INTRODUCTION TO COMPUTER MUSIC

SPECTRAL CENTROID

An estimate of brightness

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Project 3

- Goal: Use spectral centroid to control FM synthesis parameters

- What’s a spectral centroid?
- Example code
Discrete Fourier Transform

\[ R_k = \sum_{i=0}^{N-1} x_i \cos\left(\frac{2\pi ki}{N}\right) \]

\[ X_k = -\sum_{i=0}^{N-1} x_i \sin\left(\frac{2\pi ki}{N}\right) \]

How to Interpret a Discrete Spectrum

- These points \( X_k \) and \( R_k \) are evenly (linearly) spaced in frequency.
- Point \( R_{N/2} \) is at \( SR / 2 \).
- Points \( X_k \) and \( R_k \) are at \( \left(\frac{k}{(N/2)}\right) \times \left(\frac{SR}{2}\right) = k \times \frac{SR}{N} \) Hz.
- Frequency spacing (width of “bins”) is \( SR / N \) Hz – the “bin width”
- Example: \( SR=44100 \) Hz, FFT size = 1024 points, bin size = \( 44100/1024 = 43.0664 \) Hz
- FFT takes in \( N \) samples and outputs \( N \) values
- This must be because FFT and Inverse FFT preserve information: \( N \)-dimensions in, \( N \)-dimensions out
- The output values are:
  - \( R_0 \) – the “DC” component
  - \( X_0 \) – always zero, not in output
  - \( R_1, R_2, R_3, \ldots, R_{N/2-1}, X_{N/2-1} \)
  - \( R_{N/2} \) – the “Nyquist” component
  - \( X_{N/2} \) – always zero, not in output
- Note there are \( N \) points as expected
Discrete Magnitude (or Amplitude) Spectrum

- Magnitude \( A_k = \sqrt{R_k^2 + X_k^2} \)
- The magnitude spectrum is:
  - \( A_0, A_1, \ldots, A_{N/2} \)
- Note there are \( N/2+1 \) points.
- How can this be? There are only \( N/2-1 \) non-zero phases, so we still have \( N \) total dimensions.

Spectral Centroid

- Weighted average of the magnitude (amplitude) spectrum:
  \[
  \text{spectral centroid} = \frac{\sum_{i=0}^{N} i \cdot w \cdot A_i}{\sum_{i=0}^{N} A_i}
  \]
- \( w \) is the width of each spectral bin in Hz
- \( w = \text{sample rate} / \text{size of the FFT in samples} \)
Spectral Centroid

Time-Varying Spectral Centroid
Review Project 3 Code Examples