Energy-Based Optimal Step Planning for Humanoids

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***Abstract*—Step planning is becoming an increasingly important research topic for humanoid robots. Most cost functions for step planning in the literature are designed based on terrain information. The energy cost to perform each step action is usually ignored. In walking, energy consumption depends on gait features such as step length and width. In this paper, we use three simple and intuitive energy cost functions for different step lengths, widths, and the turning angle. These functions are inspired by literature on human walking energy analysis, and the function parameters are tuned to match computed costs for optimal humanoid walking motions obtained by simulation. The energy cost and the terrain cost are combined to obtain an optimal step planning sequence using A* search.**

I. INTRODUCTION

Humans have a preferred method of locomotion. Generally, we walk at a particular speed and with a particular gait pattern [1]. Studies suggest that minimizing energy consumption is an important factor in human gait selection [2]–[4]. Energy efficiency has often been considered in walking motion generation [5]–[7], but this important factor has not been well addressed in online footstep planning for humanoid robots due to the computational cost of evaluating an energy cost for whole body motion.

Most existing methods for path and footstep planning have mainly focused on environment modeling such as terrain and obstacles. In path planning, a common approach is to consider the humanoid robot as a bounding box. The search algorithm uses the bounding box to generate a collision-free path [8]–[10]. The problem with this approach is that every obstacle, even a small one that can easily be stepped over, must be avoided. To use the ability of stepping over and on, obstacles are classified based on their geometric features, such as height, and the robot applies different strategies to overcome the obstacles depending on their types [11]–[13]. Once a feasible path is found, an appropriate step sequence along the path is searched for footstep planning. The problem with this planning strategy is that the path planning stage may produce an awkward or even unexecutable step sequence.

To alleviate this issue, Chestnutt et. al proposed an algorithm which directly plans the step sequence [14]. Their method uses an action model to describe possible foot placements. Using the action model reduces the dimensionality of the search space, but also limits the resolution. An adaptive action model is used to increase this resolution [15]. A guide path is incorporated into the planning process as a heuristic for the search [16] [17].

A human normally has a preferred step length and step width for walking, and this is believed to be related to minimizing metabolic energy consumption [1], [18], [19]. Kuo et al. explored a relationship between energy consumption and certain gait characteristics, such as step length, using a simple bipedal model, and they predicted a preferred speed-step relationship for forward walking [3], [4], [20]. Inspired by their work, we designed a simple cost function for estimating energy consumption for each step, and applied it to our humanoid footstep planning. In addition to the cost terms dependent on the step length and width, we consider the energy cost for body rotation, or turning the walking direction, which has not been well addressed in biomechanical literature but is necessary for the footstep planning. To improve the fidelity of the cost function, its parameters are tuned to match the computed costs for humanoid walking motions which were obtained by using optimization. To our knowledge, this is the first work to consider energy efficiency in online humanoid footstep planning.

In Section II and III, we describe the terrain cost and step energy cost used in our footstep planning based on an A* search algorithm. We compare our planning method with an existing method considering only the terrain cost in Section IV, and conclude this work in Section V.

II. MODELING THE TERRAIN COST

Terrain cost is a common cost included in step planning. To evaluate a location's cost, we would like to know information about the terrain. In this paper, four features are considered in the cost function: slope, altitude, roughness and bumpiness. These feature values are calculated based on the terrain information at the touching down location.

The slope feature indicates the slope angle of the candidate foot location. Here, the slope angle is calculated by fitting a plane, $p(x, y) = b_0+b_1*x+b_2*y$, to the candidate foot location region, $\Re$. The feature value is the norm of the gradient.

$$C_{slope} = \sqrt{b_1^2 + b_2^2} \tag{1}$$

The altitude indicates the average height of the obstacle. Stepping up or down may result in a change of center of mass

(CoM) height, which affects the energy consumption. Here, we focus on the ground walking. Walking on a higher plane is not considered.

$$C_{altitude} = \overline{h(x, y)} \qquad (x, y) \in \Re \tag{2}$$

where $h(x, y)$ is the actual terrain height at the x and y position.

