Lecture 8

Induction Variables and Strength Reduction

I Overview of optimization
II Algorithm to find induction variables

Reference: Muchnick 14.1

Example

FOR i = 0 to 100
    A[i] = 0;

    i = 0
    L2: IF i>=100 GOTO L1
    t1 = 4 * i
    t2 = &A + t1
    *t2 = 0
    i = i+1
    GOTO L2

L1:
Definitions

1. A basic induction variable is a variable X
   • whose only definitions within the loop are assignments of the form $X = X + c$ or $X = X - c$,
     where $c$ is either a constant or a loop-invariant variable.

2. An induction variable is
   • a basic induction variable
   • a variable defined once within the loop,
     whose value is a linear function of some basic induction variable at the time of the definition.
     $$A = c_1 \times B + c_2$$

3. The FAMILY of a basic induction variable B
   • the set of induction variables A such that each time A is assigned in the loop, the value of A is a linear function of B.

Optimizations

1. Strength reduction:
   Let A be an induction variable in family of basic induction variable B
   $(A = c_1 \times B + c_2)$
   • Create new variable: $A'$
   • Initialization in preheader: $A' = c_1 \times B + c_2$
   • Track value of B: add after $B = B + \times$: $A' = A' + x \times c_1$
   • Replace assignment to A: $A = A'$
Optimizations (cont.)

2. Optimizing non-basic induction variables
   - copy propagation
   - dead code elimination

3. Optimizing basic induction variables
   Eliminate basic induction variables used only for
   - calculating other induction variables and loop tests
   Algorithm
   - Select an induction variable $A$ in the family of $B$, preferably with simple constants ($A = c_1 \cdot B + c_2$).
   - Replace a comparison such as
     
     \[
     \text{if } B > X \text{ goto L1}
     \]
   
   by
   
   \[
   \text{if } (A' > c_1 \cdot X + c_2) \text{ goto L1, assuming } c_1 \text{ is positive}
   \]
   - if $B$ is live at any exit from the loop, recompute it from $A'$
     - After the exit, $B = (A' - c_2) / c_1$

II. Basic Induction Variables

- A BASIC induction variable in a loop $L$
  - a variable $X$ whose only definitions within $L$ are assignments of the form $X = X + c$ or $X = X - c$, where $c$ is either a constant or a loop-invariant variable.
  
  - Algorithm: can be detected by scanning $L$
  - Example:
    
    ```
    k = 0;
    for (i = 0; i < n; i++) {
      k = k + 3;
      ... = m
    if (x < y)
      k = k + 4;
    if (a < b)
      m = 2 * k
    k = k - 2
    ... = m
    ```
    
    Each iteration may execute a different number of increments/decrements!!
**Strength Reduction Algorithm**

- **Key idea**
  - For each induction variable $A$, $(A = c_1B + c_2$ at time of definition)
    - variable $A'$ holds expression $c_1B + c_2$ at all times
    - replace definition of $A$ with $A = A'$ only when executed

- **Result**
  - Program is correct
  - Definition of $A$ does not need to refer to $B$

**Finding Induction Variable Families**

- **Let $B$ be a basic induction variable**
  - Find all induction variables $A$ in family of $B$:
    - $A = c_1 \cdot B + c_2$
      (where $B$ refers to the value of $B$ at time of definition)

- **Conditions**
  - If $A$ has a single assignment in the loop $L$, and assignment is one of:
    
    \[
    \begin{align*}
    A &= B \cdot c \quad A &= c \cdot B \\
    A &= B / c \quad \text{(assuming $A$ is real)} \\
    A &= B + c \quad A &= c + B \\
    A &= B - c \\
    A &= c - B
    \end{align*}
    \]

  - OR, ... (next page)
Finding Induction Variable Families (cont)

- Let $D$ be an induction variable in the family of $B$
  \[ D = c_1 * B + c_2 \]

- If $A$ has a single assignment in the loop $L$, and assignment is one of:
  \[
  \begin{align*}
  A &= D * c & A &= c * D \\
  A &= D / c & (\text{assuming } A \text{ is real}) \\
  A &= D + c & A &= c + D \\
  A &= D - c \\
  A &= c - D
  \end{align*}
  \]

- No definition of $D$ outside $L$ reaches the assignment to $A$

- Between the lone point of assignment to $D$ in $L$ and the assignment to $A$, there are no definitions of $B$

Conclusions

- Precise definitions of induction variables
- Systematic identification of induction variables
- Strength reduction
- Clean up:
  - eliminating basic induction variables
  - used in other induction variable calculations
  - replacement of loop tests
  - eliminating other induction variables
  - standard optimizations