Particle Filter with Mode Tracker (PF-MT) for Visual Tracking Across Illumination Changes

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Abstract—In this correspondence, our goal is to develop a visual tracking algorithm that is able to track moving objects in the presence of illumination variations in the scene, and that is robust to occlusions. We treat the illumination and motion (x-y translation and scale) parameters as the unknown “state” sequence. The observation is the entire image and the observation model allows for occasional occlusions (modeled as outliers). The nonlinearity and multimodality of the observation model necessitates the use of a particle filter (PF). Due to the inclusion of illumination parameters, the state dimension increases, thus making regular PF impractically expensive. We show that the recently proposed PF with Mode Tracker (PF-MT) approach can be used here since, even in most occlusion cases, the posterior of illumination conditioned on motion and the previous state is unimodal and quite narrow. The key idea is to importance sample on the motion states while approximating importance sampling by posterior Mode Tracking for estimating illumination. Experiments demonstrate the advantage of the proposed algorithm over existing PF based approaches for various face and vehicle tracking. We are also able to detect illumination model changes, e.g. those due to transition from shadow to sunlight or vice versa by using the gELL statistic and successfully compensate for it without ever loosing track.

Index Terms—Visual tracking, Particle Filter, tracking, Monte Carlo methods

I. INTRODUCTION

In recent works [3], [4], [5], we developed practically implementable particle filtering (PF) approaches for tracking on large dimensional state spaces with multimodal likelihoods. An approach called PF with posterior mode tracking (PF-MT) was introduced. The focus of [3] was only on the algorithm design and analysis and we only showed one simulated temperature field tracking problem. In this correspondence, we look at the problem of moving objects' tracking across illumination change, which is a key practical application where the above problem occurs. We explain how to use the PF-MT approach to design an efficient PF based tracker for this problem. Significantly improved performance of PF-MT over existing PF approaches as well as over some other illumination tracking approaches is demonstrated. We note that this is the first work where the PF-MT approach is exhaustively compared against existing PF methods for a real visual tracking application. The only other work where PF-MT was used for a real application is [6], but that only compared two different contour deformation models and not PF algorithms. We also briefly demonstrate the use of the ELL statistic [7] to detect illumination model change and to automatically adapt to that change. This is needed for example when the moving target (vehicle or persons) moves from a lighted to a shadowed region or vice versa.

In the absence of illumination change, the motion of a rigid object moving in front of a camera that is far from the scene can be tracked using a three dimensional vector consisting of x-y translation and uniform scale or more generally using a six dimensional affine model as in Condensation [8]. Condensation was the first work to beautifully demonstrate the use of a PF for tracking through multimodal observation likelihoods resulting from background clutter or occlusions. Now, if illumination also changes over time and if different parts of the object experience different lighting conditions, then many more dimensions get added to the state space, thus making it a much larger dimensional problem. As the state space dimension increases, the number of particles required to track using a PF increases [9], thus making PF impractical. But, as we explain later in Sec. III, in most cases of practical interest, the posterior of illumination change, conditioned on motion and on the previous state, is unimodal. Also, illumination change is usually very gradual and this causes the posterior to be also narrow. Under these two assumptions, one can replace importance sampling of illumination by just posterior mode tracking (MT), i.e. we can use a PF-MT for this problem. This one step, reduces the importance sampling dimension to three and thus drastically reduces the number of particles required. We refer to the resulting PF as Illumination PF-MT.

Early work on illumination modeling for object recognition and illumination invariant tracking include [10], [11], [12], [13]. However, learning these models from low resolution videos might pose serious practical challenges. Template adaptation approaches [14], [15], [16] suffer from problems of drift [17], e.g. if you adapt when the tracker has latched onto clutter, it will lead to tracking failure. In [18], it was assumed that a small number of centroids in the illumination space can be used to represent the illumination conditions existing in the scene. The six centroids method of [18] does not suffice given complex illumination patterns often encountered in reality. Also, it is unclear how standard trackers like mean-shift tracker [19] can be adapted for illumination invariance. We show examples of failure of template adaptation, mean-shift and six centroids method in Fig. 3 of Supplementary material. Some recent work on jointly handling appearance change due to illumination variations, as well as other factors like 3D pose change, include [20], [21], [22], [23], [24], [25].