The roughness feature indicates the deviation of the surface from the fitting plane $p(x, y)$. The roughness is computed by averaging the difference between actual terrain height and fitting plane height.

$$C_{rough} = \frac{1}{N}\Sigma|h(x, y) - p(x, y)| \qquad (x, y) \in \Re \tag{3}$$

Among current humanoid robots, most feet are a flat plate which are very sensitive to the roughness of the ground. A small roughness threshold value is given to avoid falling.

The bumpiness feature gives the maximum height change in the area of foot placement. Similar to roughness, bumps above the fitted plane are very dangerous which result rocking of foot. Even a low bump will result in falling during walking.

$$C_{bump} = max(|h(x, y) - p(x, y)|) \qquad (x, y) \in \Re \tag{4}$$

Total terrain cost is the weighted sum of these four cost features.

$$C_{Terrain} = w_1 C_{slope} + w_2 C_{altitude} + w_3 C_{rough} + w_4 C_{bump} \tag{5}$$

To avoid dangerous terrain, we set a threshold for each feature cost. The maximum slope tolerance is 30 degree; the maximum altitude change is 0.2m; the maximum roughness is 0.01m; the maximum bumpiness is 0.05m. We choose the weights manually $[w_1, w_2, w_3, w_4] = [2, 5, 100, 20]$ to make the cost terms of these four features at the same scale and balance the total terrain cost for each foot plane with single step cost described in section III. However, higher weights for the terrain cost can be chosen if safety is more important.

Fig. 1 shows an example of randomly generated terrain and the cost maps of the four features. The map size is 4mx4m. The foot size is 12x21cm with $\frac{\pi}{2}$ orientation, forward in the y direction (vertical axes). In the terrain design, three types of obstacles are used: low and wide cylinder (LWC), high and narrow cylinder (HNC), low and thin bar (LTB). The size and shape of the obstacles are chosen to ensure that the robot can step on the LWC and step over the LTB. Due to height limitations, the robot cannot step over the HNC. This property is not shown in the cost function. To enable this property, a threshold is specified to the height of the obstacle along the stride line of walking. The height limit is set to be 0.2m.

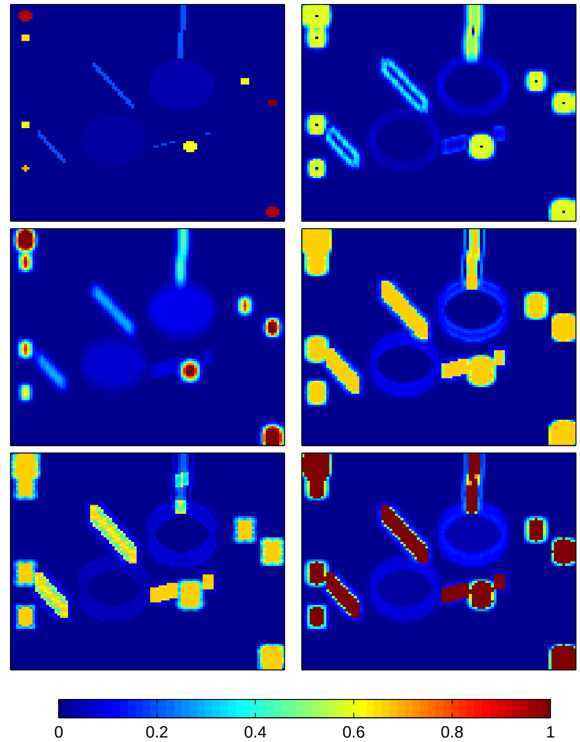

Fig. 1. Top left: terrain map with 4mx4m size; Top right: C_{slope}; Middle left: $C_{altitude}$; Middle right: C_{rough}; Bottom left: C_{bump}; Bottom right: $C_{Terrain}$.

