1. Find the best answer for the following multiple choice questions. (20 points)
   a. Java is
      i. Procedural
      ii. Object oriented
      iii. Functional
      iv. None of the above
   b. A Java interface is
      i. Extended by a class
      ii. Implemented by a class
      iii. Contains methods that are already implemented
      iv. None of the above
   c. The runtime of an algorithm depends on
      i. Data size
      ii. Processor speed
      iii. Language
      iv. RAM
      v. All of the above
      vi. None of the above
   d. Inserting an element to the beginning of an array (that is A[0] element) is more difficult than inserting an element to the beginning of a linked list.
      i. TRUE
      ii. FALSE
   e. Suppose Node is a class that contains two integer fields and a pointer to next Node. Consider the following definitions Node M = new Node(); Node N = M; The number of bytes required to hold M and N are
      i. M = 4 bytes, N = 8 bytes
      ii. M = 8 bytes, N = 4 bytes
      iii. M = 12 bytes, N = 4 bytes
      iv. M = 4 bytes, N = 12 bytes
      v. None of the above
f. Given a collection of algorithms that runs on $O(1)$, $O(n \log n)$, $O(n)$, $O(n^2)$, $O(\log n)$, $O(n!)$, order the algorithms from fastest to slowest
   i. $O(1)$, $O(n \log n)$, $O(n)$, $O(n^2)$, $O(\log n)$, $O(n!)$
   ii. $O(1)$, $O(\log n)$, $O(n)$, $O(n \log n)$, $O(n^2)$, $O(n!)$
   iii. $O(1)$, $O(\log n)$, $O(n \log n)$, $O(n)$, $O(n^2)$, $O(n!)$
   iv. None of the above

g. Suppose that the complexity of an algorithm is $O(n^2)$. Suppose that the program that uses the algorithm run in 10 seconds for a data set of size $n$. If the data size is doubled, how long will it take (approximately) to run the program?
   i. 10 seconds
   ii. 100 seconds
   iii. 6-7 minutes
   iv. None of the above

h. What is the best data structure to solve the following problem? A list needs to be built dynamically. Data must be easy to find, preferably in $O(1)$. The user does not care about any order statistics such as finding max or min or median.
   i. Use an Array
   ii. Use a Singly LL
   iii. Use a Stack
   iv. Use a Queue
   v. None of the above

i. Finding the max element in an unordered stack would require
   i. $O(1)$ operations
   ii. $O(\log n)$ operations
   iii. $O(n)$ operations
   iv. None of the above

j. Suppose that recursive function foo(n) calls itself twice within its body (assume foo(n-1) called twice). Also assume the recursion would end when n=0. How many function calls does foo initiates?
   i. $O(n)$
   ii. $O(n^2)$
   iii. $O(2^n)$
   iv. $O(n!)$
   v. None of the above
2. [10 points] Complexity of Algorithms. For each of the following Algorithms, find the complexity of the algorithm using big O notation. You must justify your answer with 1-2 lines of explanation. Choose from $O(1)$, $O(n \log n)$, $O(n)$, $O(n^2)$, $O(\log n)$, $O(n!)$. Note that you can ignore any constant factors in the expression, That is $O(4n)$ is equivalent to $O(n)$.

a. Determining all duplicates in a singly LL.

$$O(n^2) \quad \text{if not sorted} \quad O(n \log n)$$

b. An algorithm that determines all possible paths between $n$ cities.

$$O(n!)$$

For each of the following, count the number of operations where some_statement is executed based on the loops

c. for (int $i = 0; i < n; i += 2$)
   for (int $j = 1; j < n; j++$)
     {some_statement;}

   $O(n^2)$

   $O(\frac{n}{2})$  

   $O(n)$

   $O(n^2)$

d. for (int $j = 1; j < n; j *= 2$)
   for (int $i = 1; i < n; i++$)
     {some_statement;}

   $O(n \log n)$

   $O(n)$

   $O(n \log n)$
3. [15 points] Assume that LL is a DOUBLY linked list with the head node and at least one other internal node M which is not the last node. Write few lines of code to accomplish the following. You may assume that each node has a `next` pointer and `prev` pointer. You may NOT swap data to accomplish any of the following operations. For each operation, assume the original list as described above. You are encouraged to draw pictures to justify your code. Note that for each operation, you need to manipulate at least two pointers, next and prev.

