10-601: HW4 Problem 1 Solution

1 D-Separation [Andy: 21 points]

Which of the following statements are true with respect to the following graphical model, regardless of the conditional probability distributions? If false, explain why by giving a path which prevents the variables from being d-separated.

\[
\begin{align*}
A & \quad I \\
B & \quad E & \quad J \\
C & \quad F & \quad K \\
D & \quad G & \quad M & \quad H \\
\end{align*}
\]

1. \(P(D, H) = P(D)P(H)\)
   *
   \textit{Solution:} True – the paths \(DCGFEIJKMH\) and \(DCGFEIJLMH\) are blocked because \(G\) is unobserved. The paths \(DCBAEIJKMH\) and \(DCBAEIJLMH\) are blocked because \(E\) is unobserved.

2. \(P(A, I) = P(A)P(I)\)
   *
   \textit{Solution:} True – the path \(AEI\) is blocked because \(E\) is unobserved. The path \(ABCGFEI\) is blocked because \(G\) is unobserved.

   *
   \textit{Solution:} False – the path \(AEI\) is not blocked because \(G\) is observed, and it is a descendant of \(E\).

4. \(P(J, G|F) = P(J|F)P(G|F)\)
   *
   \textit{Solution:} False – the path \(JIEABCG\) is not blocked because a descendant of \(E\) is observed.

   *
   \textit{Solution:} True – The paths \(JKM\) and \(JLM\) are both blocked.

6. \(P(E, C|A, G) = P(E|A, G)P(C|A, G)\)
   *
   \textit{Solution:} False – \(EFGC\) is not blocked because \(G\) is observed.

7. \(P(E, C|A) = P(E|A)P(C|A)\)
   *
   \textit{Solution:} True – \(EABC\) is blocked because \(A\) is observed, and \(EFGC\) is blocked because \(G\) is unobserved.