www.cs.cmu.edu/~15251
Check this Website OFTEN!

Course Staff
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Grading
Homework 40%
Final 25%
Participation 5%
3 In-Recitation Tests 25%
In-Class Quizzes 5%

Weekly Homework
Homework will go out every Tuesday and is due the Tuesday after
Ten points per day late penalty
No homework will be accepted more than three days late
Assignment 1: The Great 251 Hunt!
You will work in randomly chosen groups of 4
The actual Puzzle Hunt will start at 8pm tonight
You will need at least one digital camera per group
Can buy a digital camera for $8 nowadays!

Collaboration + Cheating
You may NOT share written work
You may NOT use Google, or solutions to previous years’ homework
You MUST sign the class honor code

Textbook
There is NO textbook for this class
We have class notes in wiki format
You too can edit the wiki!!!

Feel free to ask questions
Pancakes With A Problem!
Lecture 1 (August 28, 2007)

The chefs at our place are sloppy: when they prepare pancakes, they come out all different sizes.

When the waiter delivers them to a customer, he rearranges them (so that smallest is on top, and so on, down to the largest at the bottom).

He does this by grabbing several from the top and flipping them over, repeating this (varying the number he flips) as many times as necessary.

Developing A Notation:
Turning pancakes into numbers

5
2
3
4
1

How do we sort this stack?
How many flips do we need?

5
2
3
4
1
4 Flips Are Sufficient

Best Way to Sort

X = Smallest number of flips required to sort:

Upper Bound

Lower Bound

Four Flips Are Necessary

If we could do it in three flips:

Flip 1 has to put 5 on bottom
Flip 2 must bring 4 to top (if it didn’t, we would spend more than 3)

4 ≤ X ≤ 4

Lower Bound

Upper Bound

X = 4
$P_5 = \text{Number of flips required to sort the worst case stack of 5 pancakes}$

$P_5 = \text{MAX over } s \in \text{stacks of 5 of MIN # of flips to sort } s$

$P_n = \text{MAX over } s \in \text{stacks of } n \text{ pancakes of MIN # of flips to sort } s$

$P_n = \text{The number of flips required to sort the worst-case stack of } n \text{ pancakes}$
What is $P_n$ for small $n$?

Can you do $n = 0, 1, 2, 3$?

Initial Values of $P_n$

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_n$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

$P_3 = 3$

1
3
2

requires 3 Flips, hence $P_3 \geq 3$

ANY stack of 3 can be done by getting the big one to the bottom ($\leq 2$ flips), and then using $\leq 1$ flips to handle the top two

$n^{th}$ Pancake Number

$P_n = \text{Number of flips required to sort the worst case stack of } n \text{ pancakes}$

Lower Bound

$? \leq P_n \leq ?$

Upper Bound
Bracketing:
What are the best lower and upper bounds that I can prove?

\[ \leq f(x) \leq \]

Try to find upper and lower bounds on \( P_n \), for \( n > 3 \)

Bring-to-top Method

Bring biggest to top
Place it on bottom
Bring next largest to top
Place second from bottom
And so on…

Upper Bound On \( P_n \):
Bring-to-top Method For \( n \) Pancakes

If \( n=1 \), no work required — we are done!
Otherwise, flip pancake \( n \) to top and then flip it to position \( n \)

Now use:

Bring To Top Method
For \( n-1 \) Pancakes

Total Cost: at most \( 2(n-1) = 2n - 2 \) flips
Better Upper Bound On $P_n$:
Bring-to-top Method For $n$ Pancakes

If $n=2$, at most one flip and we are done!
Otherwise, flip pancake $n$ to top and then flip it to position $n$

Now use: **Bring To Top Method For $n-1$ Pancakes**

Total Cost: at most $2(n-2) + 1 = 2n - 3$ flips

$? \leq P_n \leq 2n - 3$

Bring-to-top not always optimal for a particular stack

Bring-to-top takes 5 flips, but we can do in 4 flips

$? \leq P_n \leq 2n - 3$

What other bounds can you prove on $P_n$?
Breaking Apart Argument

Suppose a stack $S$ has a pair of adjacent pancakes that will not be adjacent in the sorted stack.

Any sequence of flips that sorts stack $S$ must have one flip that inserts the spatula between that pair and breaks them apart.

Furthermore, this is true of the “pair” formed by the bottom pancake of $S$ and the plate.

Suppose $n$ is even

$S$ contains $n$ pairs that will need to be broken apart during any sequence that sorts it.

$n \leq P_n$

Suppose $n$ is odd

$S$ contains $n$ pairs that will need to be broken apart during any sequence that sorts it.

$n \leq P_n \leq 2n - 3$ for $n > 3$

Bring-to-top is within a factor of 2 of optimal!
From ANY stack to sorted stack in $\leq P_n$
From sorted stack to ANY stack in $\leq P_n$?

Reverse the sequences we use to sort
Hence, from ANY stack to ANY stack in $\leq 2P_n$

Can you find a faster way than $2P_n$ flips to go from ANY to ANY?

ANY Stack S to ANY stack T in $\leq P_n$

S: 4,3,5,1,2
1,2,3,4,5

“new T”

T: 5,2,4,3,1
3,5,1,2,4

Rename the pancakes in S to be 1,2,3,...,n
Rewrite T using the new naming scheme that you used for S
The sequence of flips that brings the sorted stack to the “new T” will bring S to T

The Known Pancake Numbers

\[
\begin{array}{cc}
n & P_n \\
1 & 0 \\
2 & 1 \\
3 & 3 \\
4 & 4 \\
5 & 5 \\
6 & 7 \\
7 & 8 \\
8 & 9 \\
9 & 10 \\
10 & 11 \\
11 & 13 \\
12 & 14 \\
13 & 15 \\
\end{array}
\]
$P_{14}$ is Unknown

1·2·3·4···13·14 = 14! orderings of 14 pancakes

14! = 87,178,291,200

Is This Really Computer Science?

Sorting By Prefix Reversal


$(17/16)n \leq P_n \leq (5n+5)/3$

William Gates and Christos Papadimitriou.
Bounds For Sorting By Prefix Reversal.
How many different stacks of $n$ pancakes are there?

$$n! = 1 \times 2 \times 3 \times \ldots \times n$$

Pancake Network: Definition For $n!$ Nodes

For each node, assign it the name of one of the $n!$ stacks of $n$ pancakes.

Put a wire between two nodes if they are one flip apart.

Network For $n = 3$
Network For n=4

Pancake Network: Message Routing Delay

What is the maximum distance between two nodes in the pancake network?

P_n

Pancake Network: Reliability

If up to n-2 nodes get hit by lightning, the network remains connected, even though each node is connected to only n-1 others.

The Pancake Network is optimally reliable for its number of edges and nodes.

Mutation Distance

Head Cabbage
(Brassica oleracea capitata)

Turnip
(Brassica rapa)
One “Simple” Problem

A host of problems and applications at the frontiers of science

High Level Point

Computer Science is not merely about computers and programming, it is about mathematically modeling our world, and about finding better and better ways to solve problems.

Today’s lecture is a microcosm of this exercise

Definitions of:
- $n^{th}$ pancake number
- lower bound
- upper bound

Proof of:
- ANY to ANY in $\leq P_n$

Important Technique:
- Bracketing

Here’s What You Need to Know…