III. Modeling the Step Energy Cost

In walking, humans have a preferred step length and step width. Much research has been conducted to analyze the reason for this behavior, with particular emphasis on energy. Humans is believed to walk in an energy efficient manner [2]–[4]. Therefore, human-like step planning requires the consideration of energy consumption. A direct method of calculating the energy consumption is to build a dynamic model and then compute energy consumption. Due to the great complexity of humanoid robots, estimating the energy cost using a detailed model is difficult. In this paper, a simple dynamic model is used to estimate energy consumption for steps of varying lengths and widths.

A. Energy Cost with Step Length

For a particular walking speed, humans tend to choose a step length and step frequency that minimizes their energy consumption [4]. Kuo [4] used a simple passive dynamic walking model to evaluate the relationship between the energy cost of muscle activity and step length. The cost of transport forward is divided into two parts: cost of pushing off with the support foot to move the body forward and the cost of moving the swing foot. The first component is hypothesized to be proportional to the third power of step length [20]. On the other hand, the swing leg contributes substantial cost as a tradeoff between high frequency walking and long step walking. In Kuo's hypotheses [4], the cost of the swing foot increases sharply with the increase of swing frequency. Assuming the swing leg moves at a constant speed, the cost of swing foot movement is inversely proportional to the step length. The energy cost with step length can be:

$$C_{stepL} \sim Al^3 + Bl^{-1} + C \tag{6}$$

where A and B are scaling factors, C is a constant cost, l is the step length, and E_{stepL} is normalized for body weight and distance traveled to yield a dimensionless energy cost of transport.

B. Energy Cost with Step Width

Humans also appear to prefer a particular step width. This preferred step width may be a result of minimized energy cost. Based on the passive dynamic walking model, the energy loss incurred in transition from one single support to the other is proportional to the square of the step width [3]. Alternatively, a narrow step width is also costly due to increased risk of collision with the other leg. Studies suggest that energy consumption of side-to-side foot motion will increase linearly as step width decreases below the preferred step width [3]. The energy cost with the step width can be written as:

$$C_{stepW} \sim Dw^2 + Collision(w) \tag{7}$$

$$Collision(w) \sim \begin{cases} 0 & w > SW; \\ E(SW - w) & \text{else} \end{cases} \tag{8}$$

where D and E are scaling factors, w is the step width, SW is the preferred step width, and E_{stepW} is normalized for body weight and travel distance.

C. Energy Cost with Body Rotation

Besides step length and step width, turning also costs energy as body rotates to a new direction. Most work on human walking analysis is based on forward walking only. Here, we create a function to describe the cost of walking for both straight-line travel and turning. We hypothesize that the energy consumption of rotation is proportional to the square of the body rotation angle.

$$C_{rotation} \sim F\theta^2/l \tag{9}$$

where F is a scaling factor and θ is a rotation angle. The body rotation angle is defined as orientation change between target foot and supporting foot. $E_{rotation}$ is normalized for body weight and travel distance.

D. Step Energy Cost Function

The total energy cost function is the combination of three factors: step length, step width and body rotation.

$$C_{Step} = C_{stepL} + C_{stepW} + C_{rotation} \tag{10}$$

We set the parameters in Equations (7)-(9) as $A = 22.0$, $B = 4.13$, $C = 10.0$, $D = 0.2$, $E = 0.23$, $F = 0.4$. based on simulated walking cost data described in Section III (E).

Generally, step length is equal to the distance between corresponding successive points of heel contact of the opposite feet. According to the Equation (6), when the candidate foot location is close to the supporting foot, the value of C_{stepL} can be very large. To avoid a rapid increase in the cost function, we use half of the stride length (distance between successive points of heel contact of the same foot) as an approximate value of step length. Considering that initial condition will

