Automata Theory: Assignment 1
Due date: August 30 (Thursday)

Problem 1 (4 points)
Consider the following sets of integer numbers:

\[ S_1 = \{4, 5, 6\} \]
\[ S_2 = \{i : i \text{ is even}\} \]
\[ S_3 = \{i : i \text{ is divisible by 3}\} \]

For each set below, specify its elements and determine whether it is finite or infinite:

\[ S_4 = S_1 \times S_1 \]
\[ S_5 = 2^{S_1} \]
\[ S_6 = S_1 \cap S_2 \]
\[ S_7 = S_2 \cap S_3 \]

Problem 2 (2 points)
Prove that, if \( S_1 \subseteq S_2 \), then \( \overline{S_2} \subseteq \overline{S_1} \).

Problem 3 (4 points)
Prove the following equalities:

(a) \( 1 + 2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2} \).

(b) \( 1 + x + x^2 + x^3 + \ldots + x^n = \frac{x^{n+1} - 1}{x-1} \) (where \( x \neq 1 \)).