

15-150: Principles of Functional Programming

Quiz 3 Practice

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Name: David M Kahn

Andrew ID: davidkahn

Circle the lab you are in:

A	Julia and Yosef	GHC 5207
B	Juhi, Nicole, and Susan	GHC 5208
C	Nancy and Shengchao	GHC 5210
D	Thea and Cindy	GHC 4303
E	Jacky, Jonathan, and Sonya	WEH 7500
F	David	GHC 6503

Question	Points	Score
Practice	0	0
Total:	0	0

Answer the questions in the spaces provided. You will have the full 80 minutes of class time to complete this quiz. Good luck!! Y'all got this.

Question 1: Practice

Consider the following code:

```
datatype 'a tritree =
  NUB
| BRANCH of 'a * 'a tritree * 'a tritree * 'a tritree

fun mirror tri =
  (case tri of
    NUB => NUB
  | BRANCH ((x,y), t1, t2, t3) =>
    BRANCH ((y,x), mirror t3, mirror t2, mirror t1)
  )
```

- (a) (0 points) Let n be the number of nodes and d be the depth of an input tritree. Which of the following are valid **work** bounds for **mirror**? Do *not* assume that the input is balanced. Select all that apply.

- ☐ $O(\log(d))$
- ☐ $O((\log(d))^3)$
- ☐ $O(d)$
- ☐ $O(d^3)$
- ☒ $O(3^d)$
- ☐ $O(\log(n))$
- ☐ $O((\log(n))^3)$
- ☒ $O(n)$
- ☒ $O(n^3)$
- ☒ $O(3^n)$

note:
 $O(n)$ tightest

- (b) (0 points) Let n be the number of nodes and d be the depth of an input tritree. Which of the following are valid **span** bounds for **mirror**? This time *do* assume that the input is balanced. Select all that apply.

- ☐ $O(\log(d))$
- ☐ $O((\log(d))^3)$
- ☒ $O(d)$
- ☒ $O(d^3)$
- ☒ $O(3^d)$
- ☒ $O(\log(n))$
- ☒ $O((\log(n))^3)$
- ☒ $O(n)$
- ☒ $O(n^3)$
- ☒ $O(3^n)$

note:
 $O(d)$ tightest

(c) (0 points) The most general type of `mirror` is $(\alpha * \beta) \text{tritree} \rightarrow (\beta * \alpha) \text{tritree}$

(d) (0 points) Give another type that `mirror` satisfies, or state that none exists. _____

$(\text{int} * \text{int}) \text{tritree} \rightarrow (\text{int} * \text{int}) \text{tritree}$

(e) (0 points) Give a value with most general type `'a tritree`, or state that none exists. NUB

(f) (0 points) True or **False** : `(bool -> bool) tritree option tritree` is an instance of the type `'a tritree`

(g) (0 points) **True** or False : `'b list tritree option` is an instance of the type `'a tritree`

(h) (0 points) Suppose we are trying to prove a theorem about `mirror tri`, and want to proceed by weak structural induction over the input `tri`. Fill in the proof outline below. You do not need to prove the BC and IS, just state what the case would be. Make sure to clearly define or *quantify and type* any variables you introduce.

BC: Want to prove the theorem for `tri` = NUB

(proof omitted)

IS: Want to prove the theorem for `tri` = BRANCH (a,b), t1, t2, t3

(proof omitted)

IH: We assume the theorem holds for mirror t1, mirror t2,
and mirror t3

↓ for values a of arbitrary type t
 b of arbitrary type t'
 $t1, t2, t3$ of type $(t * t') \text{tritree}$