Probabilistic Graphical Models

Introduction to GM

Eric Xing
Lecture 1, January 11, 2016

Reading: see class homepage
Logistics

- **Class webpage:**
  - [http://www.cs.cmu.edu/~epxing/Class/10708-16/](http://www.cs.cmu.edu/~epxing/Class/10708-16/)
Logistics

- **Text books:**
  - Daphne Koller and Nir Friedman, *Probabilistic Graphical Models*
  - M. I. Jordan, *An Introduction to Probabilistic Graphical Models*

- **Mailing Lists:**
  - To contact the instructors: 10708-instructor@cs.cmu.edu
  - Class announcements list: 10708-students@cs.cmu.edu.

- **TA:**
  - Calvin McCarter, GHC 8008, Office hours: TBA
  - Jin Kyu Kim, GHC 6219, Office hours: TBA
  - Yuntian Deng, GHC , Office hours: TBA
  - Zhiting Hu, GHC 8116, Office hours: TBA

- **Lecturers:** Eric Xing, Matt Gormley

- **Class Assistant:**
  - Mallory Deptola, GHC 8001, x8-5527

- **Instruction aids:** Piazza
Logistics

- 5 homework assignments: 40% of grade
  - Theory exercises, Implementation exercises
- Scribe duties: 10% (~once to twice for the whole semester)
- Short reading summary: 10% (due at the beginning of every lecture)
- Final project: 40% of grade
  - Applying PGM to the development of a real, substantial ML system
    - Design and Implement a (record-breaking) distributed Logistic Regression, Gradient Boosted Tree, Deep Network, or Topic model on Petuum and apply to ImageNet, Wikipedia, and/or other data
    - Build a web-scale topic or story line tracking system for news media, or a paper recommendation system for conference review matching
    - An online car or people or event detector for web-images and webcam
    - An automatic “what’s up here?” or “photo album” service on iPhone
  - Theoretical and/or algorithmic work
    - a more efficient approximate inference or optimization algorithm, e.g., based on stochastic approximation, proximal average, or other new techniques
    - a distributed sampling scheme with convergence guarantee
- 3-member team to be formed in the first three weeks, proposal, mid-way report, oral presentation & demo, final report, peer review → possibly conference submission!
Past projects:

- Award Winning Projects:
  


  Others … such as KDD 2014 best paper

- Other projects:


  M. Sachan, A. Dubey, S. Srivastava, E. P. Xing and Eduard Hovy, *Spatial Compactness meets Topical Consistency: Jointly modeling Links and Content for Community Detection*, *Proceedings of The 7th ACM International Conference on Web Search and Data Mining (WSDM 2014)*.

- We will have a prize for the best project(s) …
What Are Graphical Models?

Graph

```
M
```

Data

Model

```
\mathcal{D} \equiv \{X_1^{(i)}, X_2^{(i)}, \ldots, X_m^{(i)}\}_{i=1}^N
```

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Reasoning under uncertainty!
The Fundamental Questions

- **Representation**
  - How to capture/model uncertainties in possible worlds?
  - How to encode our domain knowledge/assumptions/constraints?

- **Inference**
  - How do I answer questions/queries according to my model and/or based on given data?

  \[ P(X_i | D) \]

- **Learning**
  - What model is "right" for my data?

  \[ \mathcal{M} = \arg \max_{\mathcal{M} \in \mathcal{M}} F(D; \mathcal{M}) \]
Recap of Basic Prob. Concepts

- **Representation:** what is the joint probability dist. on multiple variables?
  \[ P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) \]
  - How many state configurations in total? --- \( 2^8 \)
  - Are they all needed to be represented?
  - **Do we get any scientific/medical insight?**

- **Learning:** where do we get all this probabilities?
  - Maximal-likelihood estimation? but how many data do we need?
  - Are there other est. principles?
  - Where do we put domain knowledge in terms of plausible relationships between variables, and plausible values of the probabilities?