In this note, we use a template-based tracking framework because it is simple to use and to explain our key ideas; but the Illumination PF-MT approach can also be used with other representations of the target, e.g. feature based approaches. Also, the template matching framework enables illumination to be parameterized using a Legendre basis, as suggested in past works [18], [26]. We use a very simple motion and illumination change model to demonstrate how to design PF-MT for our problem. However, we note that a similar PF-MT idea can also be extended to jointly handle appearance change due illumination as well as other factors like 3D pose change, by using the more sophisticated models of recent work [20], [21], [22], [23], [25]. Similarly, illumination can also be represented using other parameterizations such as those proposed in [10], [11], [12], [13].

Paper Organization: In Sec. II, we develop the state space model for the tracker. In Sec. III, we explain how to design a PF-MT algorithm for tracking across illumination variations. In Sec. IV, we design the gELL based change detection system for detecting and tracking illumination model changes. The experimental results are given in Sec. V and we conclude the paper in Sec. VI.

II. STATE SPACE MODEL

The state model consists of simple dynamical models for illumination $\Lambda_t$ and motion parameters $U_t$ (further details to follow).
Thus the state vector is given as, $X_t = [U_t^T A_t^T]^T$. The observation $Y_t$ is the image frame at time $t$. Our observation model assumes that occlusions may occur.

A. Notation

The notation $\text{vec}(.)$ refers to the vectorization operator which operates on a $m \times n$ matrix to give vector of dimension $mn$ by cascading the rows. $[x]_n$ denotes the $n^{th}$ element of a vector $x$. $I_i$ denotes the identity matrix. The Hadamard product (the $\star$ operation in MATLAB) is denoted by $\odot$. The terms 1 and 0 refer to the column vectors with all entries as 1 and 0 respectively. $\text{mean}(.)$ gives the mean value of the entries of a vector i.e $\text{mean}(x) = 1/N \sum_{i=1}^{N} [x]_i$ for an $N$ length vector $x$. The function $\text{round}(.)$ operates on a matrix $Z$ and outputs a matrix with integer entries as $\text{round}(z_{i,j})$ which is the integer closest to $z_{i,j}$. The notation $N(x; \mu, \Sigma)$ denotes the value of the Gaussian distribution with mean $\mu$ and covariance $\Sigma$ computed at $x$ whereas $x \sim N(\mu, \Sigma)$ implies that the random variable $x$ is Gaussian distributed with mean $\mu$ and covariance $\Sigma$. Similarly, the notation $U(a; c_1, c_2)$ denotes the value of the uniform density defined over $[c_1, c_2]$ computed at $a$ while $x \sim U(c_1, c_2)$ denotes that $x$ is uniformly distributed over $[c_1, c_2]$. The term mode refers to the local maxima of a probability density function (PDF). The PDF is animodal (multimodal) if it has exactly one (multiple) mode.

B. The Observation Model

We assume the observation model of [1], [18], but include an occlusion model similar to the one introduced in condensation [8]. The target object template image, at time $t$, is denoted as $T_t$. As introduced in [18], the changed “appearance” of the template $T_t$ is represented in terms of a linear combination of the initial template $T_0$ scaled by a set of Legendre basis functions as follows.

$$\text{vec}(T_t) = AA_t$$

where the matrix $A$ is defined in Appendix equation (7). Its columns consist of the initial template scaled by the first $D$ Legendre basis functions. The $D \times 1$ vector $A_t$ is the Legendre basis coefficients at time $t$ along the first $D$ Legendre basis functions. Henceforth, we will call it the illumination vector. Theoretically, illumination can be different for each pixel and the illumination dimension would become equal to $M$. However, it is know from earlier work that in reality, the variability is not so high and the top Legendre coefficients suffice to model most of the illumination changes [18]. The Legendre basis coefficients were successfully used in [1]. In this work, we use the Legendre basis, although any other suitable basis (even data dependent basis like PCA) can be used as well and nothing in our proposed algorithm will change. It is to be noted that we only model translation and scaling of the template as the motion states. Note that the template is not updated as a whole, but we update the motion states that need to be applied to the original template to get something that matches the object in the current image.

