Signal Quality Pricing

Decomposition for Spectrum Scheduling and System Configuration

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Background: Separate Scheduling and Configuration

Scheduling

Which transmissions occur when?
- Partition transmissions into *compatible* groups.
- Assign groups to times,
- Or frequencies.

Configuration

*How* does each transmission (and reception) occur?
- Transmission power,
- Modulation / rate,
- **Antenna steering / selection,**
- Frequency *(sometimes).*
Scheduling Example

(S)TDMA schedule

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Scheduling Example

(S)TDMA schedule

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Background Decomposition System Design Experiments

Definitions Scheduling Example Limitations of Separate Scheduling & Configuration

Scheduling Example

Anderson et al. Signal Quality Pricing
Scheduling Example

(S)TDMA schedule
Scheduling Example

(S)TDMA schedule

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Background   Decomposition   System Design   Experiments
Definitions   Scheduling Example   Limitations of Separate Scheduling & Configuration

Scheduling Example

(S)TDMA schedule

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“Chicken and Egg” Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Constraints:
Link SINR,
Half-duplex,
...
“Chicken and Egg” Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Intended signal:
D→A
“Chicken and Egg” Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Intended signal:
B→C
"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Interference:
B→A
D→C

(If both links were in use)
“Chicken and Egg” Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Signal between transmitters:
Not an issue
(Would matter for CSMA)
“Chicken and Egg” Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Trivial Schedule:
Each link gets one slot (TDMA).
“Chicken and Egg” Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Trivial Schedule:
Each link gets one slot (TDMA).
Link demand:
B→C: 1 slot
D→A: 1 slot

Faster Schedule:
Concurrent links,
Interference
“Chicken and Egg” Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Per-link best
(maximum SNR)
antenna choices:
Boosts interfering
signals, too.
“Chicken and Egg” Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Scheduling-aware antenna selection:
Low gain for interference.
Integration and decomposition

Configuration and scheduling can be expressed as a combined problem — but the state space is huge: $\Theta(n^2 2^m)$ variables

Key Idea

Transform problem into many coupled subproblems.

- Individually simple to solve
- Naturally parallel
- Iterate and update (not too many times)
Decomposition Process (Idealized)

Goal: Optimize complete schedule
Given:
Subject to: Complete constraints (PHY, MAC, Network, user, ...)

Goal: Marginally improving concurrent group
Given: Current schedule
Subject to: Link (set) compatibility (PHY, MAC constraints)

Goal: Best set of active links
Given: Estimated configurations
Subject to: PHY, MAC constraints

Goal: Best configurations
Given: Estimated active link set
Subject to: PHY constraints
Decomposition Process (Idealized)

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Subj. to: PHY, MAC constraints

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Decomposition Process (Idealized)

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Goal: Best set of active links
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Subject to: PHY, MAC constraints

Goal: Best configurations
Given: Estimated active link set
Subject to: PHY constraints
Decomposition Process

... 

Goal: *Marginally* improving concurrent group

Given: Current schedule

Subject to: Link (set) compatibility (PHY, MAC constraints)

Lagrangian dual problem: *Price* PHY & MAC constraints

e.g.

- Signal to Interference and Noise Ratio (SINR) threshold
- Half-duplex requirement
Goal: Marginally improving concurrent group

Given: Current schedule
Subject to: Link (set) compatibility (PHY, MAC constraints)

Lagrangian dual problem: Price PHY & MAC constraints

* e.g.
  - Signal to Interference and Noise Ratio (SINR) threshold
  - Half-duplex requirement
What do Constraint Prices Mean?  
(Lagrangian relaxation in 60 seconds)

Original problem: Minimize objective subject to constraints. 
\[
\min_x \quad f(x) \\
\text{s.t.} \quad g_i(x) \leq c_i
\]

Lagrangian: Minimize (objective + penalty) w/o constraints. 
\[
\min_x \quad f(x) + \lambda_i(g_i(x) - c_i)
\]

Price ($\lambda_i$): For each constraint $i$, marginal cost per unit of violation.

Dual: Find the lowest prices such that the degree of violation $\approx 0$. 

