Give 0-knowledge protocols for the following problems:

1. **Graph 3-Colorability.**
   
   **INSTANCE:** Graph $G = (V, E)$.

   **QUESTION:** Is $G$ 3-colorable? *I.e.* does there exist an assignment $c : V \rightarrow \{\text{red, white, blue}\}$ such that for every $\{u, v\} \in E$, $c(u) \neq c(v)$ (that is, no two adjacent vertices have the same color)?

   *(If you Google for the answer, you’ll find it. Feel free to check your answer with the scribe notes of Trevisan and Wagner’s class.)*

2. **Graph Edge-Colorability.**
   
   **INSTANCE:** Graph $G$ such that each node has degree $\leq d$.

   **QUESTION:** Can the edges of $G$ be colored using at most $d$ colors, such that all edges incident to a particular vertex are assigned different colors?

   *(Easy, but I doubt it has been done directly, so Google won’t give this.)*

   **Aside:** It is a theorem of Graph Theory that a graph with nodes of degree at most $d$ can always be edge-colored using at most $d + 1$ colors. At least $d$ colors are necessary. Why?

3. **Graph Vertex-Cover.**
   
   **INSTANCE:** Graph $G = (V, E)$, and positive integer $B$.

   **QUESTION:** Does $G$ have a vertex cover of size at most $B$? That is, is there a subset $S$ of $V$ of at most $B$ vertices, such that all edges are incident to at least one vertex in $S$?

**Extra Credit.** Give a 0-knowledge protocol for proving that a positive integer of length $2n$ is the product of two $n$-bit numbers.