Given the motion parameter vector $U_t$ consisting of scale, horizontal translation and vertical translation ($U_t = [s_t \tau_x^h \tau_y^v]^T$) of the initial template, the ‘moved’ (translated/scalared) template region in the current frame $Y_t$ can be computed as given in Appendix equation (8). We call it region of interest or ROI ($U_t$).

At any given instant, part or all of the ROI may get covered (occluded) by some other object(s). In the absence of any knowledge about the occluding object(s) intensity or pixel locations, we assume a simple outlier noise model for occlusion [8]. At any time $t$, any ROI pixel gets occluded with probability $(1-\theta)$ independent of the others and when it does, its intensity is uniformly distributed between 0 to 255 independent of all other pixels. On the other hand, with probability $\theta$, there is no occlusion. Thus, for all $i \in \text{ROI}(U_t)$,

$$Y_t(i) = \begin{cases} [Aa]_i + [n_i]_i, & w.p. \theta \\ [O_i], & w.p. 1 - \theta \end{cases}$$

where $[n_i]_i \sim N(0, \sigma^2_i)$, $[O_i]_i \sim U(0, 255)$ and $\text{ROI}(U_t)$ is computed using (8). The pixels outside the predicted template region (ROI) are assumed to have intensities that do not depend on $U_t$, $A_t$ or $T_0(i,j)$. Thus we have the following observation likelihood given the state vector $X_t \triangleq [U_t^T A_t^T]^T$.

$$OL(X_t) = p(Y_t|U_t, A_t) \propto p(Y_t|\text{ROI}(U_t))|A_t| = \prod_{i=1}^{M} \theta \mathcal{N}([Y_t|\text{ROI}(U_t)])_i; (A_t)_n, \sigma^2_0 + (1-\theta)\mathcal{U}([Y_t|\text{ROI}(U_t)])_i; 0, 255)$$

An example of the negative-log plot of $CL(t)$ for a scalar case (i.e. $M = 1$) is shown in Fig. 1.

C. The System Model

We defined the motion parameter vector $U_t = [s_t \tau_x^h \tau_y^v]^T$ in the previous section. As mentioned earlier, the illumination vector $A_t \in \mathbb{R}^D$ correspond to the coefficients of the Legendre basis function. Thus tracking is performed over a $D+3$ dimensional motion-illumination space.

In the absence of specific information about the object motion or about illumination variation, we assume a simple random walk model on both $U_t$ and $A_t$ i.e.

$$U_t = U_{t-1} + n_u, \quad A_t = A_{t-1} + n_A$$

where $n_u \sim N(0, \Sigma_u)$, $n_A \sim N(0, \Sigma_A)$, $\Sigma_A$ is a $D \times D$ diagonal matrix and $\Sigma_u$ is a $3 \times 3$ diagonal covariance matrix. Thus the state transition prior (STP) can be given as:

$$\text{STP}(U_t; U_{t-1}, A_{t-1}) \triangleq \text{STP}(U_t; U_{t-1}) \text{STP}(A_t; A_{t-1})$$