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Look up, this is important!

**Scheduling**
Avoid using link $ij$ or interfering with $ij$.

**Configuration**
Increase gain for $ij$ to attenuate interference.

High SINR$_{ij}$ → Price
Solution of dual problem:

**Link Activation Problem**
- Choose (estimate) link sets.
- Given:
  - Estimated antenna configuration
  - Estimated prices (dual multipliers)

**Antenna Reconfiguration Problem**
- Choose (estimate) antenna configuration.
- Given:
  - Estimated link selection
  - Estimated prices (dual multipliers)
Solution of dual problem:

Link Activation Problem
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  - Estimated prices (dual multipliers)

Combined estimates may not satisfy complicating constraints.
Solution of dual problem:

**Link Activation Problem**
- Choose (estimate) link sets.
- Given:
  - Estimated antenna configuration
  - Estimated prices (dual multipliers)

**Antenna Reconfiguration Problem**
- Choose (estimate) antenna configuration.
- Given:
  - Estimated link selection
  - Estimated prices (dual multipliers)

Example (simplified)

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
</tbody>
</table>

$T = 1$

$SINR = 0$

$SINR = 0$

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Example (simplified)

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ T = 2 \]

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Example (simplified)

<table>
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<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ T = 3 \]

\[ \text{on}, 0, 0.7 \]

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Example (simplified)

\[ T = 4 \]

<table>
<thead>
<tr>
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<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \text{on, 0}^*, 0.7 \]

\[ \text{on, 5, 0.7} \]

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-5</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \text{node value}\]

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Background Decomposition System Design Experiments

Idealized Lagrangian/RPP Example

Example (simplified)

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>8</td>
</tr>
</tbody>
</table>

node value

A 8
B 0
C -5
D 0

off, 8,
off, 5,

T = 5

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>-8</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
</tbody>
</table>

node value

A -8
B 5
C 0
D 0

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Example (simplified)

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>7.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-7.2</td>
</tr>
<tr>
<td>C</td>
<td>4.5</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ T = 6 \]

Off, 7.2, 4.5, 0

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Signal Quality Pricing
### Example (simplified)

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<th>Value</th>
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</thead>
<tbody>
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<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>1.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-1.6</td>
</tr>
<tr>
<td>C</td>
<td>0.9</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ T = 15 \]
### Example (simplified)

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<th>value</th>
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</thead>
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<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>1.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-1.4</td>
</tr>
<tr>
<td>B</td>
<td>0.8</td>
</tr>
<tr>
<td>C</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0.8</td>
</tr>
<tr>
<td>D</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.4</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

\[ T = 16 \]
Example (simplified)

<table>
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<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>-0.7</td>
</tr>
<tr>
<td>D</td>
<td>1.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>value</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>-1.3</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0.7</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
</tbody>
</table>

$T = 17$
Example (simplified)

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>-0.9</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>D</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>node</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>D</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

\[ T = 20 \]
Example (simplified)

node | value  
---|---
B | -0.8  
C | 0  
D | 0.8  

$T = 21$ (done)

node | value  
---|---
A | 0  
B | 0.5  
D | -0.5  

node | value  
---|---
A | -0.8  
C | 0.5  
D | 0  

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Background Decomposition System Design Experiments

Proof-of-Concept System

Dual problem: Node-local price and configuration estimates, distributed consensus algorithm.
- Asynchronous
- Delay- and loss-tolerant
- Eventually consistent

Global “restricted master” problem, flooding updates.
- Passively observes dual problem results.
- Recomputes (global) schedule when possible.
- Local computation, but requires global data.

