Reflection & Transmission

OUTLINE:
Light Sources
Reflection
Transmission & Refraction
Shadows
Interreflection

Light is a Function of Many Variables

Light is a function of
• position x,y,z
• direction θ,φ
• wavelength λ
• time t
• polarization
• phase

In computer graphics, we typically ignore the last three by assuming static scenes, unpolarized, incoherent light, and assume that the speed of light is infinite.

But light is still a complicated function of many variables.
How do we measure light, what are the units?
Units of Light

<table>
<thead>
<tr>
<th>quantity</th>
<th>dimension</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid angle</td>
<td>solid angle</td>
<td>[steradian]</td>
</tr>
<tr>
<td>power</td>
<td>energy/time</td>
<td>[watt]=[joule/sec]</td>
</tr>
<tr>
<td>radiance</td>
<td>power/(area*solid angle)</td>
<td>[watt/(m²*steradian)]</td>
</tr>
</tbody>
</table>

In vacuum or as an approximation for air, radiance is constant along a ray.

A picture is an array of incoming radiance values at imaginary projection plane; because of radiance-constancy, these are equal to outgoing radiances at intersection of ray with first surface hit.

In general, light is absorbed and scattered along a ray.

Light Sources

Imagine a point light source that radiates light equally in all directions.

The total output of the source per unit time will have dimensions of power = energy/time, and units of [watts] = [joule/sec]. Think of power as “photons per second”.

If the source is surrounded by a sphere of radius r, its area will be $4\pi r^2$, so the power per unit area incident on the sphere is proportional to $1/r^2$.

In general, the power of light coming from a point light source drops as $1/r^2$ with distance.

The power of light coming from a linear light source drops as $1/r$ with distance.
General Reflection & Transmission

**Reflectance** is the fraction of incident (incoming) light that is reflected, **transmittance** is the fraction of incident light that is transmitted into the material. Opaque materials have zero transmittance.

In many books, reflectance is usually denoted $\rho$, and transmittance $\tau$. We’ve been using $k_d$ and $k_s$ for diffuse and specular reflectance, and $k_t$ and $k_{st}$ for transmittance.

A general reflectance function has the form of a bidirectional reflectance distribution function (BRDF): $\rho(\theta_i, \phi_i, \theta_o, \phi_o)$, where the direction of incoming light is $(\theta_i, \phi_i)$ and the direction of outgoing light is $(\theta_o, \phi_o)$, $\theta$ is the polar angle measured from perpendicular, and $\phi$ is the azimuth.

There is a similar function for bidirectional transmittance, $\tau(\theta_i, \phi_i, \theta_o, \phi_o)$.

Light is absorbed and scattered by some media (e.g. fog).

Phong’s Illumination model is an approximation to general reflectance.

Phong Illumination Model

A point light source with radiance $I_l$, illuminating an opaque surface, reflects light of the following radiance:

If surface is perfectly **diffuse** (Lambertian). It is independent of viewing direction!

$$I = I_l k_d \max \{N \cdot L, 0\}/r^2$$

where $k_d = \text{coefficient of diffuse reflection} \ [1/\text{steradian}]$

$N = \text{unit normal vector}$

$L = \text{unit direction vector to light}$

If surface is perfectly **specular**. It is not independent of viewing direction.

$$I = I_l k_s \max \{NH, 0\}/r^2$$

where $k_s = \text{coefficient of specular reflection} \ [1/\text{steradian}]$

$H = \text{unit “halfway vector”} = (V+L)/\|V+L\|$

$V = \text{unit direction vector to viewer}$

$e = \text{exponent, controlling apparent roughness: small=rough, big=smooth}$

- *There are more realistic reflection models than Phong's.*
- *Don’t confuse Phong Illumination with Phong Shading.*

Notes 15, Computer Graphics 2, 15-463
Index of Refraction

Light does not travel at the same speed through all materials; it travels at speed $c/n$ in a material of index of refraction $n$, where $c$ is the speed of light in a vacuum.

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>INDEX OF REFRACTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>air/vacuum</td>
<td>1</td>
</tr>
<tr>
<td>water</td>
<td>1.33</td>
</tr>
<tr>
<td>glass</td>
<td>about 1.5</td>
</tr>
<tr>
<td>diamond</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Transmission with Refraction

By the principle of least time in physics, light travels from point A to point B by the path that gets it there the fastest. When passing from a material with index of refraction $n_1$ into a material with index of refraction $n_2$, light bends according to Snell’s law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where $\theta_1$ and $\theta_2$ are the angles from perpendicular.

When traveling into a denser material (e.g. air into water), light bends to be more perpendicular, and when traveling into a sparser material (e.g. water into air), light bends to be less perpendicular. If the materials are the same, then the light doesn’t bend.

In the latter case, there is total internal reflection (no light is transmitted, and the coefficient of specular transmission drops to 0), if $\theta_1 > \sin^{-1}(n_2/n_1)$.

The index of refraction varies with wavelength (hence rainbows and prisms).
Shadows

Shadows are caused by the occlusion of light.

Point light sources cast “hard shadows” (step discontinuity in radiance on receiving surface). They are an idealization.

Linear and area light sources cast “soft shadows” with an umbra (fully shadowed) region, and a penumbra (partial shadow) region. The width of the penumbra is proportional to the occluder’s distance from the receiver.

Interreflection

We typically simulate just direct illumination: light traveling on a straight, unoccluded line from light source to surface, reflected there, then traveling in a straight, unoccluded line into eye.

Light travels by a variety of paths:

- light source → eye \hspace{1cm} (0 bounces: looking at light source)
- light source → surface1 → eye \hspace{1cm} (1 bounce: direct illumination)
- light source → surf1 → surf2 → eye \hspace{1cm} (2 bounces)
- light source → surf1 → surf2 → surf3 → eye \hspace{1cm} (3 bounces)
...  

Illumination via a path of 2 or more bounces is called indirect illumination or interreflection. It also happens with transmission.