$$\text{STP}(U_t; U_{t-1}) \triangleq \mathcal{N}(U_t; U_{t-1}, \Sigma_u)$$

and

$$\text{STP}(A_t; A_{t-1}) \triangleq \mathcal{N}(A_t; A_{t-1}, \Sigma_A)$$

III. ILLUMINATION PF WITH MODE TRACKER (PF-MT)

A particle filter (PF) uses sequential importance sampling [9] along with a resampling step [27] to empirically estimate the posterior distribution, $\pi_t(X_t) \triangleq p(X_t|Y_{1:t})$, of the state $X_t$. PF-MT [3] splits the state vector $X_t$ into $X_t = [X_{t,s}, X_{t,r}]$ where $X_{t,s}$ denotes the coefficients of a small dimensional “effective basis” (in which most of the state change is assumed to occur) while $X_{t,r}$ belongs to the “residual space” in which the state change is assumed “small”. It importance samples only on the effective basis dimensions, but replace importance sampling by deterministic posterior Mode Tracking (MT) in the residual space. Thus the importance sampling dimension is only dist($X_{t,s}$) (much smaller than dist($X_t$)) and this is what decides the effective particle size. PF-MT implicitly assumes that the posterior of the residual space conditioned on the effective basis (“conditional posterior”) is unimodal most of the time. Moreover it is also assumed to be narrow. Only under these two assumptions, the conditional posterior mode is a highly likely sample from the conditional posterior.
Consider the observation model given in (2). Without any prior information, due to clutter, the observation likelihood, \( OL(U_t, \Lambda_t) \) is clearly multimodal, e.g. if there is no constraint on how large illumination change can be, one may get a very strong match to the observation with a wrong object region (wrong motion estimate). This necessitates the use of a PF. Also, even with just a seven dimensional space of illumination change, the state space dimension becomes ten, which is quite large. As a result the original PF [27] will require a very large number of particles. Other efficient PFs such as PF-Doucet [9] or Gaussian PF [28] also cannot be used since these implicitly assume that the posterior conditioned on the previous state, \( p^*(X_t) \triangleq p(X_t|X_{t-1}, Y_t) \) is unimodal. But in our problem, this will not hold, since the likelihood is multimodal and the prior on motion is typically quite broad, and this will result in a multimodal \( p^* \) (as explained in [3]). In fact, PF-Doucet [9] cannot even be implemented easily because it requires finding the posterior mode (of \( OL(X_t)STP(X_t) \)). But since our \( OL(X_t) \) is not differentiable (consists of a round operation, see (8)), one cannot use standard numerical optimization algorithms to do this. Moreover, since the multimodality in the problem comes from the likelihood (and not the system model), at any time, Gaussian mixture filters or Gaussian Sum PF [29] also cannot be used (see discussion in [3] for details). Because of the occlusion term, even conditioned on motion \( U_t \), the observation model is not linear-Gaussian. Hence, Rao-Blackwellized Particle Filter (RB-PF) [30, 31] cannot be used either.

But notice that, while \( p^* \) is often multimodal, \( p^* \) conditioned on motion, i.e.
\[
p^{**}(\Lambda_t) \triangleq p^*(X_t|U_t^{(i)}) = p(\Lambda_t|X_t^{(i)}_{t-1}, Y_t, U_t^{(i)}) \propto CL^{(i)}(\Lambda_t)p(\Lambda_t|X_t^{(i)}_{t-1})
\]
is usually unimodal. Here \( CL^{(i)}(\Lambda_t), \) defined in (3), is the conditional likelihood of \( \Lambda_t \) i.e. \( p(Y_t|U_t^{(i)}, \Lambda_t) \). This happens for the following reason. Notice that \( p(\Lambda_t|X_t^{(i)}_{t-1}) \) is Gaussian and hence unimodal. When there is no occlusion, the dominant conditional likelihood mode is the observed illumination of the target. Hence the state transition prior’s mode (target illumination at \( t-1 \)) is close to it and in fact lies in its basin of attraction and so, \( p^{**}(\Lambda_t) \) is unimodal (see Fig. 1(a)). In case of occlusion, the dominant CL mode is the intensity pattern of the occluding object. But since the illumination change prior is quite narrow, the conditional posterior, \( p^{**}(\Lambda_t) \), will still be unimodal (see Fig. 1(b)), except if the occlusion intensity is very close to the targets intensity pattern (see Fig. 1(c)). This fact is proved in Theorem 1 of [3]. In both occlusion and no-occlusion cases, narrowness of \( STP(\Lambda_t) \) ensures narrowness of \( p^{**}(\Lambda_t) \). As a result we can use PF-MT for this problem with \( X_{t,s} = U_t \) and \( X_{t,r} = \Lambda_t \). We give the stepwise illumination PF-MT algorithm in Algorithm 1. The only exception where the above split up may not work is if, the occluding objects intensity pattern is very close to that of the template i.e. the case of Fig. 1(c). If in an application, this happens very often, then one should also use a part of the illumination state as \( X_{t,s} \).