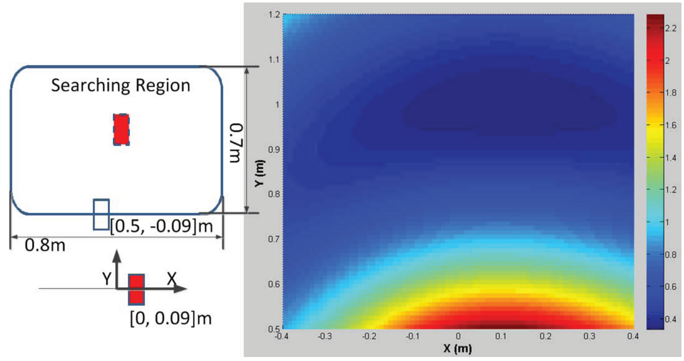

Fig. 2. Energy cost map per travel distance; $l_p = 0.5$; $SW = 0.178$; searching region in y direction [0.5, 1.2]m, x direction [-0.4,0.4]m; the optimal step location is at [0.09, 1]m; the colormap is Jet in Matlab.

affect the energy consumption, the previous step length is also included in the calculation of current step length.

$$l = \frac{s_l}{2}\left(\frac{l_r}{l_p}\right)^{0.5} \tag{11}$$

where s_l is the stride length, l_p is the distance between the support foot and previous foot location, and l_r is the optimal step length. The power of 0.5 adds a damping factor if the previous step length deviated from the optimal step length. It helps the step length return back to the preferred step length gradually. For the first step, we assume that $l_p = l_r$.

Fig. 2 shows an example of cost map showing energy cost per distance travel for different step location. The left foot is in stance at [0.5 -0.09]m. The previous foot location is [0,0.09]m. In this example, the search region is the area in front of the supporting foot [0.5m 1.2]m in the y direction and [-0.4m 0.4m] in the x direction. The resulting optimal foot location is [0.09 1]m. The map indicates that moving in the forward direction is more energy efficient than moving sideways. Turning, especially sharp turns, consumes more energy than forward motion. The expected energy cost of a sharp turn is approximately twice the value of forward walking. Therefore, given a goal location where turning is needed, the planner prefers to turn gradually to reach the goal instead of turning sharply, which matches our intuition about human-like walking

E. Comparing Our Cost Function with Simulated Humanoid Walking Cost

In order to improve the fidelity of our energy cost function for humanoid footstep planning, we tune the parameters by comparing with estimated costs for optimized humanoid walking motions. More specifically, we compare the step cost value to the commonly used torque squared optimization criteria: sum of squared joint torques $\left(\int \tau^T \tau dt\right)$. The torque squared criteria does not optimize energy. We are testing whether the results of our energy optimization are similar to a widely used optimization criteria in animation and robotics. The walking motions were obtained by using trajectory optimization.

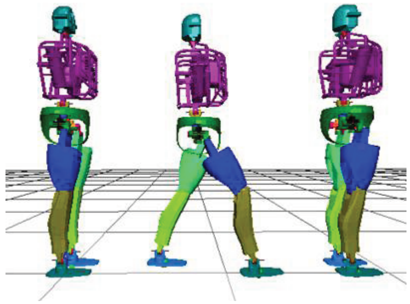

Fig. 3. Humanoid robot model in the simulation.

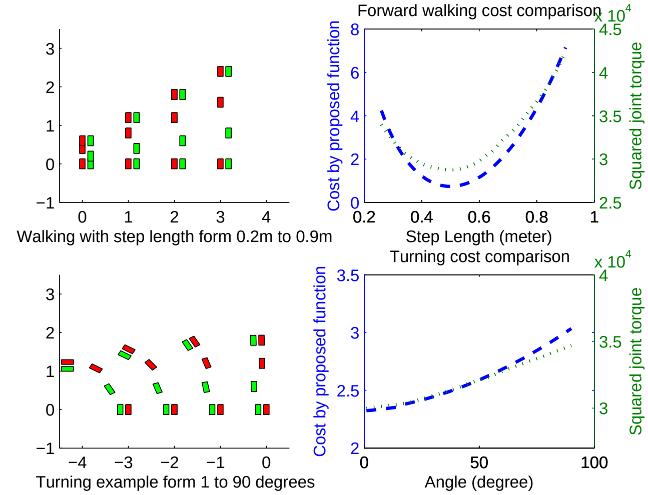

Fig. 4. Comparison between step costs by our simplified cost function and step costs by optimized humanoid walking motion. The blue line is the cost from Equation (10) and green line is the estimated cost from optimized human motion. Due to different scale size, the values are different between two cost results. However, they show a similar growth trend.

