

15 - 150

1st Quiz Friday
(bring writing implement)
open notes

1) assume • requires

• arguments are of the right type and are values

2) attack ENSURES

times (x, y) for x, y int values assume

$\Rightarrow x * y$

reduction

$\Rightarrow z$ s.t. $z = x * y$

reduction

$x = 5 \downarrow \text{in } 0$

Thm : $\text{pow}(n, k) \Rightarrow n^k$

proof: By induction on k
assume n, k are integer values
and $k \geq 0$

$\text{pow}(n, k)$

$\Rightarrow$ case k of

reduction

0 $\Rightarrow$ 1

1 $\Rightarrow n * \text{pow}(n, k-1)$

Base case k is 0:

... $\Rightarrow$ |

first clause b/c $k=0$

note that $n^0 = 1$

Inductive Step:

W.T.S. $\text{pow}(n, k) \Rightarrow n^k$

assuming $\text{pow}(n, k-1) \Rightarrow n^{k-1}$

for arbitrary $k > 0$

... $\Rightarrow n * \text{pow}(n, k-1)$

reduction

$\Rightarrow n * n^{k-1}$

IH

$\Rightarrow n^k$

$P(-)$ inductive predicate

if $P(0)$

any $n:\mathbb{N}$, $P(n) \rightarrow P(n+1)$

then any $n:\mathbb{N}$ $P(n)$

P

if $P(0)$

and $\left(\text{for all } n \in \mathbb{N} \text{ s.t. } 0 \leq n \leq k \ P(n) \right)$
 $\rightarrow P(k+1)$

then all n . $P(n)$

Thm : $\text{power}(n, k) \Rightarrow n^k$
for n, k int values

proof: By strong induction on k

Base Case ...

Inductive step ...

Case $k \bmod 2 \neq 0$...

Case $k \bmod 2 = 0$

w.i.t.s. $\text{power}(n, k) \Rightarrow n^k$

assuming $k \neq 0$, $k \bmod 2 = 0$

$\text{power}(n, j) \Rightarrow n^j$ for $0 \leq j < k$

$\Rightarrow$ let ... end 2nd clause b/c $k \neq 0$
 $k \bmod 2 = 0$

$\Rightarrow$ let root = $n^{k/2}$ in ... end I'll

(note that $0 \leq k/2 < k$ for $k > 0$)

$\Rightarrow n^{k/2} * n^{k/2}$ reduction

$\Rightarrow n^k$ arithmetic

Lemma: $\text{powHelp}(n, k, a) \Rightarrow a * n^k$

proof: By induction on k

Base case: $k = 0$

$$\text{powHelp}(n, 0, a) \Rightarrow a = n^0 * a$$

Inductive step:

$$\text{w.t.s } \text{powHelp}(n, k, a) \Rightarrow a * n^k$$

$$\text{assuming } \text{powHelp}(n, k-1, b) \Rightarrow b * n^{k-1}$$

for any $b : \text{int}$ $k > 0$

$$\text{pow Help } (n, k, a)$$

$$\Rightarrow \text{pow Help } (n, k-1, n * a) \quad \text{2nd clause}$$

$$\Rightarrow (n * a) * n^{k-1}$$

$$\text{IH } b = n * a$$

$$\Rightarrow a * n^k$$

arith.

$$\text{Thm : } \text{powerT}(n, k) \Rightarrow n^k$$

$$\text{Proof : } \text{powerT}(n, k)$$

$$\Rightarrow \text{powHelp}(n, k, 1)$$

reduction

$$\Rightarrow n^k$$

by Lemma

$\text{fn } f \ x = e \quad : \quad t \rightarrow t'$

if $e : t'$ assuming $x : t \quad f : t \rightarrow t'$

$f \ v = \langle \text{env}, \text{fn } x \Rightarrow e \rangle v$

$\Rightarrow [\text{env}, v/x, C/f] e$

$C = \langle [C/f], \text{fn } x \Rightarrow e \rangle$

$\langle [\langle [\langle [\dots /f] /f \rangle \dots] /f \rangle \dots] /f \rangle$

$$\langle \text{env}, \text{fn } x \Rightarrow e \rangle_f \quad v$$

$$\Rightarrow [\text{env}, v/x, \langle \text{env}, \text{fn } x \Rightarrow e \rangle_{f/f}] e$$

$$\langle \text{env}, \text{fn } x \Rightarrow e \rangle_v$$

$$\Rightarrow [\text{env}, v/x] e$$

$[1, 2, 3] : \text{int list}$

1) hold 1 type only

2) don't have arbitrary access

linked list

$[] : + \text{list}$

$\text{hd} :: \text{tl} : + \text{list} \quad \text{if} \quad \text{hd} : +$
 $\text{tl} : + \text{list}$

$[1, 2, 3]$

$1 :: 2 :: 3 :: []$

$\text{fn len lst} =$
 case lst of
 $[\] \Rightarrow 0$

$\text{fn } x \Rightarrow c$

$| \text{hd} :: \text{tl} \Rightarrow 1 + \text{len } \text{tl}$

$\left\langle \begin{array}{l} \text{fn lst} \Rightarrow \text{case lst of} \\ [\] \Rightarrow 0 \\ | \text{hd} :: \text{tl} \Rightarrow 1 + \text{len } \text{tl} \end{array} \right\rangle_{\text{len}}$

$$\langle env, fn\ x \Rightarrow e \rangle\ v$$

$$\Rightarrow [env, v/x]e$$