Implemented on top of 802.11 PHY with STDMA MAC using switched-beam phased array antennas.
Experimental Test Bed

Phase array antennas installed around C.U. campus
Test Load

- TDMA: 2 slots
- Best case: 1 slot
- Incompatible when using "obvious" antennas
- Algorithm achieves best case
Test Load

- TDMA: 2 slots
- Best case: 1 slot
- Incompatible when using "obvious" antennas
- Algorithm achieves best case
Test Load

- TDMA: 2 slots
- Best case: 1 slot
- Incompatible when using “obvious” antennas
- Algorithm achieves best case
Execution Trace at Node C

State Evolution at Node C

(price, Lambda, Gain)
Execution Trace at Node C

State Evolution at Node C

price

activation

gain
Numerical Results

Numerical experiments: 1400 varying scenarios

- Number of nodes (2 - 48)
- Link density (1/2 - 3 per node)
- Size of simulated area (1 - 16 sq. km)
- Random seed
**Improvement**

**Achieved Speedup in Numerical Simulations**

The diagram shows the achieved speedup in numerical simulations. The x-axis represents speedup, while the y-axis represents the fraction of experiments. The graph plots the results of various experiments, indicating the achieved speedup in numerical simulations.
Running Time 1

Iterations to Specified Fraction of Optimality

Fraction of Experiments

Number of Minor Iterations

optimal
95%
90%
80%
Running Time 2

Time to Optimal Solution vs. Problem Size

Number of Nodes vs. Iterations

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Conclusions

- Tractable solution to optimal joint beam steering and scheduling
- Mean 234% speedup over simple TDMA
- Mean 150 iterations to optimality (90th %ile: 500)

- Dual-decomposition based scheduling works in practice
- More responsive on-line MAC in progress
Thank you!
Erdal Arikan.
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Master Problem (JBSS-MP)

Minimize total time

\[
\begin{align*}
\text{min} & \quad \sum_{l \in L_A} x_l \\
\text{s.t.} & \quad \sum_{l \in L_A} S_{ijl} x_l \geq a_{ij} \quad \forall i, j \\
& \quad \sum_{j: (i,j) \in A} S_{ijl} + \sum_{j: (j,i) \in A} S_{jil} \leq 1 \quad \forall i, l \\
& \quad \frac{P_{il} D_{ijl} D_{jil}}{L_{b(i,j)N_r}} S_{ijl} + \gamma_1 (1 + M_{ijl}) (1 - S_{ijl}) \geq 0 \quad \forall i, j, l \\
& \quad \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{L_{b(k,j)N_r}} V_{kl}\right) \quad \forall i, j, l \\
& \quad S_{ijl} \leq V_{il} \quad \forall i, j, l \\
& \quad \sum_{p \in P} B_{jpl} = 1 \quad \forall j, l \\
& \quad D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall i, k, l \\
& \quad x_l \geq 0 \quad \forall l \in L_A \\
& \quad S_{ijl}, B_{jpl} \in \{0, 1\}
\end{align*}
\]

- Allocate sufficient time to each link
- Half-duplex unicast operation
- SINR on active links
- Antenna selection convexity
- Gain-antenna coupling
Master Problem (JBSS-MP)

Minimize total time

\[
\min \sum_{l \in L_A} x_l
\]

s.t.

\[
\sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall i,j
\]

\[
\sum_{j: (i,j) \in A} S_{ijl} + \sum_{j: (j,i) \in A} S_{jil} \leq 1 \quad \forall i,l
\]

\[
\frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j)N_r} S_{ijl} + \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j)N_r} V_{kl} \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \geq \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j)N_r} V_{kl}\right) S_{ijl} \leq V_{il} \quad \forall i,j,l
\]

\[
\sum_{p \in P} B_{jpl} = 1 \quad \forall j,l
\]

\[
D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall i,k,l
\]

\[
x_l \geq 0 \quad \forall l \in L_A
\]

\[
S_{ijl}, B_{jpl} \in \{0, 1\}
\]

Allocate sufficient time to each link

Half-duplex unicast operation

SINR on active links

Antenna selection convexity

Gain-antenna coupling
Master Problem (JBSS-MP)

Minimize total time

\[ \text{min} \quad \sum_{l \in LA} x_l \]

s.t.