The robot model (Figure 3) has 16 revolute joints – each leg has 6 joints (3 at the hip, 1 at the knee, and 2 at the ankle), and the waist and neck have 3 joints and 1 joint respectively. The arms are not included in the model. The total mass is about 67 kg and the height is about 170 cm.

In our optimization setting, the trajectory of each joint is represented with a series of many quintic polynomials whose parameters are the optimization variables. Given a sequence of foot step positions, we find an optimal walking trajectory that minimizes a cost function while satisfying the foot step constraints and other constraints such as physical limits as mentioned below.

The objective function we used for the trajectory optimization penalizes the joint torques and the impulse to the swing foot at the moment of collision with the ground. We also penalize the lateral deviation of the swing foot trajectory from the straight line connecting the previous and target swing positions to avoid the self collision. The joint angle and torque limits, the height clearance for the swing foot, and the friction cone constraints for the contact force at the stance foot and the impulse at the swing foot are considered as the inequality constraints of the optimization problem. The robot feet are constrained to be parallel to the flat ground during walking. The double support phase was ignored in our walking model for problem simplification; only single support phase – one foot is the stance foot and the other is the swing foot – was considered. We used SNOPT, an off-the-shelf sequential quadratic programming solver [21], in our optimization.

In this comparison, two types of walking trajectories are tested: 1) forward walking with various step lengths from 0.2m to 0.9m (Fig. 4 top left); 2) turning motion from 1 to 90 degrees to the left (Fig. 4 bottom left). The parameters in Equation (7)-(9) are tuned to match the cost from optimized humanoid walking. Fig. 4 shows the computed costs for these two types of walking trajectories. The comparison provides us a good reference to tune the parameters of the cost function. Also, our cost function is validated by the optimal humanoid walking cost where we see similar trends. The cost value of forward walking with optimal step length 0.5m is chosen to be 0.5 to match the distance travel.

Analysis of step length and step width are often based on the metabolic cost of human walking [3] [4]. Due to the difference between humans and humanoid robots, the results of such analysis may not be directly applicable to humanoid robots.. In our analysis, the cost function is compared with the cost of an optimized humanoid walking motion. This makes the cost function more applicable to the humanoid robot.

IV. Simulation Results

A. A* Planning

In step planning, A* search is often used to find the optimal step sequence from a starting point to a goal point. A* search includes two main parts: 1) expand the search by generating children nodes from the parent node 2) compute the cost of each child's node.

For the first part, we generate a child node from an action pool. Fig. 5 shows the full set of our actions which enable the robot to walk forward, walk sideways, turn left and turn right.

Large action sets in the pool will dramatically increase the number of nodes in the A* search, which results in a long computation time. Small action sets limit the options available to the A* search, which generally results in finding higher cost trajectories.. To address this problem, we adopt an adaptive step location for a given action. As shown in Fig. 6 (a), if there is an obstacle under the given step location, the adaptive step will check the surrounding area to find an obstacle-free stepping region. The searching areas include $[-0.05, 0, 0.05]$cm in x and y directions and $[-\frac{1}{18}\pi, 0, \frac{1}{18}\pi]$ in orientation. The overall coverage is given in Fig. 6 (b).

