Flawed Proof 5: All Objects are the Same Color

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Claim 1 Any set of \( n \) objects are all the same color.

Proof: We proceed by induction on the number of elements \( n \). For \( n = 1 \), this is trivially true (Any object is the same color as itself).

We assume this to be true for \( n = k \), and prove that it is true for \( n = k + 1 \). Consider any set of \( k + 1 \) elements \( S \), and remove an arbitrary element \( c \in S \) from the set. We have \( S - \{c\} \) is a set of \( k \) elements, and so by our inductive assumption, all elements in \( S - \{c\} \) are the same color. Now fix any arbitrary element \( d \in (S - \{c\}) \), and consider the set \((S - \{c\}) - \{d\}) \cup \{c\} = S - \{d\} \). This is again a set of \( k \) elements, and so by our inductive assumption, all elements in \( S - \{d\} \) have the same color. In particular, for any \( e \in S - \{d\} \), we have that \( c \) and \( e \) have the same color. But we also have \( e \in S - \{c\} \), and so \( e \) and \( d \) (and therefore \( c \)) have the same color. Therefore, each of the \( k + 1 \) elements in \( S \) have the same color. This completes the proof. \( \blacksquare \)