Announcements

Homework 3 due on Thursday before class.

Submit programming part on blackboard and hand in written part.

Midterm – March 9

Syllabus – until and including Lightness and Retinex

Closed book, closed notes exam in class.

Time: 3:00pm – 4:20pm

Midterm review class next Tuesday (March 7)
(Email me by March 6 specific questions)

If you have read the notes and readings, attended all classes, done assignments well, it should be a walk in the park😊
## Course Schedule

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## Physics-based Methods in Vision

We need to understand the relation between the lighting, surface reflectance and medium and the image of the scene.
Why study the physics (optics) of the world?

Let's see some pictures!

Light and Shadows
Reflections
Refractions
Interreflections
Scattering
More Complex Appearances
For in-depth study of Appearance, take fall Graduate class “Physics-based methods in Vision” (previously “Appearance Modeling”)

Radiometry and Image Formation

• To interpret image intensities, we need to understand Radiometric Concepts and Reflectance Properties.

• Topics to be Covered:

  1) Image Intensities: Overview
  2) Radiometric Concepts:
     Radiant Intensity
     Irradiance
     Radiance
     BRDF
  3) Image Formation using a Lens
  4) Diffuse and Specular Reflectance
Image Intensities

Image intensities = \( f(\) normal, surface reflectance, illumination \) 
Note: Image intensity understanding is an under-constrained problem!

Solid Angle

Solid Angle:
\[ d\omega = \frac{dA'}{R^2} = \frac{dA \cos \theta_i}{R^2} \quad \text{(steradian)} \]

What is the solid angle subtended by a hemisphere?
Radiant Intensity of Source

Radiant Intensity of Source: \( J = \frac{d\Phi}{d\omega} \) (watts / steradian)

Light Flux (power) emitted per unit solid angle

Surface Irradiance

Surface Irradiance: \( E = \frac{d\Phi}{dA} \) (watts / m\(^2\))

Light Flux (power) incident per unit surface area.

Does not depend on where the light is coming from!
Surface Radiance (tricky!)

\[ L = \frac{d^2\Phi}{(dA \cos \theta_r)} \, d\omega \] (watts / m\(^2\) steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- \( L \) depends on direction \( \theta_r \)
- Surface can radiate into whole hemisphere.
- \( L \) depends on reflectance properties of surface.

Radiometric concepts – boring…but, important!

(1) Solid Angle : \( d\omega = \frac{dA'}{R^2} = \frac{dA \cos \theta}{R^2} \) (steradian)

What is the solid angle subtended by a hemisphere?

(2) Radiant Intensity of Source : \( J = \frac{d\Phi}{d\omega} \) (watts / steradian)

Light Flux (power) emitted per unit solid angle

(3) Surface Irradiance : \( E = \frac{d\Phi}{dA} \) (watts / m)

Light Flux (power) incident per unit surface area.
Does not depend on where the light is coming from!

(4) Surface Radiance (tricky) : \( L = \frac{d^2\Phi}{(dA \cos \theta_r)} \, d\omega \) (watts / m\(^2\) steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- \( L \) depends on direction \( \theta_r \)
- Surface can radiate into whole hemisphere.
- \( L \) depends on reflectance properties of surface.
The Fundamental Assumption in Vision

Lighting

No Change in Surface Radiance

Surface Camera

Radiance property

- Radiance is constant as it propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.

\[ d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2 \]

\[ d\omega_1 = dA_1/r^2 \quad d\omega_2 = dA_2/r^2 \]

\[ d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2 \]

\[ \therefore L_1 = L_2 \]
Relationship between Scene and Image Brightness

- Before light hits the image plane:

  \[
  \text{Scene} \xrightarrow{\text{Scene Radiance } L} \text{Lens} \xrightarrow{\text{Image Irradiance } E}
  \]

  Linear Mapping!

- After light hits the image plane:

  \[
  \text{Image Irradiance } E \xrightarrow{\text{Camera Electronics}} \text{Measured Pixel Values, } I
  \]

  Non-linear Mapping!

Can we go from measured pixel value, \( I \), to scene radiance, \( L \)?

Relation between Image Irradiance \( E \) and Scene Radiance \( L \)

- Solid angles of the double cone (orange and green):

  \[
  d\omega_s = d\omega_t \quad \frac{dA_s \cos \alpha}{(f / \cos \alpha)^2} = \frac{dA_t \cos \theta}{(z / \cos \alpha)^2}
  \]

- Solid angle subtended by lens:

  \[
  d\omega_l = \frac{\pi d^2}{4} \frac{\cos \alpha}{(z / \cos \alpha)^2}
  \]

  (1)

  \[
  \frac{dA_s}{dA_t} = \frac{\cos \alpha}{\cos \theta} \left( \frac{z^2}{f} \right)
  \]

  (2)
Relation between Image Irradiance \( E \) and Scene Radiance \( L \)

- Flux received by lens from \( dA_i \) = Flux projected onto image \( dA_i \)

\[
L (dA_i \cos \theta) d\omega_k = E dA_i \quad (3)
\]

- From (1), (2), and (3):

\[
E = L \frac{\pi}{4} \left( \frac{d}{f} \right)^2 \cos \alpha^4
\]

- Image irradiance is proportional to Scene Radiance!
- Small field of view \( \rightarrow \) Effects of 4th power of cosine are small.

Relation between Pixel Values \( I \) and Image Irradiance \( E \)

- The camera response function relates image irradiance at the image plane to the measured pixel intensity values.

\[
g : E \rightarrow I
\]

(Grossberg and Nayar)
Radiometric Calibration - RECAP

• Important preprocessing step for many vision and graphics algorithms such as photometric stereo, invariants, de-weathering, inverse rendering, image based rendering, etc.

\[ g^{-1} : I \rightarrow E \]

• Use a color chart with precisely known reflectances.