- **Inference:** If not all variables are observable, how to compute the conditional distribution of latent variables given evidence?
  - Computing \( p(H|A) \) would require summing over all \( 2^6 \) configurations of the unobserved variables
What is a Graphical Model?
--- Multivariate Distribution in High-D Space

- A possible world for cellular signal transduction:

- Receptor A \( x_1 \)
- Receptor B \( x_2 \)
- Kinase C \( x_3 \)
- Kinase D \( x_4 \)
- Kinase E \( x_5 \)
- TF F \( x_6 \)
- Gene G \( x_7 \)
- Gene H \( x_8 \)
GM: Structure Simplifies Representation

- Dependencies among variables

![Diagram showing relationships between Receptor A, Receptor B, Kinase C, Kinase D, Kinase E, TF F, Gene G, and Gene H with variables X1 to X8.](image-url)
If $X_i$'s are **conditionally independent** (as described by a PGM), the joint can be factored to a product of simpler terms, e.g.,

\[
P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = P(X_1) P(X_2) P(X_3 \mid X_1) P(X_4 \mid X_2) P(X_5 \mid X_2) P(X_6 \mid X_3, X_4) P(X_7 \mid X_6) P(X_8 \mid X_5, X_6)
\]

Stay tune for what are these independencies!

**Why we may favor a PGM?**

- Incorporation of domain knowledge and causal (logical) structures

  1+1+2+2+2+4+2+4=18, a 16-fold reduction from $2^8$ in representation cost!
GM: Data Integration

[Diagram of biological pathways:
- Receptor A
- Kinase C
- TF F
- Gene G
- Receptor B
- Kinase D
- Kinase E
- Gene H

Nodes and edges indicating connections between biological entities.]
More Data Integration

- Text + Image + Network ➔ Holistic Social Media

- Genome + Proteome + Transcritome + Phenome + … ➔ PanOmic Biology
If $X_i$'s are **conditionally independent** (as described by a PGM), the joint can be factored to a product of simpler terms, e.g.,

\[
P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = P(X_2) P(X_4|X_2) P(X_5|X_2) P(X_7|X_5) P(X_8|X_7)
\]

Why we may favor a PGM?

- Incorporation of domain knowledge and causal (logical) structures
  
  \[2+2+4+4+4+8+4+8=36,\text{ an 8-fold reduction from } 2^8 \text{ in representation cost!}\]

- Modular combination of heterogeneous parts – data fusion
Rational Statistical Inference

The Bayes Theorem:

\[ p(h \mid d) = \frac{p(d \mid h)p(h)}{\sum_{h' \in H} p(d \mid h')p(h')} \]

- Posterior probability
- Likelihood
- Prior probability

Sum over space of hypotheses

- This allows us to capture uncertainty about the model in a principled way

- But how can we specify and represent a complicated model?
  - Typically the number of genes need to be modeled are in the order of thousands!
GM: MLE and Bayesian Learning

- Probabilistic statements of $\Theta$ is conditioned on the values of the observed variables $A_{\text{obs}}$ and prior $p(\Theta | \chi)$

$$
\Theta_{\text{Bayes}} = \int \Theta \ p(\Theta | A, \chi) \ d\Theta
$$

$p(\Theta; \chi) \propto p(A | \Theta)p(\Theta; \chi)$

| C | D | $P(F | C,D)$ |
|---|---|-------------|
| c | d | 0.9 0.1     |
| c | d | 0.2 0.8     |
| c | d | 0.9 0.1     |
| c | d | 0.01 0.99   |

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If $X_i$'s are **conditionally independent** (as described by a PGM), the joint can be factored to a product of simpler terms, e.g.,

$$P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = P(X_1) P(X_2) P(X_3| X_1) P(X_4| X_2) P(X_5| X_2)$$

$$P(X_6| X_5, X_4) P(X_7| X_6) P(X_8| X_5, X_6)$$

**Why we may favor a PGM?**

- Incorporation of domain knowledge and causal (logical) structures
  
  2+2+4+4+4+8+4+8=36, an 8-fold reduction from $2^8$ in representation cost!

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- **Bayesian Philosophy**
  
  - Knowledge meets data

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So What Is a PGM After All?

In a nutshell:

$$\text{PGM} = \text{Multivariate Statistics} + \text{Structure}$$

$$\text{GM} = \text{Multivariate Obj. Func.} + \text{Structure}$$
So What Is a PGM After All?

- The informal blurb:
  - It is a smart way to \textit{write/specify/compose/design} exponentially-large probability distributions without paying an exponential cost, and at the same time endow the distributions with \textit{structured semantics}.