\[ \sum_{l \in LA} S_{ijl} x_l \geq q_{ij} \quad \forall i, j \]

\[ \sum_{j: (i,j) \in A} S_{ijl} + \sum_{j: (j,i) \in A} S_{jil} \leq 1 \quad \forall i, l \]

\[ \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j)N_r} S_{ijl} + \gamma_1(1 + M_{ijl})(1 - S_{ijl}) \geq \gamma_1 \left( 1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j)N_r} V_{kl} \right) \quad \forall i, j, l \]

\[ S_{ijl} \leq V_{jl} \quad \forall i, j, l \]

\[ \sum_{p \in P} B_{jpl} = 1 \quad \forall j, l \]

\[ D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall i, k, l \]

\[ x_l \geq 0 \quad \forall l \in LA \]

\[ S_{ijl}, B_{jpl} \in \{0, 1\} \]

- Allocate sufficient time to each link
- Half-duplex unicast operation
- SINR on active links
- Antenna selection convexity
- Gain-antenna coupling

Anderson et al. | Signal Quality Pricing
Master Problem (JBSS-MP)

Minimize total time

\[ \min \sum_{l \in L_A} x_l \]

subject to

\[ \sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall i, j \]

\[ \sum_{j: (i,j) \in A} S_{ijl} + \sum_{j: (j,i) \in A} S_{jil} \leq 1 \quad \forall i, l \]

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\[ \forall i, j, l \]

\[ S_{ijl} \leq V_{il} \quad \forall i, j, l \]

\[ \sum_{p \in P} B_{jpl} = 1 \quad \forall j, l \]

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\[ x_l \geq 0 \quad \forall l \in L_A \]

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Master Problem (JBSS-MP)

Minimize total time

\[
\text{min} \quad \sum_{l \in L_A} x_l
\]

s.t.

\[
\sum_{l \in L_A} s_{ijl} x_l \geq a_{ij} \quad \forall i, j
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\]

\[
\frac{p_{il} d_{ijl} d_{jil}}{L(b(i,j))} s_{ijl} + \gamma_1 (1 + M_{ijl} (1 - s_{ijl})) \geq \gamma_1 \left( 1 + \sum_{k \in N \setminus \{i,j\}} \frac{p_{kl} d_{kjl} d_{jkl}}{L(b(k,j))} v_{kl} \right) \quad \forall i, j, l
\]

\[
s_{ijl} \leq v_{il} \quad \forall i, j, l
\]

\[
\sum_{p \in P} b_{jpl} = 1 \quad \forall j, l
\]

\[
d_{ik} = \sum_{p \in P} g_{ikp} b_{ipl} \quad \forall i, k, l
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Anderson et al. | Signal Quality Pricing
Master Problem (JBSS-MP)

Minimize total time

\[
\begin{align*}
\min & \quad \sum_{l \in L_A} \chi_l \\
\text{s.t.} & \quad \sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall i,j \\
& \quad \sum_{j: (i,j) \in A} S_{ijl} + \sum_{j: (j,i) \in A} S_{jil} \leq 1 \quad \forall i,l \\
& \quad \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j)N_r} S_{ijl} + \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \geq \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j)N_r} V_{kl}\right) \quad \forall i,j,l \\
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& \quad \chi_l \geq 0 \quad \forall l \in L_A \\
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\end{align*}
\]

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Anderson et al. | Signal Quality Pricing
Master Problem (JBSS-MP)

Minimize total time

\[
\text{min} \quad \sum_{l \in L_A} x_l
\]

\[
\text{s.t.} \quad \sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall i,j
\]

\[
\sum_{j:(i,j) \in A} S_{ijl} + \sum_{j:(j,i) \in A} S_{jil} \leq 1 \quad \forall i,l
\]

\[
\frac{P_{il} D_{ijl} D_{jil}}{L_b(i,j) N_r} S_{ijl} + \left(1 + M_{ijl}(1 - S_{ijl}) \right) \gamma_1 \geq \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{L_b(k,j) N_r} V_{kl} \right)
\]

\[
\forall i,j,l
\]

