Recitation 1 – *Big-O, Θ, and Ω, and Sequences (Worksheet)*

Parallel and Sequential Data Structures and Algorithms, 15-210 (Fall 2014)

August 26th, 2014

1 Welcome to 15-210!

- Lab 0 – *MiniLab*, has been released! It is due next Tuesday, September 2nd. It's something light to get you started, and shouldn't take too long. You'll get practice implementing and using our SEQUENCE library, and do a few Big-O proofs.

- In this class, we will be using SML as our default programming language, which you should be familiar with if you have taken 15-150 previously. For those who haven't (graduate students, for example), or if you need a bit of a refresher, we will be holding a crash course this week (see Piazza for updates).

- Consider functions \( f : \mathbb{N} \to \mathbb{R}^+ \) and \( g : \mathbb{N} \to \mathbb{R}^+ \). We say that \( f \in O(g) \) if and only if there exist constants \( N_0 \in \mathbb{N} \) and \( c \in \mathbb{R}^+ \) such that for all \( n \geq N_0 \), \( f(n) \leq c \cdot g(n) \).

2 Problems

1. Use the definition of big-O to prove that \( n^2 \in O(n^3 - 100) \).

2. Order the following functions by their big-O class from smallest to largest.
   
   (a) \((n!)^2\)
   (b) \(100n + \log n\)
   (c) \(3n^2\)
   (d) \((n^2)!\)
   (e) \(1/n\)
   (f) \(\log_{\left(\frac{100100100}{100100100}\right)} n\)
3. Prove or disprove the following statement: $O$ is an anti-symmetric relation on functions, i.e. for any functions $f$ and $g$, if $f \in O(g)$ and $g \in O(f)$, then $f = g$.

4. Use SML to implement the function \texttt{transpose} : \texttt{'a seq seq $\rightarrow$ 'a seq seq} that swaps the rows and columns of a 2D sequence.

\begin{verbatim}
fun transpose (M : 'a seq seq) : 'a seq seq =
\end{verbatim}

5. Rewrite the above function using pseudocode and sequence notation.

\begin{verbatim}
fun transpose (M : 'a seq seq) : 'a seq seq =
\end{verbatim}
6. **Bonus:** For a sorted sequence of numbers \( S = (x_1, \ldots, x_n) \), find and prove a lower bound on the number of comparisons required to determine if \( S \) contains duplicates, i.e. \( i \neq j \land x_i = x_j \).

7. **Bonus:** Prove that \( 10^n \in \Omega(n^{10}) \).