a. Delete the head node
   ```c
   head = head->next;
   head->prev = NULL;
   ```

b. Insert a node P immediately after M
   ```c
   P->next = M->next;
   M->next = P;
   (P->next)->prev = P;
   P->prev = M;
   ```

c. Swap head and the node M (you may not swap data)
   ```c
   Q = head->next;
   P = m->prev;
   N = M->next;
   ```

   ```c
   P->next = head;
   head->prev = P;
   head->next = N;
   N->prev = head;
   ```

   ```c
   M->next = Q;
   Q->prev = M;
   M->prev = NULL;
   ```

   ```c
   head = M;
   ```
4. [10 points] Complete the following method in the LinkedList class. The method contains is supposed to return true if there is a node in the list that is equal to the given Comparable c. You can assume the Node class has the public fields, data (a Comparable) and next (a pointer to another Node)

```
public boolean contains(Comparable c) {
    Node ptr = head;
    while (ptr != null) {
        if (ptr.data.compareTo(c) != 0) {
            ptr = ptr.next;
        } else {
            return true;
        }
    }
    return false;
}
```

5. [10 points] Suppose you were asked to write a method that will take two sorted stacks A and B (min on top) and create one stack that is sorted (min on top). You are allowed to use only the stack operations such as pop, push, size and top. No other data structure such as arrays are not allowed. You are allowed to use stacks. Note that elements on the stack can be compared using compareTo.

```
public Stack mergeSortedStacks(Stack A, Stack B) {
    Stack C = new Stack();
    while (!A.empty() || !B.empty()) {
        if ((A.top()).compareTo(B.top()) < 0) {
            C.push(A.pop());
        } else {
            C.push(B.pop());
        }
    }
    return C;
}
```
6. [10 pts] Consider the following recursive method.

```java
public int foo(int a, int b) {
    if (a % b == 0)
        return b;
    else
        return foo(b, a % b);
}
```

a. What is the output given by `foo(17, 3)`?

b. What is the output given by `foo(3, 9)`?

c. Explain briefly the purpose of the `foo` function. (example: it is finding the max of x and y)

7. [10 pts] Given a Queue Q, write a method that will find the max element in the queue. You may only use queue operations such as enqueue, dequeue, size etc.. No other data structure can be used other than queues. Queue must remain intact after finding the max. The elements in the queue can be compared using compareTo method.

```java
public Comparable findMax(Queue Q) {
    Comparable max = Q.dequeue();
    Q.enqueue(max);
    for (int i = 1; i < Q.size(); i++) {
        Comparable nxt = Q.dequeue();
        if (nxt.compareTo(max) >= 0) max = nxt;
        Q.enqueue(nxt);
    }
    return max;
}
```
8. [5 pts] Write a method `findLast` that will find the last node of a LL. Return a pointer (reference) to the last node.

```java
public Node findLast(LinkedList myLL) {
    Node ptr = myLL.head;
    while (ptr != null && ptr.next != null) {
        ptr = ptr.next;
    }
    if (ptr != null) return null;
    return ptr;
}
```

9. [10 pts] For each of the following scenarios choose the “best” data structure from the following list or a combination of data structures: an unsorted array, linked list, DLL, circular LL, stack, queue. In each case, justify your answer briefly.

   a. Suppose that a grocery store decided that customers who come first will be served first

   - Queue

   b. A list must be maintained so that any element can be accessed randomly

   - Array

   c. A program needs to remember operations it performed in opposite order

   - Stack

   d. The size of a file is unknown. The entries need to be entered as they come in. Entries must be deleted when they are no longer needed. It is important that structure has flexible memory management

   - LL

   e. A list must be maintained so that elements can be added to the beginning or end in O(1)

   - Circular DLL
Extra Credit: (10 pts) write a method `createArrayOfLL` that takes a file of words, one in each line and creates an array of linked list nodes as follows.

(a) Create an array of 26 linked lists
(b) Open the file and insert each word into the array based on its first letter, e.g.: any word that begins with ‘a’ goes to A[0] list etc..
(c) Return the reference to the array of linked lists

You must consider ALL cases. You must write reasonable comments to describe your algorithm. You may assume public fields for each node, data and next. There WILL NOT be any partial credit for this problem. The question will be graded ALL or NOTHING. [Use extra paper, if necessary to write the answer]. Node class is defined as follows.

Public Node[] createArrayOfLL(String inputfile) {