CMU 15-896
Social choice 1: The basics

Teacher: Ariel Procaccia
Social choice theory

• A mathematical theory that deals with aggregation of individual preferences
• Origins in ancient Greece
• Formal foundations: 18th Century (Condorcet and Borda)
• 19th Century: Charles Dodgson
• 20th Century: Nobel prizes to Arrow and Sen
The voting model

- Set of voters $N = \{1, \ldots, n\}$
- Set of alternatives $A$, $|A| = m$
- Each voter has a ranking over the alternatives
- **Preference profile** = collection of all voters’ rankings

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<thead>
<tr>
<th>1</th>
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Voting rules

• Voting rule = function from preference profiles to alternatives that specifies the winner of the election

• Plurality
  o Each voter awards one point to top alternative
  o Alternative with most points wins
  o Used in almost all political elections
More voting rules

• Borda count
  o Each voter awards \( m - k \) points to alternative ranked \( k \)’th
  o Alternative with most points wins
  o Proposed in the 18\(^{th}\) Century by the chevalier de Borda
  o Used for elections to the national assembly of Slovenia
  o Similar to rule used in the Eurovision song contest

Lordi, Eurovision 2006 winners
More voting rules

• Positional scoring rules
  - Defined by vector \( (s_1, \ldots, s_m) \)
  - Plurality = \( (1,0,\ldots,0) \), Borda = \( (m - 1, m - 2, \ldots, 0) \)
• \( x \) beats \( y \) in a pairwise election if the majority of voters prefer \( x \) to \( y \)
• Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between these two alternatives
More voting rules

• Single Transferable vote (STV)
  o $m - 1$ rounds
  o In each round, alternative with least plurality votes is eliminated
  o Alternative left standing is the winner
  o Used in Ireland, Malta, Australia, and New Zealand (and Cambridge, MA)
### STV: Example

<table>
<thead>
<tr>
<th>2 voters</th>
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<tbody>
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Carnegie Mellon University
Social choice axioms

• How do we choose among the different voting rules? Via desirable properties!

• Majority consistency = if a majority of voters rank alternative \( x \) first, then \( x \) should be the winner

Which of the rules we talked about is not majority consistent?
Marquis de Condorcet

• 18\textsuperscript{th} Century French Mathematician, philosopher, political scientist
• One of the leaders of the French revolution
• After the revolution became a fugitive
• His cover was blown and he died mysteriously in prison
Condorcet winner

- Recall: $x$ beats $y$ in a pairwise election if a majority of voters rank $x$ above $y$
- Condorcet winner beats every other alternative in pairwise election
- Condorcet paradox = cycle in majority preferences
Condorcet consistency

- Condorcet consistency = select a Condorcet winner if one exists

Which of the rules we talked about is Condorcet consistent?
Poll: What is the relation between majority consistency and Condorcet consistency?

3. Equivalent
4. Incomparable
More voting rules

• Copeland
  o Alternative’s score is \( \# \text{alternatives it beats} \) in pairwise elections
  o Why does Copeland satisfy the Condorcet criterion?

• Maximin
  o Score of \( x \) is \( \min_y |\{i \in N: x \succ_i y\}| \)
  o Why does Maximin satisfy the Condorcet criterion?
Application: Web Search

- Generalized Condorcet: if there is a partition $X, Y$ of $A$ such that a majority prefers every $x \in X$ to every $y \in Y$, then $X$ is ranked above $Y$
- Assumption: spam website identified by a majority of search engines
- When aggregating results from different search engines, spam websites will be ranked last [Dwork et al., WWW 2001]
Application: Web Search

Google

bing

DuckDuckGo

overall
Metamorphosis

Charles Lutwidge Dodgson

Carroll

15896 Spring 2015: Lecture 1
Dodgson’s Rule

• Distance function between profiles: #swaps between adjacent candidates

• Dodgson score of $x = \text{the min distance from a profile where } x \text{ is a Condorcet winner}$

• Dodgson’s rule: select candidate that minimizes Dodgson score

• The problem of computing the Dodgson score is NP-complete!
Dodgson Unleashed

Voter 1: a, b, c, d, e
Voter 2: b, a, c, d, e
Voter 3: e, b, c, a, d
Voter 4: e, c, d, b, a
Voter 5: b, e, d, a, c
**Awesome example**

- Plurality: \(a\)
- Borda: \(b\)
- Condorcet winner: \(c\)
- STV: \(d\)
- Plurality with runoff: \(e\)

<table>
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<th>33 voters</th>
<th>16 voters</th>
<th>3 voters</th>
<th>8 voters</th>
<th>18 voters</th>
<th>22 voters</th>
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Is social choice practical?

- UK referendum: Choose between plurality and STV as a method for electing MPs
- Academics agreed STV is better...
- ... but STV seen as beneficial to the hated Nick Clegg
- Hard to change political elections!
Computational social choice

• However:
  o in human computation systems...
  o in multiagent systems...
  the designer is free to employ any voting rule!

• Computational social choice focuses on positive results through computational thinking
Example: Robobees

- Robobees need to decide on a joint plan (alternative)
- Many possible plans
- Each robobee (agent) has a numerical evaluation (utility) for each alternative
- Want to maximize sum of utilities = social welfare
- Communication is restricted
Example: Robobees

• Approach 1: communicate utilities
  o May be infeasible
• Approach 2: each agent votes for favorite alternative (plurality)
  o $\log m$ bits per agent
  o May select a bad alternative

\[ n/2 - 1 \text{ agents} \]

\[ n/2 + 1 \text{ agents} \]
Example: Robobees

• Approach 3: each agent votes for an alternative with probability proportional to its utility

• Theorem [Caragiannis & P 2011]: if \( n = \omega(m \log m) \) then this approach gives almost optimal social welfare in expectation
### Example: Pnyx

A powerful & user-friendly preference aggregation tool

<table>
<thead>
<tr>
<th>Unique winner</th>
<th>Most preferred alternative</th>
<th>Approved alternatives</th>
<th>Linear rankins</th>
<th>Rankings with ties</th>
<th>Pairwise comparisons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plurality rule</td>
<td>Approval voting</td>
<td>Borda's rule</td>
<td>Bucket</td>
<td>Borda's rule</td>
<td>Young's generalization of Borda's rule</td>
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<table>
<thead>
<tr>
<th>Lottery</th>
<th>Random dictatorship</th>
<th>Nash's rule</th>
<th>Maximal lotteries</th>
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<table>
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<tr>
<th>Ranking without ties</th>
<th>Plurality scores</th>
<th>Approval voting scores</th>
<th>Kemeny's rule</th>
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