![Figure 1](image_url)

**Fig. 1.** In this figure, we plot the negative log of the conditional likelihood \( p^{**}(\Lambda_t) \) with the negative log of \( p^{**}(\Lambda_t) \) for a simple 1D scalar case. Fig. 1(a) shows the no occlusion case where CL mode is very close to the STP mode leading to a unimodal \( p^{**}(\Lambda_t) \). Fig. 1(b) demonstrates that the occlusion case with unimodal \( p^{**}(\Lambda_t) \) (the CL mode is very far from the STP mode). In Fig. 1(c), it is shown that when the CL mode is close to the STP mode (but not in the basin of attraction of the STP), this gives rise to a multimodal \( p^{**}(\Lambda_t) \).

**Algorithm 1 Illumination PF-MT.** Going from \( \pi_{t-1|t-1}^N \) to \( \pi_{t|t}^N(X_t) = \sum_{i=1}^N w_t^{(i)}\delta(X_t - X_t^{(i)}) \)

For each \( t > 0 \),

1) **Importance Sample (IS) on motion:** For all \( i \), sample \( U_t^{(i)} \sim STP(U_t; U_t^{(i)}, \Lambda_t) \) (defined in (5)). Use \( U_t^{(i)} \) to compute the corresponding ROI using (8).

2) **Mode Tracking (MT) on illumination:** Use the current observation to get \( Y_t(ROI(U_t^{(i)})) \) and compute the mode \( m_t^{(i)} \) of \( p^{**}(\Lambda_t) \) by solving the following convex optimizing problem,

\[
m_t^{(i)} = \arg \min_{\Lambda_t} \log p^{**}(\Lambda_t) = \min_{\Lambda_t} L^{(i)}(\Lambda_t)
\]

where \( L^{(i)}(\Lambda_t) = [-\log CL^{(i)}(\Lambda_t)] + [-\log STP(\Lambda_t; A_t^{(i)}, \Lambda_t)] \)

where \( CL^{(i)}(\Lambda_t) \) is defined in (3) and STP(.) in 5. Generate illumination particle as \( \Lambda_t^{(i)} = m_t^{(i)} \).

3) **Weighting and Resampling:** Compute the weights using \( w_t^{(i)} \propto w_t^{(i)}CL(U_t^{(i)}, A_t^{(i)})STP(\Lambda_t^{(i)}, A_t^{(i)}) \) and resample

4) Increment \( t \) and go back to Step 1

IV. ILLUMINATION PF-MT WITH ILLUMINATION MODEL CHANGE

In most cases, the illumination changes gradually over time and hence the illumination change variance takes a small value. The exception is when a car or a person transitions from shadow to sunlight or vice versa or in an indoor scenario if the light bulb is switched on or off. During these transitions, if we track with a small illumination variance model, the tracker will gradually lose track. Thus there is a need to detect model change and to assign a high illumination change variance temporarily during the transition period and to change it back once the transition is over. If we allow the illumination change to have a larger variance all the time, then the PF-MT algorithm as designed in the previous section will no longer be applicable (since it will become more likely that \( p^{**} \) is multimodal). We propose to detect model change using the recently proposed generalized Expected (negative) Log Likelihood (gELL) statistic [7]. The gELL is designed to detect model changes before complete loss of track, which is what our application needs. In fact it works by using the partly tracked part of the change to detect it. Standard approaches, like tracking error use loss of track to detect change and hence take longer.