We have analyzed the second part in section II and III. The estimated cost of a foot location is given by:

$$C_E = C_{Step} + C_{Terrain} + Heuristic + C_{previous} \quad (12)$$

where C_E is the estimated cost of this foot location; $C_{Terrain}$ is given by Equation (5); C_{Step} is given by Equation (10);

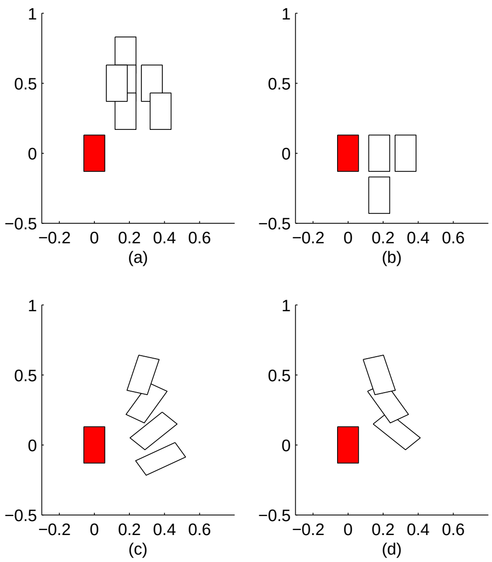

Fig. 5. Footstep action set with forward (a), sideways and backward (b), turn right (c) and turn left (d). The actions displayed are only those for the right foot (relative to the left foot shown in red).

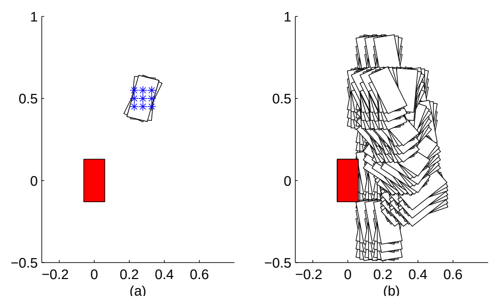

Fig. 6. Foot action with adaptive steps (a), adaptive steps with $[-0.05, 0, 0.05]$cm in x and y directions and $[-\frac{1}{18}\pi, 0, \frac{1}{18}\pi]$ in orientation (b), overall coverage for 16 foot actions with adaptation.

$C_{previous}$ is the accumulated cost of previous steps. Heuristic cost is the estimated cost to the goal which guides the searching direction. Here, the Heuristic cost is calculated by the Euclidean distance from the foot location to the goal location.

B. Comparing Path Planning Results

In this sectopm, we test four planning methods with 1) fixed action set and fixed step cost (FSFC) 2) adaptable action set and fixed step cost (ASFC) 3) fixed action set and energy cost (FSEC) 4) adaptable action set and energy cost (ASEC). For the fixed step cost used in FSFC and ASFC, we used the constant cost of 0.5 for each step in planning. We used Equation (10) to evaluate the energy cost for each step action in planning with FSEC and ASEC.

The test environment is 10mx10m in size where the three types of obstacles (LWC, HNC, LTB) described in section II are randomly placed inside. Fig. 7 shows the results of our first experiment. Green and pink color foot step sequence is computed by FSFC. The total step cost is 16.9 based on Equation (10) and total terrain cost is 0.10 based on Equation (5). Gray and white color foot step sequence is computed by FSEC with total step cost 12.3 and total terrain cost 0.13. Red and yellow color foot step sequence is computed by ASEC with total step cost 12.1 and total terrain cost 0.23. In this experiment, we compared how well step planners with and without an energy-based optimization criterion perform using the energy-based criterion. From the result, both ASEC and FSEC have better performance than the FSFC in step cost.

