Rigid Body Dynamics
Part II
A rigid body

- Collection of particles
- Distance between any two particles is always constant
Forces and Torques

- Forces on individual particles generate torques
  - (consequence of constant inter-particle distance)

Net torque on body: \( \boldsymbol{\tau} = \sum \boldsymbol{\tau}_i = \sum \boldsymbol{r}_i \times \boldsymbol{f}_i \)
Linear and Angular Accelerations

◆ From conservation of linear momentum:

\[ p = Mv; \dot{p} = F; \dot{v} = \frac{1}{M} F \]

◆ From Conservation of angular momentum:

\[ L = I\omega; \dot{L} = \tau; \dot{\omega} = I^{-1}(\tau - \omega \times I\omega) \]
Numerical Integration

◆ COM Acceleration \( \rightarrow \) Velocity \( \rightarrow \) Position
  • Easy: \( v_{t+1} = v_t + \Delta t \dot{v}; \ x_{t+1} = x_t + \Delta t v_{t+1} \)

◆ Angular Acceleration \( \rightarrow \) Angular Velocity
  • Easy: \( \omega_{t+1} = \omega_t + \Delta t \dot{\omega} \)

◆ Angular Velocity to Rotations?
  • A bit trickier: \( R_{t+1} = R_t + \Delta t \dot{R}_{t+1} \)?
Given a set of forces, you know how to compute the motion of a rigid body.

Where do forces come from?
- User interaction
- Gravity
Rigid Bodies
Computing forces

- Given a set of forces, you know how to compute the motion of a rigid body
- Where do forces come from?
  - User interaction
  - Gravity
  - Collisions and contacts
Collision Response

“Nonconvex Rigid Bodies with Stacking”, Guendelman et al., SIGGRAPH 2003
Collision Response

Collision Process

Δt

no force

no force
Collision Response

A Soft Collision

- Force
- Velocity

$\Delta t$
Collision Response

A Harder Collision

force

velocity

$\Delta t$
Collision Response

A Very Hard Collision

force

\[ \Delta t \]

velocity
Collision Response

A Rigid Body Collision

impulsive force

velocity

\[ f_{imp} = \infty \]
\[ \Delta t = 0 \]
Collision Response

- An impulse changes velocities instantaneously

\[ J = \delta t F \]

- They can therefore be used to model rigid body collisions

- Derivation on the whiteboard…
Computing forces

Given a set of forces, you know how to compute the motion of a rigid body

Where do forces come from?

• User interaction
• Gravity
• Collisions and contacts
• Articulation
Articulated Rigid Body Dynamics

Derivation on the whiteboard…
Artistic control over rigid body simulations

Many-Worlds Browsing for Control of Multibody Dynamics Twigg and James, 2007
Many Worlds Browsing...

Sampling Plausible Worlds

compute and apply impulse

[O’Sullivan et al., 2003]
Many Worlds Browsing…

Interactive Browsing – various criteria

Input

Positive

Negative
Many Worlds Browsing