Generalized ELL (gELL) is the Kerridge inaccuracy [33] between the posterior at time \( t \), \( \pi_{t|t} \) and the \( \Delta \)-step ahead prediction distribution \( \pi_{t|t-\Delta}, \) i.e. \( gELL(t, \Delta) \triangleq E_{\pi_{t+\Delta}}[-\log \pi_{t+\Delta|t-\Delta}(X_t)] \) where \( E_{p[\cdot]} \) denotes expectation w.r.t pdf \( p(X) \) and \( \pi_{t+\Delta|t-\Delta}(X_t) \triangleq p(X_t|Y_{t+\Delta}) \). In practical applications, it is not clear how to choose \( \Delta \). One option is to compute the maximum of gELL over all \( \Delta \), i.e. to compute
The goal of this correspondence is to show how to design PF-MT for illumination tracking problem and to demonstrate that it provides a much more efficient solution (efficient in terms of number of particles needed) to visual tracking under illumination change, compared to other PFs. Hence, here, we only show comparisons with other PF methods. In Fig. 3 of Supplementary materials and also in [34], we also show comparisons with some other approaches from recent work.

In all of our experiments, we used a set of labeled video sequences for learning the dynamical model parameters for $A_t$ and $U_t$. First, manually hand-mark the target centroids in the training sequence and then use these to learn the motion vectors $U_t$. The corresponding illumination vector $A_t$ is computed from the image frame $Y_t$ as,

$$A_t = (A^T A)^{-1}A^T Y_t (ROI(U_t)).$$

The covariance matrices of the change of $U_t$ and of $A_t$, $\Sigma_A$ and $\Sigma_U$ are estimated using standard maximum likelihood estimation applied to $(U_t - U_{t-1})$ and $(A_t - A_{t-1})$. For learning the illumination model, we used $D = 7$.
Fig. 3. Performance comparison of various PFs while tracking across illumination changes for face (left) and vehicle tracking (right) application (visuals in the supplementary section). We show the location error from the ground truth for different particle filters. PF-MT correspond to Particle Filter with Mode Tracker (i.e. Illumination PF-MT), FULL-PF correspond to PF-Gordon [27], Full-PF-AUX correspond to Auxiliary-PF [32] and PF-NO-ILL corresponds to PF-Gordon without illumination model. It can be seen that Illumination PF-MT outperforms the rest. It is to be noted that Auxiliary-PF has some negligible performance improvement over PF-Gordon with standard resampling strategy; but it is far worse than Illumination PF-MT. In all of these experiments we used $N=100$ particles.

Fig. 4. An instance of face tracking under large illumination variation when someone switches the lighting conditions in a room.

Fig. 5. This Figure shows the results using gELL based change detection statistics. It can be see that the tracker can track through drastic illumination changes. The top row demonstrates that we are able track through illumination model changes when the car moves from sunlight to shadow area. During the transition from sunlight to shadow area (around frame 19), the gELL value goes above threshold indicating a model change (see gELL plot in the top row). If we do not detect the transition and increase $\Sigma_\Lambda$ then, tracker fails (second row).

(i.e. used Legendre functions up to order 3). For all the PF algorithms, we used a fixed particle size of $N=100$. In our experiments, we handmark the approximate target ROI in the first frame. The tracking performance of illumination PF-MT was compared with several other PF-based algorithms like - PF without the illumination model, Auxiliary-PF [32] and PF-Gordon [27]. Auxiliary-PF [32] uses look-ahead resampling strategy to improve effective particle size. PF-Doucet [9] cannot be implemented for our problem because it is not possible to use numerical convex optimization techniques to find the mode of $p^*(U_t, \Lambda_t) \propto OL(U_t, \Lambda_t)STP(U_t, \Lambda_t)$. This is due to fact that the observation likelihood $OL(U_t, \Lambda_t)$ is not continuously differentiable due to the involvement of round() operations in the mapping from $U_t$ to $Y_t$ (refer to (8) and (2)). However, the same is not true for the conditional likelihood of $\Lambda_t$ i.e. $CL^{(3)}(\Lambda_t)$ which enables us to implement PF-MT for our problem.

