Exercise 1. Use the construction given in class or in Theorem 1.39 to convert each of the two NFAs at the right into an equivalent DFA.

Exercise 2. Are the following languages regular? For each language, either give an automaton\(^1\) that recognizes it or give a proof that it is not regular.

a. \(\{a^n b^n c^n\}\), for \(\Sigma = \{a, b, c\}\)

b. \(\{ww^R | w \in \Sigma^*\}\), for \(\Sigma = \{0, 1\}\)

c. \(\{ww | w \in \Sigma^*\}\), for \(\Sigma = \{0, 1\}\)

Exercise 3. Consider the set of strings that have an equal number of occurrences of “01” and “10”. (For example, “010” is in this language but “0101” is not.) Under each of the following alphabets, is this language regular? For each alphabet, give an automaton that recognizes it or prove that it is not regular.

a. \(\Sigma = \{0, 1\}\)

b. \(\Sigma = \{0, 1, \#\}\)

Exercise 4. If \(A\) is a regular language, let \(A_{\frac{1}{3}}\) be the set of all first thirds of strings in \(A\) so that

\[ A_{\frac{1}{3}} = \{x | \text{for some } yz, |x| = \frac{1}{3}|xyz| \text{ and } xyz \in A\}\]

Similarly, let \(A_{\frac{2}{3}}\) be the set of all last thirds of strings in \(A\) so that

\[ A_{\frac{2}{3}} = \{z | \text{for some } xy, |z| = \frac{1}{3}|xyz| \text{ and } xyz \in A\}\]

Finally, let \(A_{\frac{1}{3}, \frac{2}{3}}\) be the set of strings with their middle third deleted so that

\[ A_{\frac{1}{3}, \frac{2}{3}} = \{xz | \text{for some } y, |x| = |z| = \frac{1}{3}|xyz| \text{ and } xyz \in A\}\]

a. Give an automaton that recognizes \(A_{\frac{1}{3}}\) or prove that it is not regular.

b. Give an automaton that recognizes \(A_{\frac{2}{3}}\) or prove that it is not regular.

c. Give an automaton that recognizes \(A_{\frac{1}{3}, \frac{2}{3}}\) or prove that it is not regular.

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\(^1\)Give the state-transition diagram of the automaton; don’t give a tuple.