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S_{ijl} \leq V_{il} \quad \forall i,j,l
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\]

\[
x_l \geq 0 \quad \forall l \in L_A
\]

\[
S_{ijl}, B_{jpl} \in \{0, 1\}
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Master Problem (JBSS-MP)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{ij}$</td>
<td>Activation of link $ij$</td>
</tr>
<tr>
<td>$V_i$</td>
<td>Node $i$ is active (in current link set)</td>
</tr>
<tr>
<td>$D_{ij}$</td>
<td>Directivity of node $i$ toward $j$</td>
</tr>
<tr>
<td>$B_{jp}$</td>
<td>Indicator: beam $p$ used at node $j$</td>
</tr>
</tbody>
</table>

Table: Key Notation

\[
\begin{align*}
\min & \quad \sum_{l \in LA} x_l \\
\text{s.t.} & \quad \sum_{l \in LA} S_{ijl} x_l \geq q_{ij} \quad \forall i, j \\
& \quad \sum_{j: (i, j) \in A} S_{ijl} + \sum_{j: (j, i) \in A} S_{jil} \leq 1 \quad \forall i, l \\
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& \quad x_l \geq 0 \quad \forall l \in LA \\
& \quad S_{ijl}, B_{jpl} \in \{0, 1\}
\end{align*}
\]

Anderson et al. | Signal Quality Pricing
Decomposition Approach – Detailed

Master problem
Dantzig-Wolfe decomposition
Lagrangian Relaxation on SINR constraint
Lagrangian Relaxation on duplex constraint
Block separation
Block separation
Quadratic approximation
Dantzig-Wolfe Decomposition

Restricted Master Problem (RMP)
Given feasible link sets, allocates time to each.
Produces capacity constraint dual values ($\beta$).

Subproblem
Given $\bar{\beta}$, finds improving link set.

Subproblem Complexity

<table>
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<tr>
<th>Functional</th>
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<tr>
<td>Objective: Reduced-Cost Column</td>
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Anderson et al.: Signal Quality Pricing
Dantzig-Wolfe Decomposition

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Anderson et al.
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Lagrangian Dual Problem

SINR and duplex constraints relaxed; multipliers are $\lambda$, $\mu$. Constraint functionals are $d^s(\cdot)$ and $d^d(\cdot)$.

$$L'(S, D, V, \lambda, \mu) = \bar{\beta}^T S - \lambda^T d^s(S, D, V) - \mu^T d^d(S)$$

$$\phi'(\lambda, \mu) = \max_{S, D, V} L'(S, D, V, \lambda, \mu)$$

[CLAP-dual-2]

$$\begin{align*}
\min_{\lambda, \mu} & \quad \phi'(\lambda, \mu) \\
\text{s.t.} & \quad S_{ij} \leq V_i \quad \forall i \\
& \quad D_{ik} = \sum_{p \in P} G_{ikp} B_{ip} \quad \forall i, k \\
& \quad \sum_{p \in P} B_{jp} = 1 \quad \forall j
\end{align*}$$
Decomposition Approach – Detailed

- Master problem
- Dantzig-Wolfe decomposition
- Lagrangian Relaxation on SINR constraint
- Lagrangian Relaxation on duplex constraint
- Block separation
- Block separation
- Quadratic approximation
Block Separation

\[
\begin{align*}
\min_x & \quad f(x_1, x_2) \\
\text{s.t.} & \quad g_1(x) \leq c_1 \\
& \quad g_2(x) \leq c_2
\end{align*}
\]

\[\iff\]

\[
\begin{align*}
\min_{x_1} & \quad f_1(x_1) \\
\text{s.t.} & \quad g_1(x_1) \leq c_1 \\
\min_{x_2} & \quad f_2(x_2) \\
\text{s.t.} & \quad g_2(x_2) \leq c_2
\end{align*}
\]
Relaxed Primal Fixed-antenna Link Assignment Problem (RP-FLAP)

\[
\max_{S, V} \quad \beta^T S + \lambda^T d^s(S, V) - \mu^T d^d(S)
\]

s.t.
\[
S_{ij} \leq V_i \quad \forall ij
\]
Fixed-Link Antenna Reconfiguration Problem (FARP)