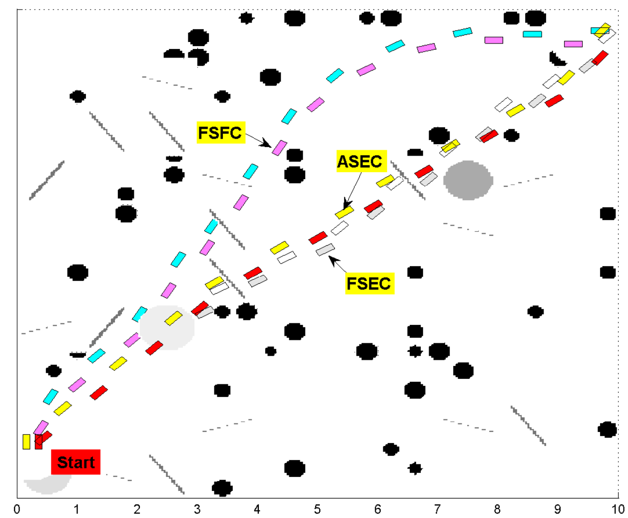

Fig. 7. Path planning in a large environment with size 10mx10m; green and pink color step sequences is computed by FSFC with total step cost 16.9 and terrain cost 0,10; gray and white color step sequences is computed by FSEC with total step cost 12.3 and terrain cost 0,13; red and yellow color step sequences is computed by ASEC with total step cost 12.1 and terrain cost 0,23; the color map is an inverted gray scale color.

In our next experiments, we run the planner 40 times in different environments which are randomly generated. In each round, all four methods are tested. Fig. 8 shows the average costs for these 4 methods in 40 rounds. To give a clear view, the step cost and the terrain cost are separated. Average costs for the methods which consider the energy cost (FSEC and ASEC) are significantly smaller than the methods which use fix step cost (FSFC and ASFC). This is unsurprising because the step cost is included in the A* search. The step cost function can improve the energy cost of step planning.

The data also shows that the adaptable action method helps reduce the total terrain cost. The terrain costs of footstep sequences generated by the ASEC and ASFC methods are significantly smaller than those generated by the FSEC and FSFC methods. The adaptable action allows the planner to evaluate a larger number of potential footstep locations which reduces the terrain cost. Also, the terrain costs of FSEC and ASEC are higher than the ones of FSFC and ASFC. This is because, in the case of fixed step cost (FSFC and ASFC), the planner tries to find a path with low terrain cost. However, when the step energy cost is considered (FSEC and ASEC), the planner tries to find a path which minimizes both terrain cost and step cost. Therefore, the terrain costs of FSEC and ASEC are higher than the ones of FSFC and ASFC. However, their total costs are lower.

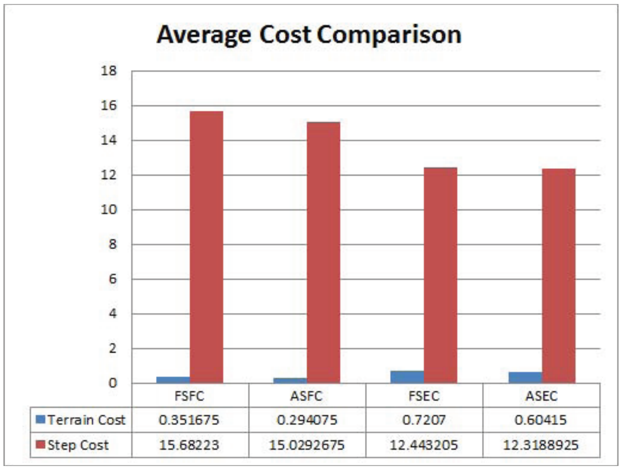

Fig. 8. Bar graph of average cost comparison between four types of methods.

V. Conclusion

In this paper, we designed a simple energy cost function for estimating energy consumption effectively, and applied it to humanoid foot step planning. A new cost term which addresses turning is added to the total cost function. To improve the fidelity of the cost function, its parameters are tuned to match the simulated humanoid walking cost which is obtained by trajectory optimization. The simulation results show that the proposed step planner has better energy efficiency compared to a fixed step cost.

Generally, avoiding failure (falling in the case of humanoid walking) is the first priority in motion planning. When the walking condition is bad, attention is given to prevent falling. However, in steady state human walking or avoiding obstacles, which can be handled with confidence, the walking patterns are more likely to be determined by energetics. One limitation of the current cost function is that the velocity is not considered. Humanoids may have different walking speed in different situations. Another future research area would involve comparison of the footstep choices to human footstep choices in actual experiments.

Acknowledgment

This material is based upon work supported in part by the US National Science Foundation (EEC-0540865, ECCS-0824077, and IIS-0964581), the DARPA M3 program, and the DARPA Robotics Challenge.

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