• Use more camera exposures to fill up the curve.
• Method assumes constant lighting on all patches and works best when source is far away (example sunlight).
• Unique inverse exists because \( g \) is monotonic and smooth for all cameras.

Surface Appearance

Image intensities = \( f(\text{normal, surface reflectance, illumination}) \)

Surface reflection depends on both the viewing and illumination directions.
BRDF: Bidirectional Reflectance Distribution Function

\[
E_{\text{surface}}(\theta, \phi) \quad \text{Irradiance at Surface in direction } (\theta, \phi)
\]

\[
L_{\text{surface}}(\theta, \phi) \quad \text{Radiance of Surface in direction } (\theta, \phi)
\]

\[
\text{BRDF } f(\theta_{i}, \phi_{i}; \theta_{r}, \phi_{r}) = \frac{L_{\text{surface}}(\theta_{r}, \phi_{r})}{E_{\text{surface}}(\theta_{i}, \phi_{i})}
\]

Important Properties of BRDFs

- Conservation of Energy:

\[
\int_{\text{hemisphere}} f(\theta_{i}, \phi_{i}; \theta_{r}, \phi_{r})d\omega_{i} \leq 1
\]
Important Properties of BRDFs

- **Helmholtz Reciprocity**: (follows from 2nd Law of Thermodynamics)

  BRDF does not change when source and viewing directions are swapped.

  \[ f(\theta_s, \phi_s; \theta_r, \phi_r) = f(\theta_r, \phi_r; \theta_s, \phi_s) \]

- **Rotational Symmetry (Isotropy):**

  BRDF does not change when surface is rotated about the normal.

  Can be written as a function of 3 variables: \( f(\theta_s, \phi_s; \theta_r, \phi_r - \phi_s) \)
From the definition of BRDF:

\[ L_{\text{surface}}(\theta_r, \phi_r) = E_{\text{surface}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \]

Write Surface Irradiance in terms of Source Radiance:

\[ L_{\text{surface}}(\theta_r, \phi_r) = L^{\text{src}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i d\omega_i \]

Integrate over entire hemisphere of possible source directions:

\[ L_{\text{surface}}(\theta_r, \phi_r) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} L^{\text{src}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \sin \theta_i \, d\theta_i d\phi_i \]
### Angles and Solid Angles

- **Angle** \( \theta = \frac{l}{r} \)
  - \( \Rightarrow \) circle has \( 2\pi \) radians
- **Solid angle** \( \Omega = \frac{A}{R^2} \)
  - \( \Rightarrow \) sphere has \( 4\pi \) steradians

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### Differential Solid Angle and Spherical Polar Coordinates

Volume element
\[ dV = r^2 \sin \theta \, d\theta \, d\phi \, dr \]
Reflectance Models

Reflection: An Electromagnetic Phenomenon

Two approaches to derive Reflectance Models:

- Physical Optics (Wave Optics)
- Geometrical Optics (Ray Optics)

Geometrical models are approximations to physical models
But they are easier to use!
Reflectance that Require Wave Optics

Mechanisms of Reflection

- Body Reflection:
  - Diffuse Reflection
  - Matte Appearance
  - Non-Homogeneous Medium
  - Clay, paper, etc

- Surface Reflection:
  - Specular Reflection
  - Glossy Appearance
  - Highlights
  - Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection
Example Surfaces

Body Reflection:
- Diffuse Reflection
  - Matte Appearance
  - Non-Homogeneous Medium
  - Clay, paper, etc

Surface Reflection:
- Specular Reflection
  - Glossy Appearance
  - Highlights
  - Dominant for Metals

Many materials exhibit both Reflections:

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Diffuse Reflection and Lambertian BRDF

- Surface appears equally bright from ALL directions! (independent of \( \vec{v} \))

- Lambertian BRDF is simply a constant:
  \[
  f(\theta, \phi; \theta_i, \phi_i) = \frac{\rho_d}{\pi} \text{ albedo}
  \]

- Surface Radiance:
  \[
  L = \frac{\rho_d}{\pi} I \cos \theta_i = \frac{\rho_d}{\pi} I \vec{n} \cdot \vec{s}
  \]

- Commonly used in Vision and Graphics!
Diffuse Reflection and Lambertian BRDF

Lambert's Cosine Law

White-out: Snow and Overcast Skies

CAN'T perceive the shape of the snow covered terrain!

CAN perceive shape in regions lit by the street lamp!!

WHY?
Diffuse Reflection from Uniform Sky

\[ L_{\text{surface}}(\theta_r, \phi_r) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} L_{\text{src}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \sin \theta_i d\theta_i d\phi_i \]

• Assume Lambertian Surface with Albedo = 1 (no absorption)

\[ f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{1}{\pi} \]

• Assume Sky radiance is constant

\[ L_{\text{src}}(\theta_i, \phi_i) = L^{\text{sky}} \]

• Substituting in above Equation:

\[ L_{\text{surface}}(\theta_r, \phi_r) = L^{\text{sky}} \]

Radiance of any patch is the same as Sky radiance !! (white-out condition)

Specular Reflection and Mirror BRDF

- Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when \( \vec{v} = \vec{r} \)).
- Mirror BRDF is simply a double-delta function:

\[ f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v) \]

- Surface Radiance:

\[ L = I \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v) \]
Combing Specular and Diffuse: Dichromatic Reflection

Observed Image Color = a x Body Color + b x Specular Reflection Color

Klinker-Shafer-Kanade 1988

Does not specify any specific model for Diffuse/specular reflection

Diffuse and Specular Reflection

diffuse  specular  diffuse+specular
Next Class

- Photometric Stereo
- Reading: Horn, Chapter 10.