- A more formal description:
  - It refers to a family of distributions on a set of random variables that are compatible with all the probabilistic independence propositions encoded by a graph that connects these variables.
Two types of GMs

- Directed edges give causality relationships (Bayesian Network or Directed Graphical Model):

  \[
P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = P(X_1) P(X_2) P(X_3|X_1) P(X_4|X_2) P(X_5|X_2) \\
P(X_6|X_3, X_4) P(X_7|X_6) P(X_8|X_5, X_6)
\]

- Undirected edges simply give correlations between variables (Markov Random Field or Undirected Graphical model):

  \[
P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = \frac{1}{Z} \exp\{E(X_1)+E(X_2)+E(X_3, X_1)+E(X_4, X_2)+E(X_5, X_2) \\
+ E(X_6, X_3, X_4)+E(X_7, X_6)+E(X_8, X_5, X_6)\}
\]
Bayesian Networks

Structure: **DAG**

- Meaning: a node is **conditionally independent** of every other node in the network outside its Markov blanket.

- Local conditional distributions (CPD) and the DAG completely determine the **joint** dist.

- Give causality relationships, and facilitate a **generative** process.
Markov Random Fields

Structure: *undirected graph*

- Meaning: a node is *conditionally independent* of every other node in the network given its *Directed neighbors*

- Local contingency functions (*potentials*) and the *cliques* in the graph completely determine the *joint* dist.

- Give *correlations* between variables, but no explicit way to generate samples
Towards structural specification of probability distribution

- Separation properties in the graph imply independence properties about the associated variables
- For the graph to be useful, any conditional independence properties we can derive from the graph should hold for the probability distribution that the graph represents

- The Equivalence Theorem
  For a graph $G$,
  Let $\mathcal{D}_1$ denote the family of all distributions that satisfy $I(G)$,
  Let $\mathcal{D}_2$ denote the family of all distributions that factor according to $G$,
  Then $\mathcal{D}_1 \equiv \mathcal{D}_2$. 
GMs are your old friends

Density estimation
  Parametric and nonparametric methods

Regression
  Linear, conditional mixture, nonparametric

Classification
  Generative and discriminative approach

Clustering
An (incomplete) genealogy of graphical models

(Picture by Zoubin Ghahramani and Sam Roweis)
Fancier GMs: reinforcement learning

- Partially observed Markov decision processes (POMDP)
Fancier GMs: machine translation

The HM-BiTAM model (B. Zhao and E.P Xing, ACL 2006)
Fancier GMs: genetic pedigree

An allele network
Fancier GMs: solid state physics

Ising/Potts model
Application of GMs

- Machine Learning
- Computational statistics

- Computer vision and graphics
- Natural language processing
- Informational retrieval
- Robotic control
- Decision making under uncertainty
- Error-control codes
- Computational biology
- Genetics and medical diagnosis/prognosis
- Finance and economics
- Etc.
Why graphical models

- A language for communication
- A language for computation
- A language for development

Origins:
- Wright 1920’s
- Independently developed by Spiegelhalter and Lauritzen in statistics and Pearl in computer science in the late 1980’s
Why graphical models

- **Probability theory** provides the glue whereby the parts are combined, ensuring that the system as a whole is consistent, and providing ways to interface models to data.

- The **graph theoretic** side of graphical models provides both an intuitively appealing interface by which humans can model highly-interacting sets of variables as well as a data structure that lends itself naturally to the design of efficient general-purpose algorithms.

- Many of the classical multivariate probabilistic systems studied in fields such as statistics, systems engineering, information theory, pattern recognition and statistical mechanics are special cases of the general graphical model formalism.

- The graphical model framework provides a way to view all of these systems as instances of a common underlying formalism.
Plan for the Class

- Fundamentals of Graphical Models:
  - Bayesian Network and Markov Random Fields
  - Discrete, Continuous and Hybrid models, exponential family, GLIM
  - Basic representation, inference, and learning
  - ...

- Advanced topics and latest developments
  - Approximate inference
    - Monte Carlo algorithms
    - Variational methods and theories
  - “Infinite” GMs: nonparametric Bayesian models
  - Optimization-theoretic formulations for GMs, e.g., Structured sparsity
  - Nonparametric and spectral graphical models, where GM meets kernels and matrix algebra
  - Alternative GM learning paradigms,
    - e.g., Margin-based learning of GMs (where GM meets SVM)
    - e.g. Regularized Bayes: where GM meets SVM, and meets Bayesian, and meets NB ...

- Case studies: popular GMs and applications
  - Multivariate Gaussian Models
  - Conditional random fields
  - Mixed-membership, aka, Topic models
Questions ?