\[
\begin{align*}
\text{max}_{D,B} & \quad \beta^T \bar{S} - \sum_{ij} \lambda_{ij} \left( \frac{P_i D_{ij} D_{ji}}{Lb(i,j) N_r} \bar{S}_{ij} + \gamma_1 (1 + M_{ij}) (1 - \bar{S}_{ij}) \right) \\
\text{s.t.} & \quad D_{ik} - \sum_{p \in P} G_{ikp} B_{ip} = 0 \quad \forall i,k \\
& \quad \sum_{p \in P} B_{ip} = 1 \quad \forall i
\end{align*}
\]
Let \( x \) denote the vector of all antenna gains \( D \). Now let \( i \) partition \( x \) as:

\[
x_i = \bigcup_{k \neq i} D_{ik}.
\]

\[
g_i(x) = \begin{cases} 
\sum_j \left( \frac{1}{2} \bar{\lambda}_{ij} \bar{S}_{ij} \frac{P_i}{Lb(i, j)N_r} D_{ij} \bar{D}_{ji} \right) + \frac{k}{|N|} & \text{if } i \text{ is a transmitter} \\
\sum_j \left( \frac{1}{2} \bar{\lambda}_{ji} \bar{S}_{ji} \frac{P_j}{Lb(j, i)N_r} \bar{D}_{ji} D_{ij} \right) + \frac{k}{|N|} & \text{if } i \text{ is a receiver}
\end{cases}
\]

\[
h_i(x) = \begin{cases} 
\sum_j \left( \sum_{k, l \in N \setminus \{i, j\}} \left( \frac{1}{2} \gamma_1 \bar{S}_{ij} \bar{\lambda}_{kl} \frac{P_i}{Lb(i, l)N_r} D_{il} \bar{D}_{li} \right) \right) & \text{if } i \text{ is a transmitter} \\
\sum_j \left( \sum_{k, l \in N \setminus \{i, j\}} \left( \frac{1}{2} \gamma_1 \bar{S}_{ji} \bar{\lambda}_{ji} \frac{P_k}{Lb(k, i)N_r} \bar{D}_{ki} D_{ik} \right) \right) & \text{if } i \text{ is a receiver}
\end{cases}
\]

\[
f_i(x) = g_i(x_i) - h_i(x)
\]

\[
f(x) = \sum_i f_i(x) \text{ given } \sum_j \bar{S}_{ij} \leq V_i \quad \forall i
\]
Single-Node Antenna Reconfiguration Problem (SNARP) II

[SNARP$_i$]

\[
\begin{align*}
&\max_{D,B} \quad 1 - f_i(D) \\
\text{s.t.} \quad & D_{ik} - \sum_{p \in P} G_{ikp} B_{ip} = 0 \quad \forall k \\
& \sum_{p \in P} B_{ip} = 1 \\
& B_{ip} \leq 1 \quad \forall p \in P \\
& B_{ip} \geq 0 \quad \forall p \in P
\end{align*}
\]
Mathematical Components

Per-node:
- Link activation problem
- Antenna configuration problem
- Incremental subgradient calculation
- Primal estimate sequence

Inter-node exchange of:
- Primal and dual estimates

Distributed, asynchronous, incremental optimization process
Interference-Limited Wireless Networks

Shannon capacity of a narrowband Gaussian channel is given by:

\[ C = B \log_2 (1 + \frac{P}{N}) \]  \hspace{1cm} (1)

- \(B\) is a fixed resource.
- \(P\) has practical and regulatory limits.
- Your \(P\) may be someone else's \(N\).
Interference-Limited Wireless Networks

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Aggregate capacity of $n$ interacting interference–limited Gaussian channels

Interference power relative to signal power
- $0.05 \times P$
- $0.1 \times P$
- $0.2 \times P$
- $0.4 \times P$
- $0.8 \times P$
Absent some other bottleneck, Signal-to-Interference and Noise Ratio (SINR) limits throughput.

- Concurrent links increase total capacity,
- *If* the links don’t unduly interfere with each other.
- **Identify** or **create** low mutual-interference link sets.