In the first experiment, we evaluate the tracking performance of illumination PF-MT for face tracking in the presence of illumination change and occlusions. Here, the lighting conditions variations could be attributed to two factors - a) the target’s distance from the window and variable ambient lighting coming through it, and b) occasional switching off and on of the light sources inside the room. The visual tracking results are given in Fig. 2. It can be clearly seen that illumination PF-MT (top row) clearly outperforms the rest of the PFs (i.e. PF-Gordon, FULL-PF, FULL-PF-AUX) for a limited particle budget of just 100 particles. The other PFs lose track within the first few frames and are unable to recover.

For quantitative tracking performance analysis, we did some further experiments with face and vehicle tracking from surveillance videos. The quantitative performance comparison plots for face tracking is shown in Fig. 3(a). The car dataset was generated from a camera observing a road from above as the cars approach an intersection (shown in Supplementary material Fig. 1). The illumination variations were due to the variations in the ambient lighting conditions. The corresponding quantitative tracking accuracy plots...
are given Fig. 3(b). It can be seen that with just 100 particles, our algorithm has the best performance in terms of tracking accuracy for both face and vehicle tracking scenarios. Another instance of face tracking under illumination variations using PF-MT has been demonstrated in Fig. 2 (dataset taken from http://www.eecs.qmul.ac.uk/~andrea/avis2007_d.html). We also show visual tracking results on the standard CAVIAR data set [35] in the Supplementary material Fig. 2.

We demonstrate the utility of illumination model change detection and compensation in Fig. 5. Notice that the tracker fails during a sunlight to shadow transition if we do not detect and adapt to the change.

VI. CONCLUSIONS

In this correspondence, we have tackled the difficult problem of visual tracking under variable illumination by reformulating it as a problem of large dimensional tracking with multimodal observation likelihood and using the PF-MT approach to design an efficient PF algorithm. We show exhaustive experiments to demonstrate the superior performance of our algorithm in handling large illumination variations and severe occlusions for both face and vehicle tracking videos. We also use the recently proposed idea of generalized ELL (gELL) to detect and adapt to changes in the illumination model. In future works, sparse representation of the illumination vector could be leveraged to replace the posterior mode tracking part by recently proposed particle filtered modified compressed sensing (PaFiMoCS) [36].

APPENDIX

The changed ‘appearance’ of the template \( T_i \) is represented in terms of a linear combination of the initial template \( T_0 \) scaled by a set of Legendre basis functions as follows [18].

\[
vec(T_i) = A \Lambda_i, \quad A \triangleq \begin{bmatrix} vec(T_0 \circ P_0), & \ldots, & vec(T_0 \circ P_{D-1}) \end{bmatrix};
\]

\[
P_n(i,j) = \begin{cases} p_n(i) & n = 1, \ldots, k \\ p_{n-k}(j) & n = k + 1, \ldots, D - 1 \end{cases}
\]

where \( p_n(\cdot) \) is the Legendre polynomial of \( n \)th order and \( \Lambda_i \) is the vector of Legendre basis coefficients at time \( t \). Henceforth, we will call it the illumination vector. Here, \( A \) is an \( M \times D \) matrix with \( D = 2k + 1 \) with \( k \) being the highest degree of the Legendre polynomials being used and \( M \) is the number of pixels in the initial template \( T_0 \). Now, given the motion parameter vector \( U_t \) consisting of scale, horizontal translation and vertical translation \( (U_t = [\sigma_t \quad T_x \quad T_y]^T) \) of the initial template, \( ROI(U_t) \) can be computed as [18],

\[
ROI(U_t) \triangleq \text{round}\left( J_i U_t + i_0, J_j U_t + j_0 \right)
\]

with \( J_i \triangleq \left[ (i_0 - \hat{i}_0) \ 1 \ 0 \right], J_j \triangleq \left[ (j_0 - \hat{j}_0) \ 0 \ 1 \right]
\]

The terms \( i_0 \) and \( j_0 \) are the \( M \) dimensional vectors containing the \( x \) and \( y \) coordinates of all the pixels in the initial template \( T_0 \), \( i_0 = \text{mean}(i_0) \) and \( j_0 = \text{mean}(j_0) \) denote the center of the initial template. Notice that equation (8) essentially is a geometric transformation that maps the pixels in the initial template to the current template region.

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