Iterative vs. recursive factorial

Consider the following implementations of the factorial function, and try to prove that it satisfies its postcondition.

```c
int factIter(int n)
{
    // You can assume that this function is correctly implemented.
    // That is, you can assume factIter(n) is equal to n!
}
```

```c
int factRec(int n)
{//@requires n >= 0;
//@ensures \result == factIter(n);
{
    if (n == 0) {
        return 1;
    }
    else {
        return n * factRec(n - 1);
    }
}
```

Solution:

Partial correctness.

Base case First, we consider the base case. When \( n = 0 \), we know that we return 1, which is 0!, so it's equal to \( \text{factIter}(0) \).

Inductive hypothesis Next, we assume that \( \text{factRec}(k) \) satisfies the postcondition for some \( \text{int } k \) where \( k >= 0 \), or in other words that the result of \( \text{factRec}(k) \) is equal to \( \text{factIter}(k) \).

Inductive step Now, we consider \( \text{factRec}(k + 1) \). Since \( k >= 0 \), we know \( k + 1 > 0 \).

Therefore, we'll be in the else case and will return \( (k + 1) * \text{factRec}(k + 1 - 1) \), which is equal to \( (k + 1) * \text{factRec}(k) \). We're allowed to make this call since we know that \( k + 1 > 0 \) and so \( k >= 0 \).

By the inductive hypothesis, \( \text{factRec}(k) \) is equivalent to \( \text{factIter}(k) \) and by the definition of factorial (and the assumption that \( \text{factIter} \) is correct) \( (k + 1) * \text{factIter}(k) \) is equal to \( \text{factIter}(k + 1) \).

Thus, the function has partial correctness.

Termination:

We've shown that if the function terminates, it is correct, but we need to show that the function terminates.
By the precondition, we know that \( n \geq 0 \).

**Base case** We also know that if \( n = 0 \) then we terminate immediately.

**Inductive hypothesis** Assume that \( \text{factRec}(k) \) terminates for some \( k \geq 0 \), where \( k \) is an int.

**Inductive step** Then, consider \( \text{factRec}(k + 1) \). We recurse and call \( \text{factRec}(k) \). By our inductive hypothesis, \( \text{factRec}(k) \) terminates, so therefore \( \text{factRec}(k + 1) \) terminates as well.

Thus, for all \( n \geq 0 \), this function terminates.