![Graph showing aggregate capacity of n interacting interference-limited Gaussian channels](image)
Absent some other bottleneck, Signal-to-Interference and Noise Ratio (SINR) limits throughput.

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Spatial-Reuse TDMA (STDMA)

Goal: Select sets of links or broadcasts such that spatial separation minimizes interference.

- Old idea: (goes back to [Nelson 85]).
- Which sets?
- How much time for each?
- What configuration?
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Optimal scheduling is NP-Hard.

Responses:
- Relax objective ✓
- Relax constraints ✗
- Tighten constraints ✓
Optimal scheduling is NP-Hard.

Responses:
- Relax objective ✓
- Relax constraints X
- Tighten constraints ✓
Steerable, Switchable and Smart Antennas

**FIG. 2**
(Prior Art)

**FIG. 3**

Complication

If each node has $p$ patterns, each set of $m$ links has $p^{2m}$ configurations.

Hairier than other adaptations:

- Power change affects signal and interference equally.
- Modulation change affects only the link in question.
- Antenna change affects everyone arbitrarily.
Complication

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Hairier than other adaptations:

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- Antenna change affects everyone arbitrarily.
**Goal:** Partition links into concurrently-feasible sets to achieve desired throughput (delay, jitter, BER, etc.)

- **Ignoring RF interference:** Nodes can only participate in one link at a time. → Graph coloring-like algorithms (polynomial), e.g. [Hajek 88].

- **Pair-wise RF interference:** Link pairs are either compatible or not; any combination of links not including a forbidden pair is OK. → Polynomial graph algorithms, e.g. [Chlamtac 87, Ephremides 90, Chen 06, Liu 09].

- **Cumulative RF interference:** Combined interference from all other links must be acceptable for every link. Optimality is NP-hard [Arikan 84]. Greedy algorithms by, e.g. [Grönkvist 00, Brar 06]. Optimization algorithms by e.g. [Björklund 03, Johansson 06]. *

- **Continuous Interference Effect:** Link capacity as a function of SINR, not a threshold, e.g. [Radunović 04].
Antenna Capabilities

- Omnidirectional & Fixed Directional.

- Switched Beam
  - Sectorized antennas or arrays with pre-computed patterns.
  - Control consists of selecting among available patterns.

- Adaptive Array
  - Synthesizes beam patterns using on-line techniques.
  - Generally involves active measurement *e.g.* pilot tones.

- NO wedges, cones, pencil beams, etc.

- ... and the environment would distort them if there were.
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Controllable Antennas in Wireless Networks

- CSMA Protocols (not going to talk about)
  - “Deafness” problem, mixed directional/omni RTS-CTS, directional NAV, etc..
- Cellular (telephone or data)
  - One smart base station with many dumb clients.
  - ≈ No client-client interference.
  - Linear problem size, information & control all at BS.
  - (Some limited inter-cell interference mitigation exists.)
- STDMA
Controllable Antennas in STDMA

- Schedule then configure
  - [Lin 04]
- Configure then schedule
  - [Sánchez 99, Dyberg 02] and others. Special case: [Sundaresan 07]
- Schedule with assumed capabilities
  - Infinitesimal beam width [Cain 03]
  - Geometric rules e.g. significant signal propagates only in a wedge [Deopura 07].
  - Arbitrary $k$ nulls [Sundaresan 06].
- Joint Scheduling and Configuration *
  - Pairwise configuration considered in scheduling [Sundaresan 07], *DIRC* [Liu 09].

* Anderson et al. Signal Quality Pricing
"Bad Neighbor" links

Bad Neighbor SIR at Receiver

Proportion of link pairs

-20 0 20 40

SIR (dB)

Anderson et al. | Signal Quality Pricing
Achieved Speedup in Numerical Simulations

Speedup by Node Density

Node Density (nodes/m²)

- 0.002
- 0.003
- 0.004
- 0.005
- 0.006
- 0.007
- 0.008
- 0.009
- 0.01
- 0.012
- 0.014
- 0.016
- 0.018
- 0.024

Speedup

1 2 3 4 5 6