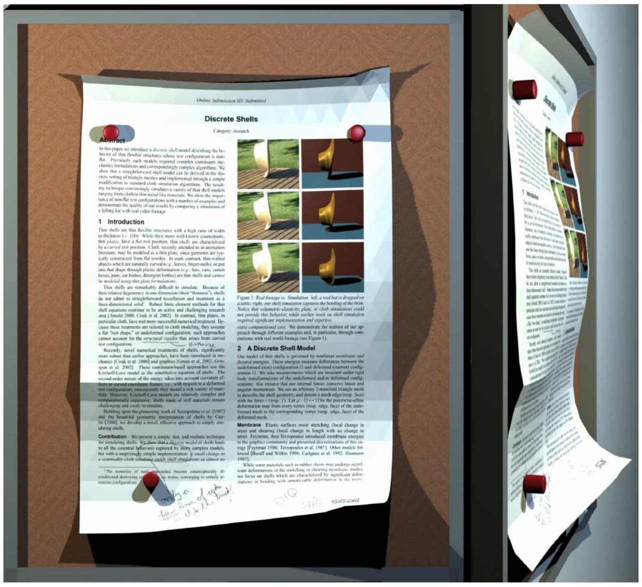

Figure 1: *Example paper modeled as a thin shell* (from [Grinspun et al. 2003])

Instructions

Cloth animation is an important component of modern computer animation [Baraff and Witkin 1998; Choi and Ko 2002; Bridson et al. 2002]. The goal of this assignment is to animate a deforming thin shell or cloth in a simple setting. We will use the "Discrete Shells" formulation described by [Grinspun et al. 2003] (also see [Bridson et al. 2003]); an example is shown in Figure 1. For simplicity, you need not consider any mechanical damping models or complicated collision processing. The requirements of the assignment are as follows:

1. **Energy:** Support deformation energy functions of the form,

$$W = k_B W_B + k_L W_L + k_A W_A, \tag{1}$$

   on a triangle mesh.

2. **Differentiation:** Use automatic and/or symbolic differentiation of the deformation energy (and forces) to obtain first and second derivatives w.r.t. particle positions.

3. **Newmark Integration:** Use the energy derivatives to implement the Newmark time-stepping scheme [Wood 1990],

$$\begin{aligned} \mathbf{x}_{i+1} &= \mathbf{x}_i + \Delta t_i \dot{\mathbf{x}}_i + \Delta t_i^2 \left((\frac{1}{2} - \beta)\ddot{\mathbf{x}}_i + \beta \ddot{\mathbf{x}}_{i+1} \right) \\ \dot{\mathbf{x}}_{i+1} &= \dot{\mathbf{x}}_i + \Delta t_i \left((1-\gamma)\ddot{x}_i + \gamma \ddot{x}_{i+1} \right) \end{aligned}$$

   where Δt_i is the duration of the i^{th} time step, and $\dot{\mathbf{x}}_i$ and $\ddot{\mathbf{x}}_i$ are configuration velocity and acceleration at the beginning of the i^{th} time step, respectively, and β and γ are parameters. Proceed in this order:

   (a) *Explicit Newmark Integration ($\beta = 0$):* This only requires first-order derivatives of W. Experiment with Newmark's numerical damping parameter, γ; how do the results look? (NOTE: $\gamma = \frac{1}{2}$ minimizes numerical damping.)

   (b) *Implicit Newmark Integration ($\beta > 0$; use $\beta = \frac{1}{4}$):* Implement implicit integration using first- and second-order derivatives. Use only a single step of Newton's method to compute the state at the next time step. You may use any numerical library you wish to represent your sparse matrix, as well as solve the system of equations; you should use the Preconditioned Conjugate Gradient method [Shewchuk 1994] with a simple preconditioner. Compare the stability of the implicit scheme to that of the explicit scheme on a concrete test example.

4. **Simple Contact:** Implement contact with the ground using a simple plane-vertex spring contact model. You do not need to use a collision detection hierarchy, and can simply test all vertices against the ground plane. This will allow you to drop some objects on the ground, post paper on the wall, etc.

5. **Create!** Be creative and see what kind of animations you can produce–provide videos of your creations. Can you make your system interactive?

6. **Document** all of your results in either an HTML webpage or PDF document. Describe any interesting steps you have taken, and any short-comings you encountered.

References

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