15-294 Rapid Prototyping Technologies:

The Pascaline

Dave Touretzky
Computer Science Dept.
Carnegie Mellon University
Blaise Pascal

- French mathematician, physicist, writer, philosopher.
- b. 1623 – d. 1662 (age 39)
- Father was a tax collector.
- Known for:
  - Theory of probability; Pascal's triangle
  - Study of fluids, pressure, and vacuum
  - Writings on philosophy and theology
  - First working mechanical calculator: the Pascaline
The Pascaline

History of the Pascaline

- Pascal invented the device in 1642 (at age 19) to help his father with his tax computations.
- Widely viewed as the first mechanical calculator. Could add and (with a trick) subtract.
- Over 40 were built over several decades, in a variety of models.
- 9 survive today in museums or private collections.
Input and Output
Input Close-Up
French Currency (17th century)
# 17th Cent. French Currency: Livres

- **Deniers**: 12 deniers = 1 sol
- **Sols**: 20 sols = 1 livre
- **Nombres Simples**: “simple numbers”
- **Dixaines**: tens
- **Centaines**: hundreds
- **Milles**: thousands
- **Dixaines de Mille**: tens of thousands
- **Centaines de Mille**: hundreds of thousands
Under the Hood

Esto probat Instruenti Emblemum

Blasius Gracal Artetenus

Anuentor

20 May 4052
Early Version, From 1642
Why Do the Read-Out Wheels Have Two Sets of Numbers?
What Makes A Device “Digital”?  

A slide rule is analog: it has a continuous state space, so an infinite number of states.

Digital devices have discrete state spaces, and a physical non-linearity to force clean transitions from one state to another.
Digits Don't Guarantee Discreteness
The Backstop Pawl
Backstop Pawl and Sautoir
Carry Operations

- Carrying is what makes addition difficult.
- How do you get one digit to affect the digit next to it?
- Carry can propagate:
  \[
  0999999 \rightarrow 1000000
  \]
- Mechanically, this is a nightmare.
- Two solutions:
  - One-toothed gear (doesn't chain well; can jam)
  - The sautoir ("jumper") – Pascal's invention
One-Tooth Gear (Purple) For Carry
Trick: Elevate Successive Digits
The Sautoir
Sautoir
Sautoir
How To Add

1. Clear the machine:
   (a) Set all digits to “9”.
   (b) Add 1 to get all zeros: tests the ripple carry.
2. Dial in the digits of the first addend.
3. Dial in the digits of the second addend: this may cause carries to occur.
4. Read the result on the output wheels.
Representing Signed Numbers

- In a 3 digit machine $999 + 1 = 000$.
- So 999 is also -1.
- Which means 998 is also -2.
- We can choose between signed and unsigned representations:
  - Unsigned: 0 to 999 represented as “000” to “999”.
  - Signed: -500 to +499 represented as “500” to “999” followed by “000” to “499”.
Converting Positive to Negative

• Subtract the value from 999, then add 1.
• Example: how do we represent -14?

\[
\begin{align*}
999 \\
- \ 14 \\
----- \\
985 \ + \ 1 \ &= \ 986 \ \text{is} \ -14
\end{align*}
\]
Nines' Complement

- To form the nines' complement of a number, subtract every digit from 9.
- Note: there is never any borrowing or carrying, so this can be computed very quickly.
- Denote the nines' complement as $C(n)$.
- Verify for yourself:
  \[ C(C(n)) = n \]
- Negation: $\text{Neg}(n) = C(n) + 1$
Tens' Complement Subtraction

\[ a - b = a + -b \]
\[ = a + \text{Neg}(b) \]
\[ = a + C(b) + 1 \text{ three operations} \]

But we can simplify this by taking the nines' complement of both sides.
Nines' Complement Subtraction

\[ C(a-b) = 9999 - (a - b) = (9999 - a) + b = C(a) + b \]

two operations
Nines' Complement of a Sum

Proof that \( C(a+b) = C(a) + C(b) + 1 \):

Neg\((a+b)\) = \( C(a+b) + 1 \)

Neg\((a+b)\) = Neg\((a)\) + Neg\((b)\)
= \([C(a) + 1]\) + \([C(b) + 1]\)
= C\((a)\) + C\((b)\) + 2

So \( C(a+b) + 1 = C\((a)\) + C\((b)\) + 2 \)

Therefore \( C(a+b) = C\((a)\) + C\((b)\) + 1 \).
Nines' Complement Subtraction

$C(a-b) = C(a + \text{Neg}(b))$

$= C(a + C(b) + 1)$

$= C((a+1) + C(b))$

$= C(a+1) + C(C(b)) + 1$

$= C(a+1) + b + 1$

$= [C(a) + C(1) + 1] + b + 1$

$= C(a) + 2 + 1 + b + 1$

$= C(a) + b$  **two operations**
Subtraction in the Pascaline

Compute $a - b$ as $C(C(a) + b)$:

1. Clear the machine.
2. Enter $C(a)$ using complement digit marks.
3. Add in $b$ using the regular digit marks.
   - This gives $C(a) + b$
4. Read the result on the complement number readout instead of the regular readout.
   - This gives $C(C(a) + b)$
Pascaline Replicas (Many)
John Napier

- Scottish mathematician (1550-1617)
- Invented “Napier's bones”, used to perform multiplication using only addition.
- Napier's bones were very successful and widely used in Europe until the mid-1960s.
- Napier is also the inventor of logarithms.
$7 \times 46785399 = ?$
Multiplying Multi-Digit Numbers

- Do single digit multiplications, shift, and add:

As with the abacus, humans do most of the work.
Wilhelm Schickard (1592 - 1635)

- Described a “calculating clock” in letters to his friend Johannes Kepler in 1623 and 1624, with sketches included.
- Claims the prototype worked, but it has not survived. Second, professionally-built version was destroyed in a fire before delivery.
- Addition by rotating wheels.
- Subtraction by moving wheels in opposite direction.
- Multiplication via Napier's bones (lookup table).
Schickard's Calculating Clock

- The surviving notes don't describe a fully functional machine.
- Requires additional wheels and springs. Did he add them?
- Used single tooth carry gear, which doesn't work for many-digit carries.
Schickard Replica
Leibniz Step Reckoner

- Successor to the Pascaline. Designed in 1673, completed in 1694.
- Could add and subtract automatically.
- Multiply and divide by shifting the carriage.
History of Computing

To learn more about the history of computing:

- Take Tom Cortina's mini-course: 15-292 History of Computing

- Visit the Computer History Museum at www.computerhistory.org or in person in Mountain View, California.
Building Our Own Pascaline

• Just laser-cut parts plus metal fasteners.
• No sawing. No drilling. No glue. Assemble with a screwdriver and pliers.
Hex Standoffs Prevent Rotation
End-Cap Holds Gear on Shaft
Slot and Tab Box Construction
Goal

• Make a Pascaline kit.
• Open source.
• Distribute via:
  – TechShop?
  – Instructables?
  – [your suggestions here]
Assignment 3: The Pascaline

• Will be released on Wednesday.
• Will be done in groups of 2.
• What you need to do today:
  – Find a partner.
  – Email your pairings to Dave.